PHY331
Magnetism

Lecture 9

Last week…

- We showed that the magnetic field from a single dipole is not sufficient to align its neighbours. E.g. would require very large fields (100's of T) to align electron spins. If such effect were the correct explanation, ferromagnetism would not exist above a few kelvin.

- We then showed that it is the energy of exchanging electrons that 'forces' dipoles to align. Find lowest ground state is always spin-parallel.

- This mechanism accounts for domain formation in 'insulating' magnetic materials. It fails in conducting materials (I.e. for conduction electrons. Will cover this in last lecture (lecture 10).

This week....

- Will discuss the Weiss Molecular Field model of ferromagnetism.
- Will show that it provides a good account of spontaneous magnetisation of a ferromagnet, and predicts the magnetic susceptibility above the Curie temperature.

a leading question…………

    dipole fields don't work

    quantum mechanics doesn't always "work"

    What can we try next?

    why not ignore the *atomic–level structure* of the ferromagnet completely?

    Pierre Weiss suggested that,

\#   Inside an individual domain a *spontaneous alignment* of the magnetic moments occurs. Then assume the alignment of any *individual magnetic moment* is due to an *internal magnetic field* which arises from all the *other magnetic moments* present in the domain
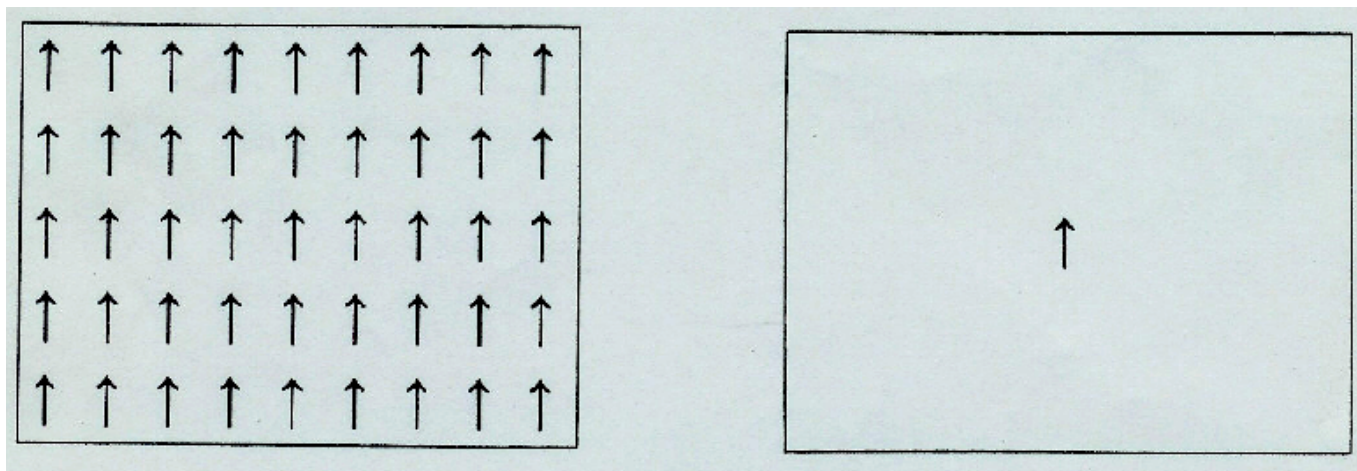
\# The magnitude of this ***internal magnetic field*** is assumed to be ***proportional to the magnetisation*** M of the sample,

$$\text{Total } B \text{ field} = B_{ext} + B_{int}$$

$$B = B_{ext} + \mu_0 H_{int}$$

$$B = B_{ext} + \mu_0 \gamma M \qquad (1)$$

Here γ the constant of proportionality is called

γ ***Weiss Molecular Field constant***

*W*hat did Weiss do?

He incorporated this "***molecular field***" into the ***existing treatment of the paramagnet***

His theory has *considerable success* in correctly *predicting* the *properties of the ferromagnet*

It is capable of *extension* to *all the other types* of *magnetic materials* that show *co-operative behaviour*

It's also so *straightforward* that *almost anyone can use it*!

The Spontaneous Magnetisation of the Ferromagnet

From the quantum treatment of the paramagnet we obtain,

$$M \quad = \quad N \mu_J B_J(y)$$

where M is a function of temperature T through,

$$y = \frac{\mu_J B}{kT} \qquad (2)$$

so we can write,

$$M(T) \quad = \quad N \mu_J B_J(y) \qquad (3)$$

N ow, as $T \to 0, \quad y \to \infty, \quad B_J(y) \to 1$ so that the *saturation magnetisation* $M(0)$ is equal to,

$$M(0) \;=\; N\mu_J \qquad (4)$$

or,
$$\frac{M(T)}{M(0)} = B_J(y) \qquad (5)$$

But for the *ferromagnet*, we must add the *molecular field term* to y. So substitute (1) into (2) to get…

$$y = \frac{\mu_J}{kT}\left(B_{ext} + \mu_0 \gamma M\right) \qquad (6)$$

However, we want the *spontaneous magnetisation*

so we let $B_{ext} \to 0$ and write $M = M(T)$
explicitly

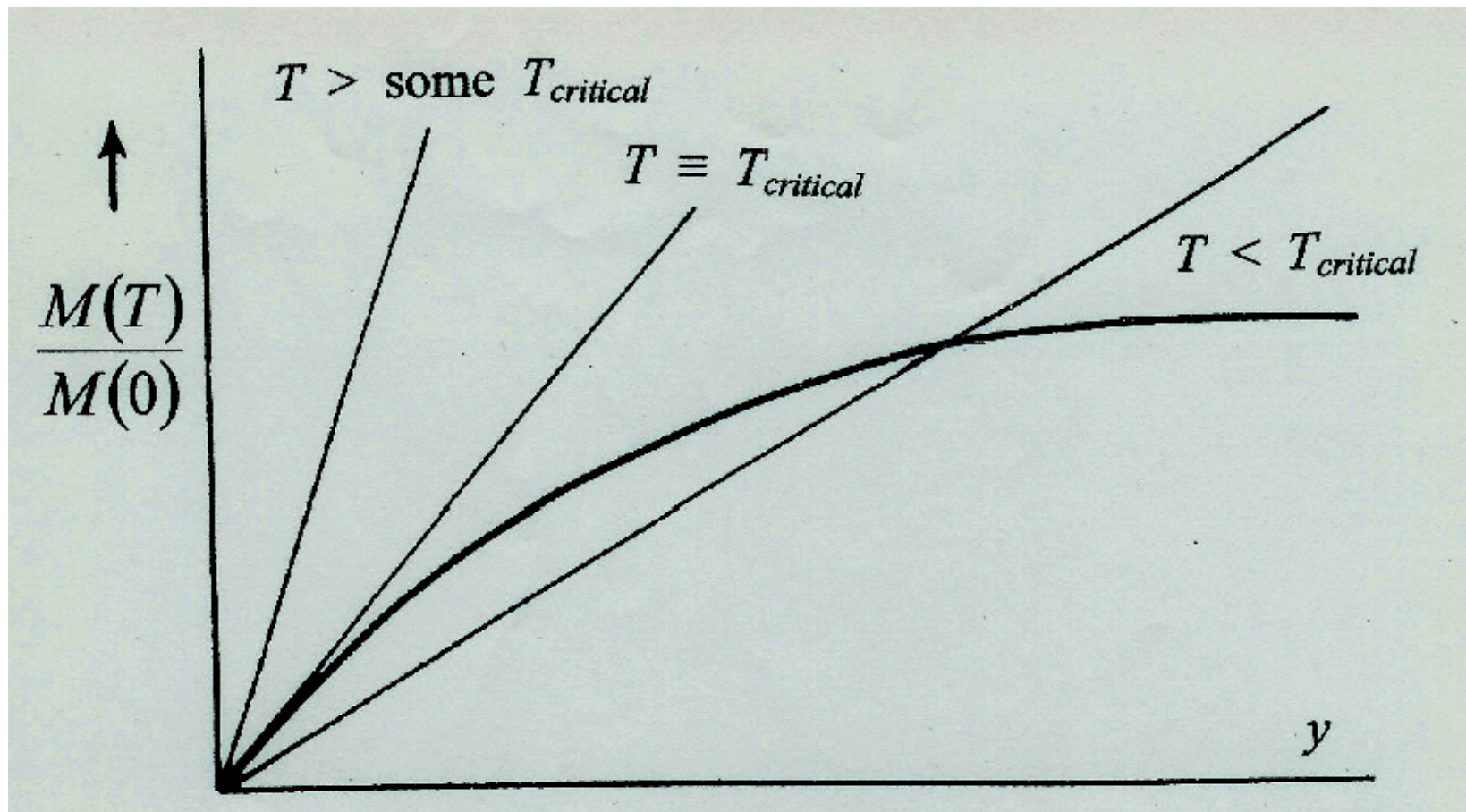that is, $$y = \frac{\mu_J}{kT} \mu_0 \gamma M(T)$$

so rearranging, and using equation (4) for $M(0)$
we get,

$$M(T) = \frac{kT}{\mu_J \mu_0 \gamma} y \quad \text{or} \quad \frac{M(T)}{M(0)} = \left[\frac{kT}{N \mu_J^2 \mu_0 \gamma} \right] y \qquad (7)$$

We now have two equations (5 and 7) for the
variation of $M(T)/M(0)$

Equation 2 is the ***Brillouin function***
Equation 3 is a ***straight line*** with a slope
proportional to *T*

