Phys 570

Lecture #4

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ELECTRICAL CONDUCTIVITY AND OHM'S LAW

The momentum of a free electron is related to the wavevector by $m\mathbf{v} = \hbar\mathbf{k}$. In an electric field E and magnetic field B the force F on an electron of charge –*e can be written as:*

$$F = m\frac{dv}{dt} = \hbar\frac{dk}{dt} = -e\left[E + \frac{1}{c}v \times B\right] \qquad (39)$$

This equation is the Newton's second law of motion for the electron of charge –e and mass m_e in both of E and B.
We want to find the Electrical Conductivity (From Ohm's Law).
Hence, we set B = 0 (no magnetic Field):

$$\hbar\frac{dk}{dt} = -e\left[E\right] \Rightarrow dk = -eEdt/\hbar$$

by intigrating both sides:

$$k(t) - k(0) = -eEt/\hbar \qquad (40)$$

If the force **F = -e*E*** *is applied at time t = 0 to an electron gas that fills* the Fermi sphere centered at the origin of k space, then at a later time *t the* sphere will be displaced to a new center at:

$$\delta k = -eEt / \hbar \qquad (41)$$

Notice that the Fermi sphere is displaced as a whole because every electron is displaced by the same δk.

Because of collisions of electrons, the displaced sphere may be maintained in a steady state in an electric field. If the collision time is τ, the displacement of the sphere is given by (41) with *t* = τ. The velocity is: **v** =**P**/m = *ħ**k*** /m = -eE τ /m.

If ***E*** = constant; there are n electrons of charge -e per unit volume, the electric current density is:

