MATH203 Calculus

Dr. Bandar Al-Mohsin

School of Mathematics, KSU

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Outline

- Definition of sequences.
- Definition of convergent sequence.
- Definition of divergent sequence.
- Definition of constant sequence.

Definition of sequences

A sequence is a function whose domain is the set of positive integers. It is denoted by $\{a_n\} = a_1, a_2, a_3, \ldots, a_n, \ldots$ (entire seq) and $\{a_n\} = a_1, a_2, a_3, \ldots, a_n$ (finite seq).

Example: Find the first four terms and nth term of each:

(a) $\{\frac{n}{n+1}\}$ (b) $\{2 + (0.1)^n\}$ (c) $\{(-1)^{n+1}\frac{n^2}{3n-1}\}$

(d) $\{4\}$ (e) $a_1 = 3$ and $a_{k+1} = 2a_k$ for $k \geqslant 1$.

Definition of convergent sequence (c$'$gt)

A sequence is $\{a_n\}$ has a limit L, or converges to L denoted by either $\lim\limits_{n\to\infty} a_n = L$ or $a_n \to L$ as $n \to \infty$.

Definition of divergent sequence (d$'$gt)

A sequence $\{a_n\}$ is called if

- $\lim\limits_{n\to\infty} a_n$ does not exist.
- $\lim\limits_{n\to\infty} a_n = +\infty$ or $\lim\limits_{n\to\infty} a_n = -\infty$.

Definition of constant sequence

A $\{a_n\}$ is constant if $a_n = c$ for every n, $c \in \mathbb{R}$ and $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c = c$.

Theorem 1

Let $\{a_n\}$ be a sequence and f be a function such that

- $f(n) = a_n$
- $f(x)$ exists for every real number $x \geqslant 1$

then

1. If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} f(n) = L$
2. If $\lim_{x\to\infty} f(x) = \infty$ (or $-\infty$), then $\lim_{n\to\infty} f(n) = \infty$ (or $-\infty$).

Examples:

(1) If $a_n = 1 + (\frac{1}{n})$, determine whether $\{a_n\}$ converges or diverges.

(2) Determine whether $\{a_n\}$ converges or diverges

(a)$\{\frac{1}{4}n^2 - 1\}$ (b) $\{(-1)^{n-1}\}$