# MATH203 Calculus

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# Power Series

## Definition

If $x$ is a variable, then an intinite series of the form $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_2 x + \cdots + a_n x^n + \ldots;\ a_i \in \mathbb{R}$ is called a power series in $x$ or. $\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_2(x-c) + \cdots + a_n(x-c)^n + \ldots;\ c \in \mathbb{R}$ is called a power series in $(x-c)$

**Remarks:**

1. We can check the convergence or divergence of a power series $\sum_{n=0}^{\infty} a_n x^n$ for different values of $x$.
2. Every power series in $x$ converges if $x = 0$.
3. To find all other values of $x$ for which $\sum_{n=0}^{\infty} a_n x^n$ is convergent, we often use **the absolute ratio test**.

## Interval of convergence

After finding values of $x$ which are convergent in the interval, say (a,b), this is called the interval of convergence for the power series $\sum_{n=0}^{\infty} a_n x^n$.

## Radius of convergence

Half of the length of interval of convergence is called the radius of convergence of the the power series $\sum_{n=0}^{\infty} a_n x^n$.

## Theorem

Every power series $\sum_{n=0}^{\infty} a_n x^n$ satisfies one of the following:

1. The series converges only when $x = 0$ and this convergence is absolute.
2. The series converges for all $x$, and this convergence is absolute.
3. There is a number $R >$ such that the series converges absolutely when $x < R$ and diverges when $x > R$. Note that the series may converge or diverge depending on the particular series.

# Examples

Find the interval of convergence and radius of convergence of the following series:

(1): $\sum_{n=1}^{\infty} \frac{n}{3^n} x^n$ (2): $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (3): $\sum_{n=0}^{\infty} (n!) x^n$

(4): $\sum_{n=0}^{\infty} (2x)^n \frac{1}{n}$ (5): $\sum_{n=0}^{\infty} x^n \frac{1}{\sqrt{n}}$ (6): $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} (x-3)^n$

**Solution**: