PHY331
Magnetism

Lecture 6

Last week…

Learned how to calculate the magnetic dipole moment of an atom.

Introduced the “Landé g-factor”. Saw that it compensates for the different contributions from the orbital motion and the electron spin.

Compare calculated values with measured data

This week….

- Will derive a quantum theory of Paramagnetism. Will obtain a new expression for paramagnetic susceptibility that we will compare with the classical value. Will see that it confirms Curie’s Law.

- Will introduce some basic concepts relating to Ferromagentism.

Quantum theory of paramagnetism

What are the ***changes*** from Langevin's classical theory? The ***magnetic moment value*** is,

$$\mu_J = g\sqrt{J(J+1)}\,\mu_B$$

and the ***resolved component*** of the moment along the field direction is, $M_J g\mu_B$ where

$$M_J = J, (J-1), \ldots\ldots, -(J-1), -J$$

the ***potential energy*** of the dipole in the field is then,

$$-M_J g\mu_B B$$

so this gives all we need to substitute into,

$$M = \sum \text{[1] Resolved component of a dipole in field direction} \times \text{[2] Number of dipoles with this orientation}$$

we now have a rather unusual result: the dipoles no longer have an ***infinite set of orientations in space***,they have a ***finite set of orientations*** described by M_J, this means that, the ***integral*** over $d\theta$ that was used in Langevin's treatment

$$\frac{\langle m\rangle}{m}=\frac{\int_0^\pi \cos\theta\cdot cmB\sin\theta\cdot\exp\left(mB\cos\theta/kT\right)d\theta}{\int_0^\pi cmB\sin\theta\cdot\exp\left(mB\cos\theta/kT\right)d\theta}$$

can be replaced by a ***summation*** over the M_J

$$\frac{\left\langle\mu_J^{\parallel}\right\rangle}{\mu_J}=\frac{\sum_{-J}^{J} M_J g\mu_B\cdot\exp\left(M_J g\mu_B B/k_BT\right)}{\sum_{-J}^{J}\exp\left(M_J g\mu_B B/k_BT\right)}$$

whose solution is:

$$\frac{\left\langle \mu_J^{\|} \right\rangle}{\mu_J} = \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}\cdot y\right) - \frac{1}{2J}\coth\left(\frac{1}{2J}\cdot y\right) \qquad (1)$$

$$\coth x = \frac{1}{\tanh x} = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \dots$$

This is the **Brillouin function** $B_J(y)$.