1) $T >$ some $T_{critical}$ $M(T)/M(0) \equiv 0$
is the only solution

2) $T \equiv T_{critical}$ the straight line (eqn 3) is a ***tangent***
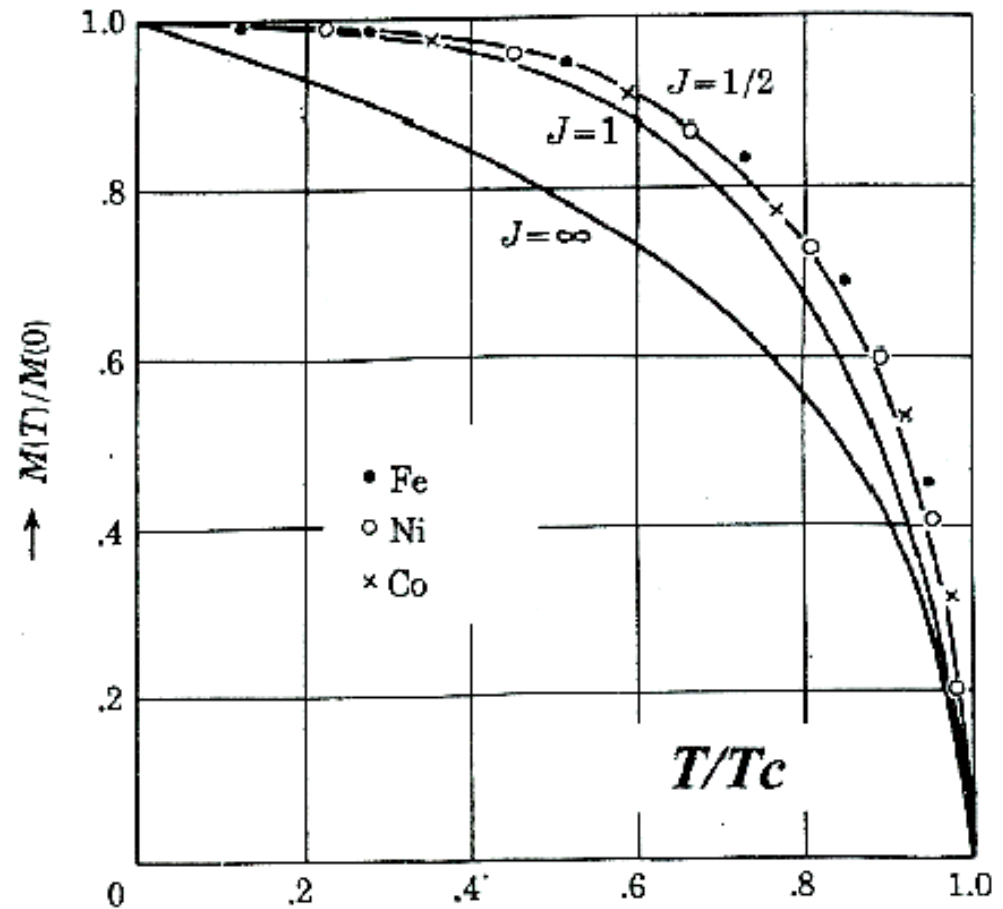to $B_J(y)$ at the ***origin***, - still only one solution $M(T)/M(0) \equiv 0$

3) $T < T_{critical}$ two solutions,
$M(T)/M(0) \equiv 0$
as before $M(T)/M(0) =$ real [***actual value*** of $M(T)$]

We can now replot the real
   solutions,

$$\frac{M(T)}{M(0)} \quad versus \quad T$$

Temperature variation of
magnetisation M of a
ferromagnet

Evaluation of the critical temperature
(Curie temperature)

At small values of y we can expand $B_J(y)$ as,

$$B_J(y) \approx \frac{J+1}{3J} y + ...$$

which is also a straight line, - so equating these two gives,

$$\frac{J+1}{3J} y = \frac{kT}{N \mu_J^2 \mu_0 \gamma} y$$

$$T_C = N \mu_J^2 \mu_0 \gamma \frac{J+1}{3Jk}$$

then using, $\mu_J = J g \mu_B$ gives,

$$T_C = \frac{\mu_0 N \; g^2 J(J+1)\mu_B^2}{3k} \gamma \qquad (8)$$

so that, $\qquad T_C = T_C(\gamma)$

- Not unexpected: the *larger* the Weiss Molecular Field constant γ the *higher* the Curie temperature T_C.

Successes of the Weiss Molecular Field model

i) it gives a *good account* of,

\# The *spontaneous magnetisation* of the feromagnet,

\# The *temperature variation* of magnetisation, $M(T)/M(0)$

\# The *magnetic susceptibility* above T_C - *called the Curie_Weiss Law!*

Summary

- Have seen that if we assume that the internal magnetic field is proportional to the magnetisation of the sample, we can get a spontaneous magnetization for temperatures less than the Curie Temperature.

- We also find the larger the field constant (relating internal field to magnetization), the higher the Curie Temperature.

- Next week…paramagnetic susceptability of free electrons - Pauli paramagnetism.