$$j = nqv = n(-e)v = n(-e)(-eE\tau / m) = ne^2\tau E / m \qquad (42)$$

ELECTRICAL CONDUCTIVITY AND OHM'S LAW

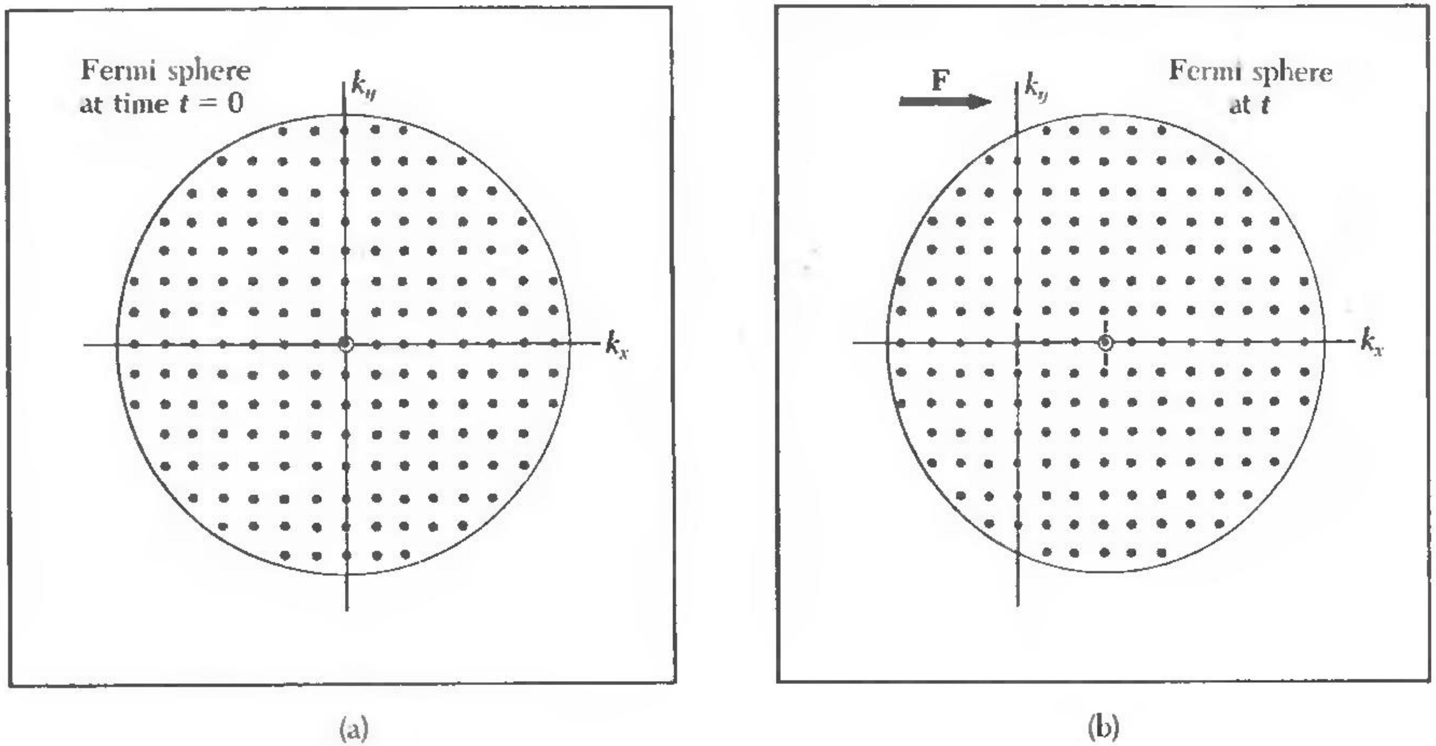


Figure 10 (a) The Fermi sphere encloses the occupied electron orbitals in **k** space in the ground state of the electron gas. The net momentum is zero, because for every orbital **k** there is an occupied orbital at $-\mathbf{k}$. (b) Under the influence of a constant force **F** acting for a time interval t every orbital has its **k** vector increased by $\delta\mathbf{k} = \mathbf{F}t/\hbar$. This is equivalent to a displacement of the whole Fermi sphere by $\delta\mathbf{k}$. The total momentum is $N\hbar\delta\mathbf{k}$, if there are N electrons present. The application of the force increases the energy of the system by $N(\hbar\delta\mathbf{k})^2/2m$.

Equation (42) is called Ohm's Law.
We can find the electrical conductivity σ defined by $\boldsymbol{j} = \sigma \boldsymbol{E}$, so by (42):

$$\sigma = \frac{ne^2\tau}{m} \qquad (43)$$

The electrical resistivity ρ is defined as the reciprocal of the conductivity, so that: (see table 3)

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad (44)$$

It is easy to understand the result (43). Charge transported is proportional to the density *ne;* e/m is because the acceleration is proportional to **e** and inversely proportional to the mass **m**.

ELECTRICAL CONDUCTIVITY AND OHM'S LAW

Conductivity in units of 10^5 (ohm-cm)$^{-1}$.
Resistivity in units of 10^{-6} ohm-cm.

Li 1.07 9.32	Be 3.08 3.25			
Na 2.11 4.75	Mg 2.33 4.30			
K 1.39 7.19	Ca 2.78 3.6	S 0 4		
Rb 0.80 12.5	Sr 0.47 21.5	Y 0 5		
Cs 0.50 20.0	Ba 0.26 39.	L 0 7		
Fr	Ra	A		

 50 9.9	Cr 0.78 12.9	Mn 0.072 139.	Fe 1.02 9.8	Co 1.72 5.8	Ni 1.43 7.0	Cu 5.88 1.70
b 69 4.5	Mo 1.89 5.3	Tc ~0.7 ~14.	Ru 1.35 7.4	Rh 2.08 4.8	Pd 0.95 10.5	Ag 6.21 1.61
a 76 3.1	W 1.89 5.3	Re 0.54 18.6	Os 1.10 9.1	Ir 1.96 5.1	Pt 0.96 10.4	Au 4.55 2.20

O	F	Ne
S	Cl	Ar
Se	Br	Kr
Te	I	Xe
Po 0.22 46.	At	Rn
Tm 0.16 62.	Yb 0.38 26.4	Lu 0.19 53.
Md	No	Lr

- According the Table 3, conductivity of cupper could be:

5.88×10^5 /Ω.cm at room temp.

- This value could be as high as **10^5** times larger at low temperatures.
- Hence: cupper crystals become more pure when cooled and vise versa. This applies to all crystals.
- This leads to large increase in relaxation time t that can reach values: **2×10^{-9} s** at very low temp.
- We have a quantity that depends on τ which is ℓ (mean free path) which represents the mean distance between every tow collisions.
- ℓ is expressed as: $\ell = v_F \tau$

if the electric field were switched off; the momentum distribution would relax back to its ground state with the net relaxation rate:

$$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i} \tag{45}$$

where τ_L and τ_i are the collision times for scattering by phonons and by imperfections, respectively.

Total resistance from phonons and impurities is:

$$\rho = \rho_L + \rho_i \tag{46}$$

First term is independent of impurities (when their concentration is small) .
and 2nd term is independent of temperature.

Resistance of Potassium

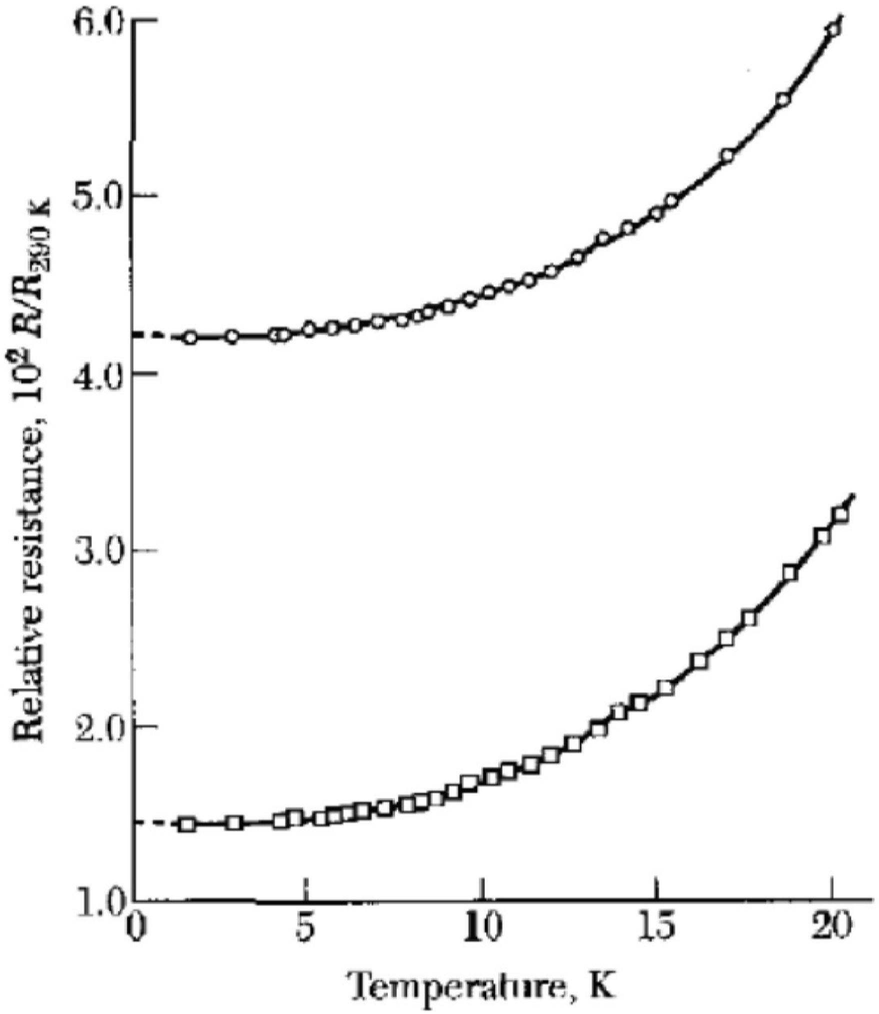


Figure 12 Resistance of potassium below 20 K, as measured on two specimens by D. MacDonald and K. Mendelssohn. The different intercepts at 0 K are attributed to different concentrations of impurities and static imperfections in the two specimens.

The relation $\ell = \mathbf{v}_F\tau$ shows clearly that Fermi velocity v_F is the same as the velocity of electrons in the conductor because all collisions involve only electrons near the Fermi surface.

From Table 1 we have $v_F = 1.57 \times 10^8$ cm S^{-1} for Cu, thus the mean free path is $\ell(4\ K) = 0.3$ *cm. Mean free paths as long as* 10 cm have been observed in very pure metals in the liquid helium temperature range.

Since v_F is very high as we showed previously; and because ℓ is large (as large as 10 cm) then we expect that τ is very small. Usually this time is opposite to Fermi velocity.

