MATH203 Calculus

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Alternating Series

Definition

The alternating Series

$$\sum_{n=1}^{\infty}(-1)^n a_n = -a_1 + a_2 + \cdots + (-1)^n a_n + \ldots$$ or

$$\sum_{n=1}^{\infty}(-1)^{n-1} a_n = a_1 - a_2 + \cdots + (-1)^{n-1} a_n + \ldots$$

Alternating Series Test (AST)

If $a_n > 0$, then the alternating Series $\sum_{n=1}^{\infty}(-1)^n a_n$ or. $\sum_{n=1}^{\infty}(-1)^{n-1} a_n$ converge if the following conditions are satisfied.

1. $a_n \geqslant a_{n+1} > 0.$
2. $\lim_{n\to\infty} a_n = 0.$

If condition (2) in AST is not satisfied then the series is d'gt.

Examples

Determine whether the alternating series converges or diverges

(1): $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n}{4n^2-3}$ (2): $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n}{4n-3}$

(3): $\sum_{n=1}^{\infty}(-1)^{n-1}n5^{-n}$ (4): $\sum_{n=1}^{\infty}(-1)^{n}\frac{n+1}{n}$

Solution:

Absolute convergence

Definition

A series $\sum_{n=1}^{\infty} |a_n|$ is called an absolutely convergent if the series $\sum_{n=1}^{\infty} |a_n| = |a_1| + |a_2| + \cdots + |a_n| + \ldots$ is convergent.

Conditionally convergent Series

Definition

A series $\sum_{n=1}^{\infty} a_n$ is called conditionally convergent if the series $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} |a_n|$ is divergent.

Theorem

If a series $\sum_{n=1}^{\infty} |a_n|$ is convergent, then the series $\sum_{n=1}^{\infty} a_n$ is convergent (converse is not true).

Examples

Determine whether the series is absolute convergent, conditionally convergent or divergent

(1): $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2}$ (2): $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n}$

(3): $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{1}{\sqrt{n}}$ (4): $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n+1}$

Solution:

Remark

For any series $\sum_{n=1}^{\infty}(-1)^n a_n$ exactly one of the following statements is true:

- It is absolutely convergent.
- It is conditionally convergent.
- It is divergent.

Absolute Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series of non-zero terms, suppose $\sum_{n=1}^{\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$, then

- the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent if $L < 1$.
- the series $\sum_{n=1}^{\infty} a_n$ is divergent if $L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$.
- If $L = 1$ (fails), the series may converge or diverge.

Examples

Determine whether the series is absolute convergent, conditionally convergent or divergent

(1): $\sum_{n=1}^{\infty}(-1)^n \frac{n^2+4}{2^n}$

(2): $\sum_{n=1}^{\infty}\frac{(-10)^n}{n!}$

(3): $\sum_{n=1}^{\infty}(-1)^n \frac{n^4}{e^n}$

Solution: