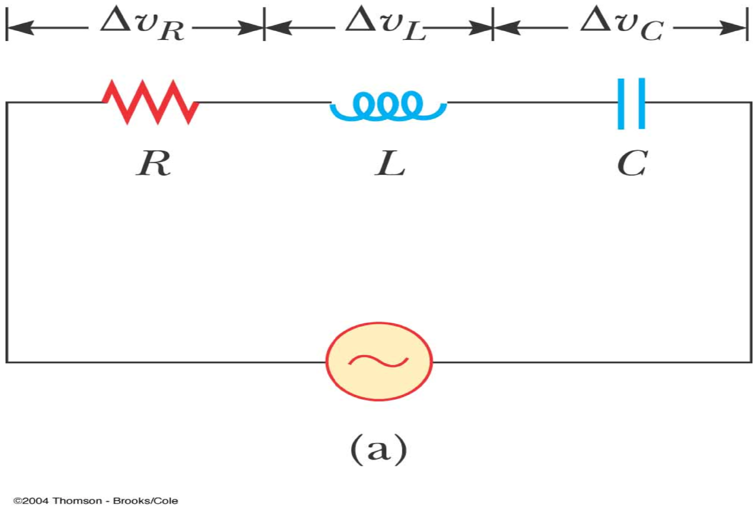
33.5 The RLC series circuit

In the previous sections we considered individual elements connected to an AC source. Here, we will adopt a circuit that contains a combination of circuit elements: a resistor, an indicator, and a capacitor.



The instantaneous applied voltage is

$$\Delta v = V_{max} \sin \omega t$$

The current also varies as

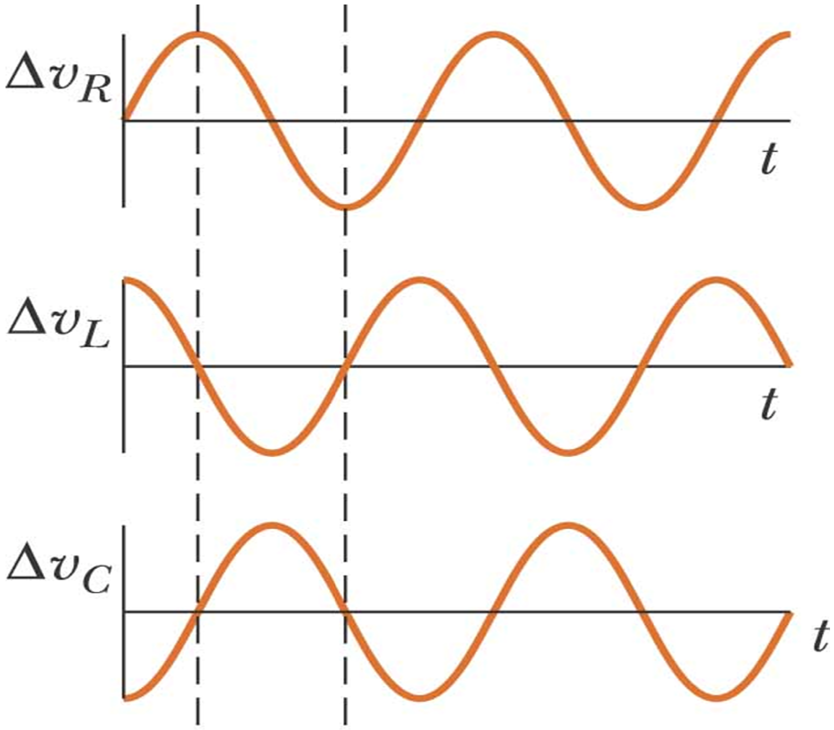
$$i = I_{max} \sin(\omega t - \phi) \qquad 33.10$$

Where φ is some phase angle between the current and the applied voltage. Based on our discussed in chapter 28, the current everywhere in the circuit must be the same while the voltage across each element has a different amplitude and phase.

$$\Delta v_R = I_{max} R = \Delta V_R \sin \omega t$$

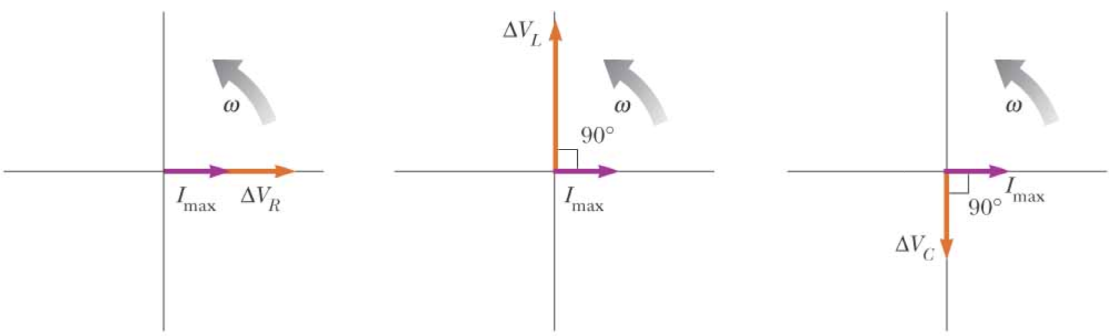
$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$



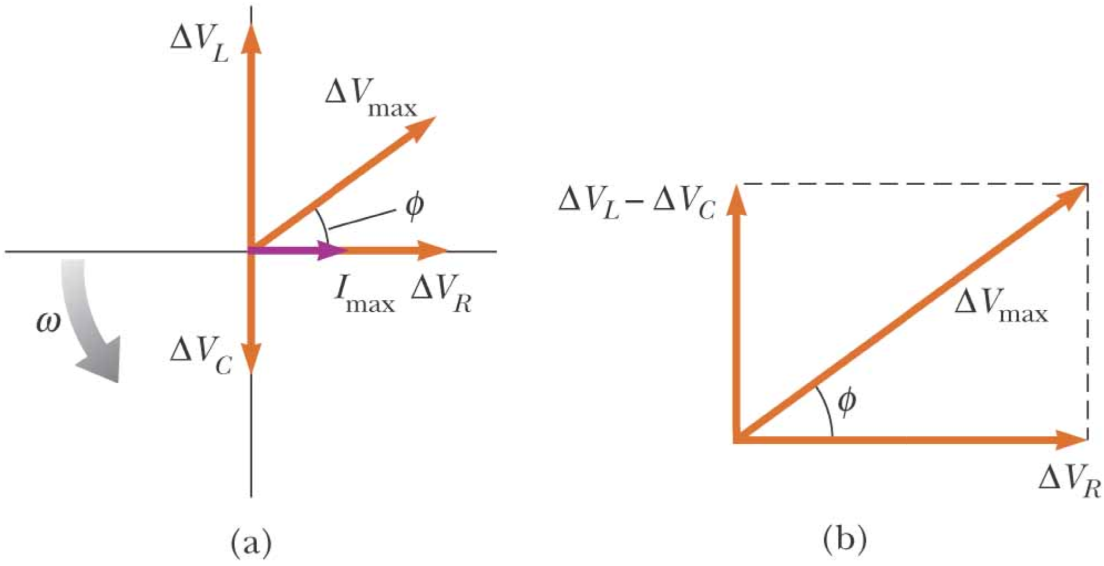
(b)

Phase relationships between the voltage and current phasors for (a) a resistor, (b) an indicator, and (c) a capacitor connected in series are shown in following figure.



(a) Resistor (b) Inductor (c) Capacitor

Because the phasors are rotating vectors, the voltage phasors can be combined using vector addition as in the following figure.



From this figure, we conclude that,

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2}$$

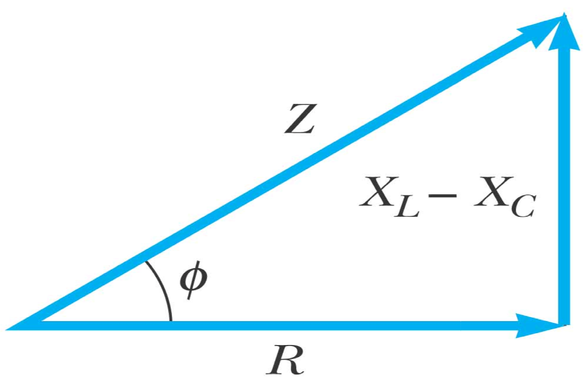
$$\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2} \qquad 33.12$$

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and is called the impedance of the circuit.

From the figure, one can also find the phase angel φ,

$$\phi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = \tan^{-1}\left(\frac{I_{max}X_L - I_{max}X_C}{I_{max}R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad 33.13$$



Exercise:

Based on the phase angle in eq. (33.13) discus the following situations:

1- When $X_L > X_C$
2- When $X_L < X_C$
3- When $X_L = X_C$