MOTION of electrons IN MAGNETIC FIELDS

When electrons move under both of B and E:

$$F = -e\left(\vec{E} + \frac{1}{c}\vec{\mathrm{v}} \times \vec{\mathrm{B}}\right) \tag{49}$$

1st term: eE is coulumbic force, 2nd term is Lorentz formce.

if mv=$\hbar\delta$k then we have: $$\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v = -e\left(\vec{E} + \frac{1}{c}\vec{\mathrm{v}} \times \vec{\mathrm{B}}\right) \tag{50}$$

if B lie along the z axis. Then the component equations of motion are:

$$\left.\begin{aligned}
\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_x &= -e\left(E_x + \frac{\mathrm{B}}{c}\mathrm{v_y}\right) \\
\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_y &= -e\left(E_y + \frac{\mathrm{B}}{c}\mathrm{v}_x\right) \\
\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_z &= -eE_z
\end{aligned}\right\} \tag{51}$$

In the steady state in a static electric field the time derivatives are zero, so that the drift velocity is:

$$\left.\begin{aligned} v_x &= \frac{-e\tau E_x}{m} - \omega_c \tau v_y \\ v_y &= \frac{-e\tau E_y}{m} + \omega_c \tau v_x \\ v_z &= \frac{-e\tau E_z}{m} \end{aligned}\right\} \qquad (52)$$

where $\omega_c = \dfrac{eB}{mc}$ is the cyclotron frequency

This means that when electron moves in the existence of magnetic field B, it will rotate with this frequency. We notice the linear dependence on B, when B increases w will increase.

Since we are talking about motion of electrons under the effect of B, let us imagine that these electrons move inside a conductor. The Hall field is the electric field developed across two faces of a conductor, in the direction jxB, when a current j flows across a magnetic field B.

** Let us consider a conductor in a form of *rectangular parallelepiped* with current flowing in x-direction. Hence we have: E_x.
We also have a B perpendicular on this conductor.
Current cannot move in y direction ➔ $v_y = 0$.
Hence, 2nd equation in (52) = 0.
Accordingly; we have:

$$0 = -\frac{e\tau}{m}E_y + \omega_c \tau v_x = -\frac{e\tau}{m}E_y + \frac{eB\tau}{mc}v_x$$

$$\Rightarrow E_y = \frac{B}{c}v_x = \frac{B}{c}\left[-\frac{e\tau}{m}E_x\right] \Rightarrow E_y = -\frac{eB\tau}{mc}E_x \qquad (53)$$

Hall coefficient is defined as:

$$R_H = \frac{E_y}{j_x B} \qquad (54)$$

using: $j_x = \frac{ne^2\tau E_x}{m}$ we can get:

$$R_H = -\frac{eB\tau E_x / mc}{ne^2\tau E_x / m} = -\frac{1}{nec} \qquad (55)$$

Hence; R_H is negative for free electron.

