10 ةرضاحم	13/12

(4) Anderson-Darling test.

$x_1, x_2, \dots, x_n$

$y_0, y_1, \dots < y_k < y_{k+1}$

$$y_0 = \begin{cases} 0 & \text{if there is no truncation} \\ d & \text{if there is.} \end{cases}$$

$$y_{k+1} = \begin{cases} \infty & \text{if there is no censoring} \\ u & \text{if there is.} \end{cases}$$

The Anderson darling test statistic:

$$A^2 = nF^*(u) + n\left[\sum_{j=0}^{k}(1-F_n(y_j))^2 \ln\left(\frac{1-F^*(y_j)}{1-F^*(y_{j+1})}\right) + \sum_{j=1}^{k}(F_n(y_j))^2 \ln\left(\frac{F^*(y_{j+1})}{F^*(y_j)}\right)\right]$$

Level of confidence α	0.1	0.05	0.01
Critical value	1.933	2.492	3.857

Example: claim amount 200, 400, 1000, 1600, 3000, 5000, 5400, 6200

$F^*(x) = 1 - e^{-\theta b}$; $\frac{1}{\theta} = 3300$

a) Find A^2

b) Determine the result of the test for $\alpha = 10\%$

Solution:

a.

j	y_j	$F_8(y_j)$	$F^*(y_j)$	a_j	b_j
0	0	0	0	0.06	0
1	200	0.125	0.058	0.046	0.010
2	400	0.25	0.114	0.102	0.051
3	1000	0.375	0.261	0.071	0.054
4	1600	0.5	0.384	0.106	0.11
5	3000	0.625	0.597	0.085	0.104
6	5000	0.75	0.780	0.007	0.017
7	5400	0.875	0.805	0.0038	0.038
8	6200	1	0.8472	0	0.1658
9	∞	1	1	0.482	0.553

total

$$a_j = (1 - F_n(y_j))^2 \ln\left(\frac{1 - F^*(y_j)}{1 - F^*(y_{j+1})}\right)$$

$$b_j = (F_n(y_j))^2 \ln\left(\frac{F^*(y_{j+1})}{F^*(y_j)}\right)$$

$$A^2 = -8 + 8(0.482 + 0.553) = 0.289$$

b.

$C = 1.933$

we accept the hypothesis.

(5) Chi-square test :

Interval	# of observations
(c_0, c_1)	n_1
(c_1, c_2)	n_2
⋮	⋮
(c_{k-1}, c_k)	n_k

$F^* \leftarrow$ data [illegible] cdf

- $\hat{P}_j = F^*(c_j) - F^*(c_{j-1})$
- $E_j = n\hat{P}_j$
- $O_j = n_j$

The Chi-square statistic :

$$\chi^2 = \sum_{j=1}^{k} \frac{(E_j - O_j)^2}{E_j} = \left(\sum_{j=1}^{k} \frac{n_j^2}{E_j}\right) - n.$$

- χ^2 is computed from the Chi-square distribution table with degree of fredom equal to $k-r-1$ where r is the number of parameters estimated in the model.

- we accept the hypotheses if:

$$\chi^2 \leq \chi^2_{k-r-1,\,1-\alpha} \quad , \quad \alpha = \text{level of confidence.}$$

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Example: F^*:

interval	$F^*(c_j)$	# of observations
$x<2$	0.035	5
$2 \leq x \leq 5$	0.135	42
$5 \leq x < 7$	0.630	137
$7 \leq x < 8$	0.83	66
$8 \leq x$	1	50
		300

a. χ^2

b. Test the null hypothesis with $\alpha = 5\%$

c. " " " $\alpha = 2.5\%$

a.

j	$\hat{P}_j$	$E_j = n\hat{P}_j$	O_j	$(E_j - O_j)^2$
1	0.035	10.5	5	30.25
2	0.095	28.5	42	182.25
3	0.5	150	137	169
4	0.2	60	66	36
5	0.17	51	50	1

$$\chi^2 = \frac{30.25}{10.5} + \frac{182.25}{28.5} + \frac{169}{150} + \frac{36}{60} + \frac{1}{51} = 11.02$$

$r = 0$, $k = 5$

b. $C = \chi^2_{4, 0.95} = 9.488 < 11.02 = \chi^2$

$\Rightarrow$ we reject the hypothesis

c. $C = \chi^2_{4, 0.975} = 11.14 > 11.02 = \chi^2$

$\Rightarrow$ we accept