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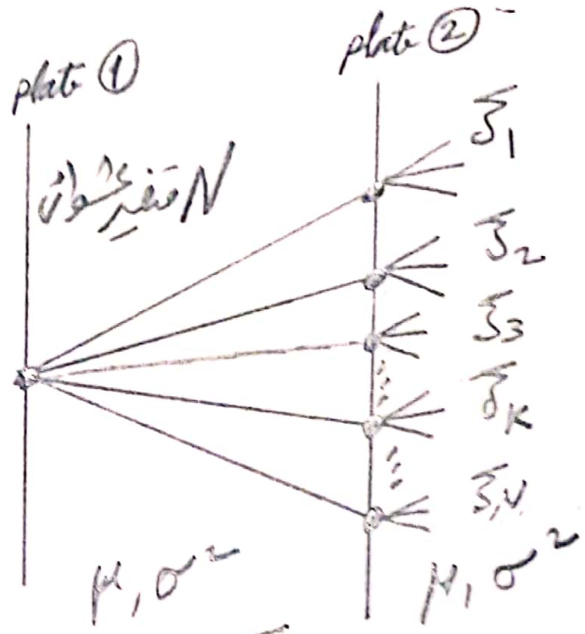
lecture (11)

Random sums

pb 2.3.3 p.64 Text book



$E(\xi_K) = \mu$, $Var(\xi_K) = \sigma^2$

$E(N) = \nu$, $Var(N) = \tau^2$

$Z = \xi_1 + \xi_2 + \dots + \xi_K + \dots + \xi_N$

is the random sum

$E(Z) = \mu(\mu) = \mu^2$

$Var(Z) = \mu\sigma^2 + \mu^2\sigma^2$

$\therefore \boxed{Var(Z) = \mu\sigma^2(1+\mu)}$ #

$E(X) = \mu\nu$

$Var(X) = \nu\sigma^2 + \mu^2\tau^2$

where $X = \xi_1 + \xi_2 + \dots + \xi_N$

• **Distribution of random sum** p.61 Textbook

Let $\xi_1, \xi_2, \dots$ are continuous r.v.s having a probability density function $f(z)$, then the prob. density fn for the fixed sum $\xi_1 + \xi_2 + \dots + \xi_n$ is the n-fold convolution $f^n(z)$ of the density $f(z)$ which is given by

$$\begin{cases} f^1(z) = f(z), & n = 1 \\ f^n(z) = \int f^{n-1}(z-u)\, f(u)\, du, & n > 1 \end{cases}$$

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Defn

let $X = \xi_1 + \xi_2 + \cdots + \xi_n$ given that $N = n \geq 1$

then X has the prob. density $f\underline{n}$

$$f_X(z) = \sum_{n=1}^{\infty} f^{n}(z)\, P_N(n)$$

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EX p.62 A geometric sum of Exponential random Variables

let $f(z) = \begin{cases} \lambda e^{-\lambda z} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$

and $P_N(n) = \beta(1-\beta)^{n-1}$ for $n = 1, 2, \ldots$

Find the prob. density $f\underline{n}$ for $X = \xi_1 + \xi_2 + \cdots + \xi_N$

Ans $$f_X(z) = \sum_{n=1}^{\infty} f^{n}(z)\, P_N(n)$$

Note that
$Z \sim \exp(\lambda)$
$N \sim \text{geom}(\beta)$

$\because$ the n-fold convolution of $f(z)$ is the gamma density $f\underline{n}$, $n \geq 1$

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$$\therefore f^{n}(z) = \begin{cases} \dfrac{\lambda^{n}}{\Gamma(n)} z^{n-1} e^{-\lambda z} & , z \geq 0 \\ 0 & , z < 0 \end{cases}$$

$$\therefore f_{X}(z) = \sum_{n=1}^{\infty} \frac{\lambda^{n}}{(n-1)!} z^{n-1} e^{-\lambda z} \cdot \beta(1-\beta)^{n-1}$$

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$n = 1, 2, 3, \ldots$

$$\therefore f_{X}(z) = \lambda \beta e^{-\lambda z} \sum_{n=1}^{\infty} \frac{[\lambda(1-\beta) z]^{n-1}}{(n-1)!} \rightarrow \sum_{m=0}^{\infty} \frac{[\lambda(1-\beta)z]^{m}}{m!}$$

$$\therefore f_{X}(z) = \lambda \beta e^{-\lambda z} \cdot e^{\lambda(1-\beta) z}$$

$$= \lambda \beta e^{\cancel{-\lambda z} + \cancel{\lambda z} - \lambda \beta z}$$

تذكر

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$$

$$\therefore f_{X}(z) = \lambda \beta e^{-\lambda \beta z} \quad , z \geq 0$$

$\therefore X \sim \exp(\lambda \beta)$

i.e X has an exponential dist$\underline{n}$ with parameter $\lambda \beta$.

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