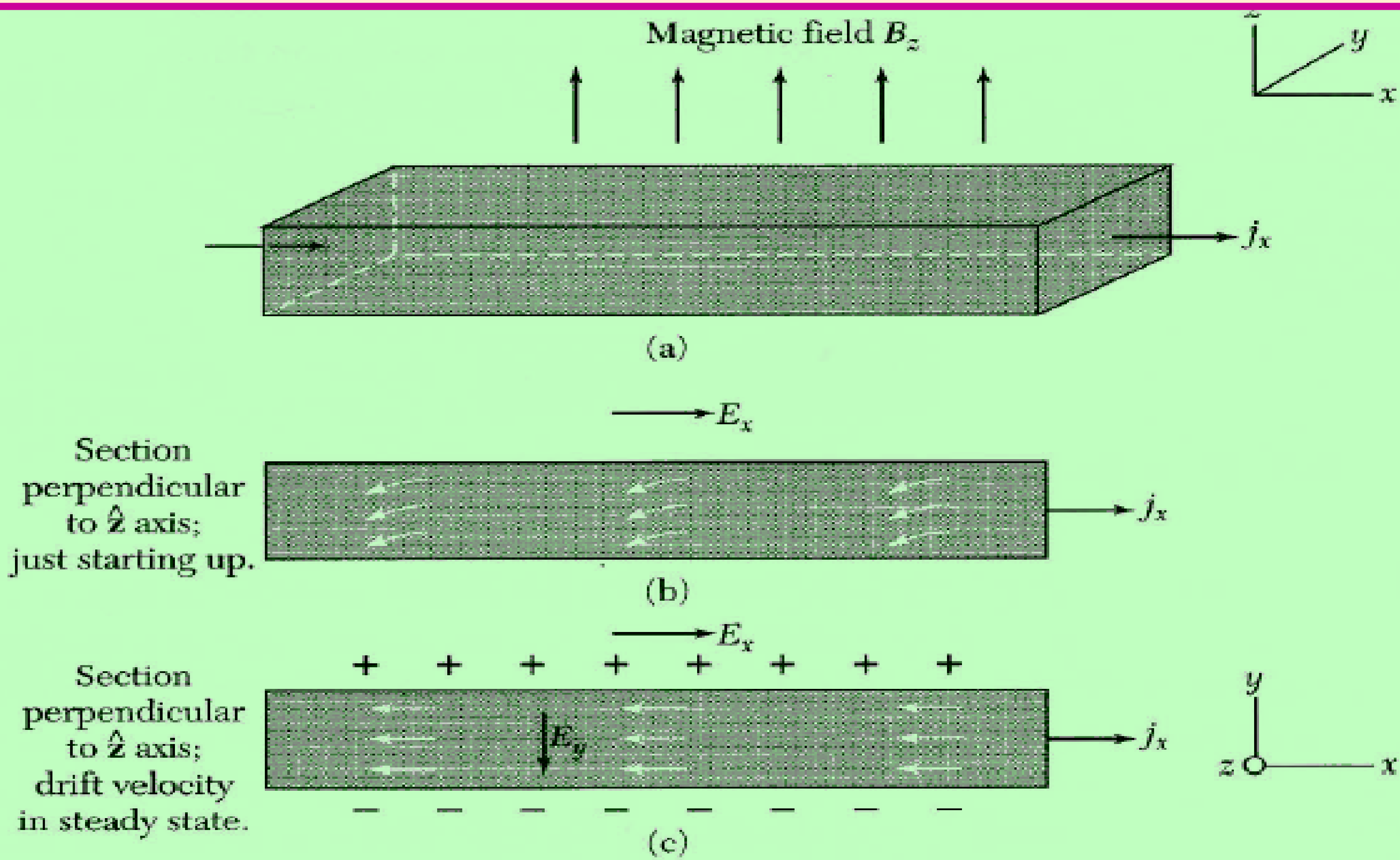


Figure 14 The standard geometry for the Hall effect: a rod-shaped specimen of rectangular cross-section is placed in a magnetic field B_z, as in (a). An electric field E_x applied across the end electrodes causes an electric current density j_x to flow down the rod. The drift velocity of the negatively-charged electrons immediately after the electric field is applied as shown in (b). The deflection in the $-y$ direction is caused by the magnetic field. Electrons accumulate on one face of the rod and a positive ion excess is established on the opposite face until, as in (c), the transverse electric field (Hall field) just cancels the Lorentz force due to the magnetic field.

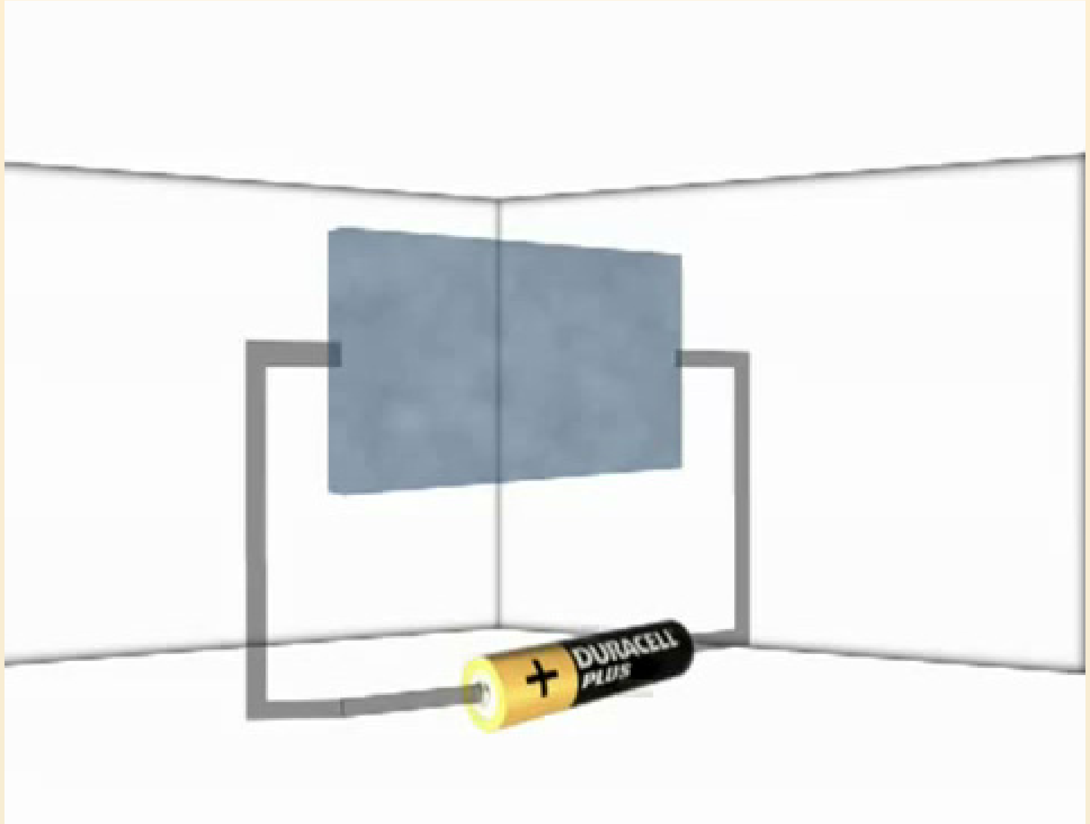
Table 4 Comparison of observed Hall coefficients with free electron theory

[The experimental values of R_H as obtained by conventional methods are summarized from data at room temperature presented in the Landolt-Bornstein tables. The values obtained by the helicon wave method at 4 K are by J. M. Goodman. The values of the carrier concentration n are from Table 1.4 except for Na, K, Al, In, where Goodman's values are used. To convert the value of R_H in CGS units to the value in volt-cm/amp-gauss, multiply by 9×10^{11}; to convert R_H in CGS to m^3/coulomb, multiply by 9×10^{13}.]

Metal	Method	Experimental R_H, in 10^{-24} CGS units	Assumed carriers per atom	Calculated $-1/nec$, in 10^{-24} CGS units
Li	conv.	−1.89	1 electron	−1.48
Na	helicon	−2.619	1 electron	−2.603
	conv.	−2.3		
K	helicon	−4.946	1 electron	−4.944
	conv.	−4.7		
Rb	conv.	−5.6	1 electron	−6.04
Cu	conv.	−0.6	1 electron	−0.82
Ag	conv.	−1.0	1 electron	−1.19
Au	conv.	−0.8	1 electron	−1.18
Be	conv.	+2.7	—	—
Mg	conv.	−0.92	—	—
Al	helicon	+1.136	1 hole	+1.135
In	helicon	+1.774	1 hole	+1.780
As	conv.	+50.	—	—
Sb	conv.	−22.	—	—
Bi	conv.	−6000.	—	—

Hall Effect Animation

- From table we notice that: the lower the concentration, the greater R_H.
- Measuring R_H is important for measuring the carrier concentration.
- Eq. (55) follows from the assumption that τ for all electrons are equal, independent of the velocity of the electron.



Thermal conductivity coefficient K is defined as:

$$j_u = -K \frac{dT}{dx}$$

J_u is the Thermal Energy Flux (Amount of thermal energy flown cross unit area in 1 sec. From previous lectures:

$$C_{el} = \frac{1}{2}\pi^2 N k_B \frac{T}{T_F} \quad with : T_F = \frac{\varepsilon_F}{k_B} \qquad \Rightarrow C_{el} = \frac{1}{2}\pi^2 N k_B \frac{T}{\varepsilon_F} k_B$$

$$\text{from Chapt. 5: } K = \frac{1}{3} C v l$$

$$\Rightarrow K_{el} = \frac{1}{3}.\frac{1}{2}\pi^2 N k_B \frac{T}{\varepsilon_F} k\, v l = \frac{1}{3}.\frac{1}{2}\pi^2 N k_B \frac{T}{m v_F^2}.2.k_B v_F l$$

$$\Rightarrow K_{el} = \frac{\pi^2 n k_B{}^2 T \tau}{3m} \quad (n\, for\, N \;\; and\; l = v_F \tau) \qquad (56)$$

THERMAL CONDUCTIVITY OF METALS

- Do the electrons or the phonons carry the greater part of the heat current? in a metal?
- In pure metals the electronic contribution is dominant at all temperatures.
- In impure metals or in disordered alloys, the electron mean free
- path is reduced by collisions with impurities, and the phonon contribution may be comparable with the electronic contribution.