It is plotted as a function of

$$y = \frac{\mu_J B}{kT}$$

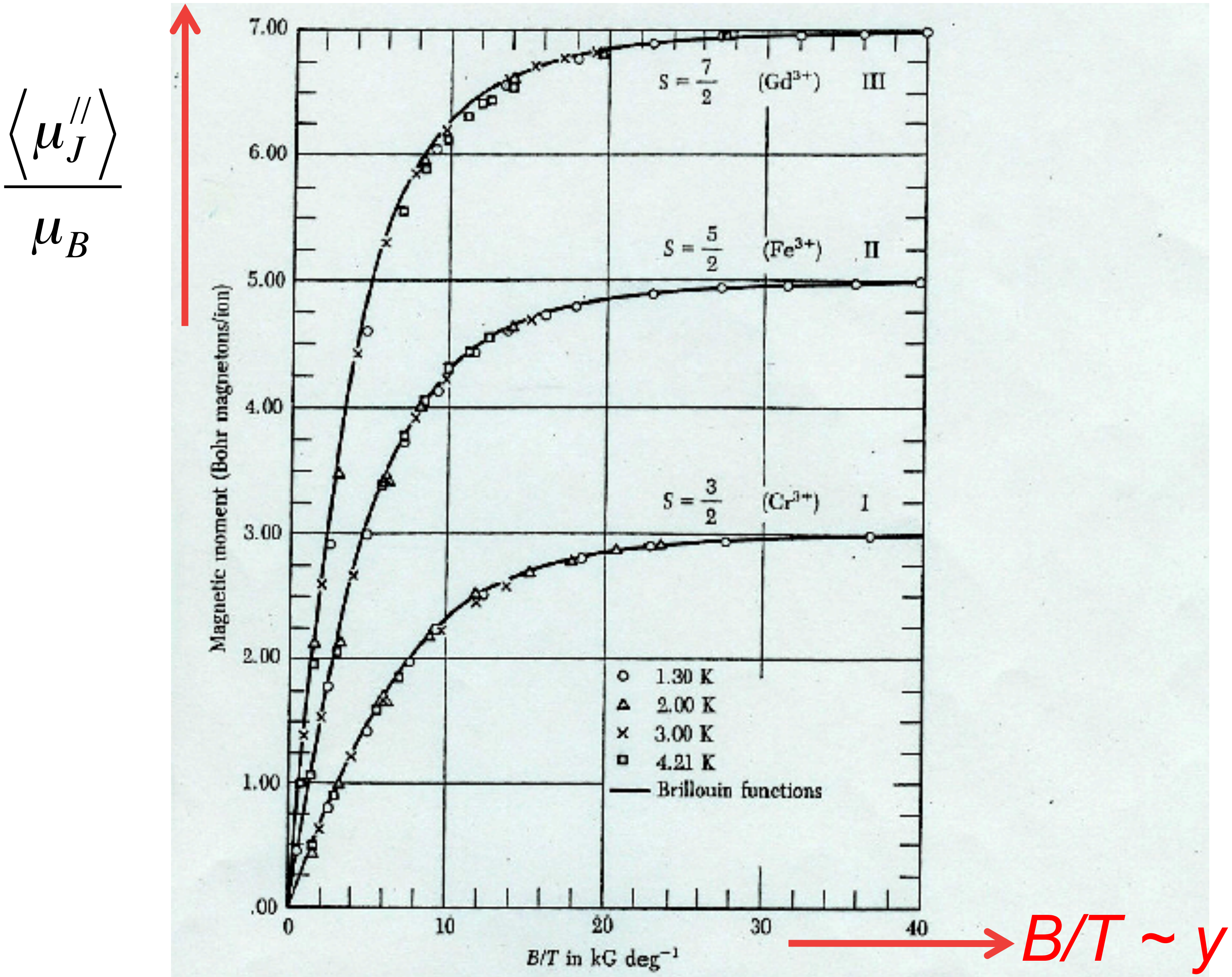


Figure 4 Plot of magnetic moment *versus* B/T for spherical samples of (I) potassium chı alum, (II) ferric ammonium alum, and (III) gadolinium sulfate octahydrate. Over 99.5% m saturation is achieved at 1.3 K and about 50,000 gauss. (5T). After W. E. Henry.

The magnetic susceptibility χ

Just as with the classical theory, we can derive an expression for the magnetic susceptibility χ in the limit of small applied fields and high temperatures,

that is $y = \dfrac{\mu_J B}{kT} << 1$ Use $\coth x = \dfrac{1}{x} + \dfrac{x}{3}$

Substitute into equation (1) to show

$$B_J(y) \approx \frac{J+1}{3J} y + \ldots\ldots$$

or

$$B_J(y) \approx \frac{J+1}{3J} \frac{\mu_J B}{kT} \qquad (2)$$

but, $$\frac{\langle \mu_J^{||} \rangle}{\mu_J} = B_J(y)$$ or $$\langle \mu_J^{||} \rangle = \mu_J \, B_J(y) \quad (3)$$

and $$M = N \langle \mu_J^{||} \rangle \quad (4)$$

so that substituting (3) into (4) gives,

$$M = N \langle \mu_J^{||} \rangle = N \mu_J \, B_J(y) \quad (5)$$

Using (1) $$M = N \mu_J \frac{J+1}{3J} \frac{\mu_J B}{kT} = N \frac{J+1}{J} \frac{\mu_J^2 \, \mu_0 H}{3kT}$$

so that, $$\chi = \frac{M}{H} = \mu_0 \, N \frac{J+1}{J} \mu_J^2 \frac{1}{3kT}$$

Finally using, $\mu_J = J g \mu_B$ gives,

$$\chi = \frac{\mu_0 N g^2 J(J+1)\mu_B^2}{3kT} = \frac{C}{T}$$

which is Curie's Law (again).

So we compare with the classical result,

$$\chi = \frac{M}{H} = \frac{\mu_0 N m^2}{3kT} = \frac{C}{T}$$

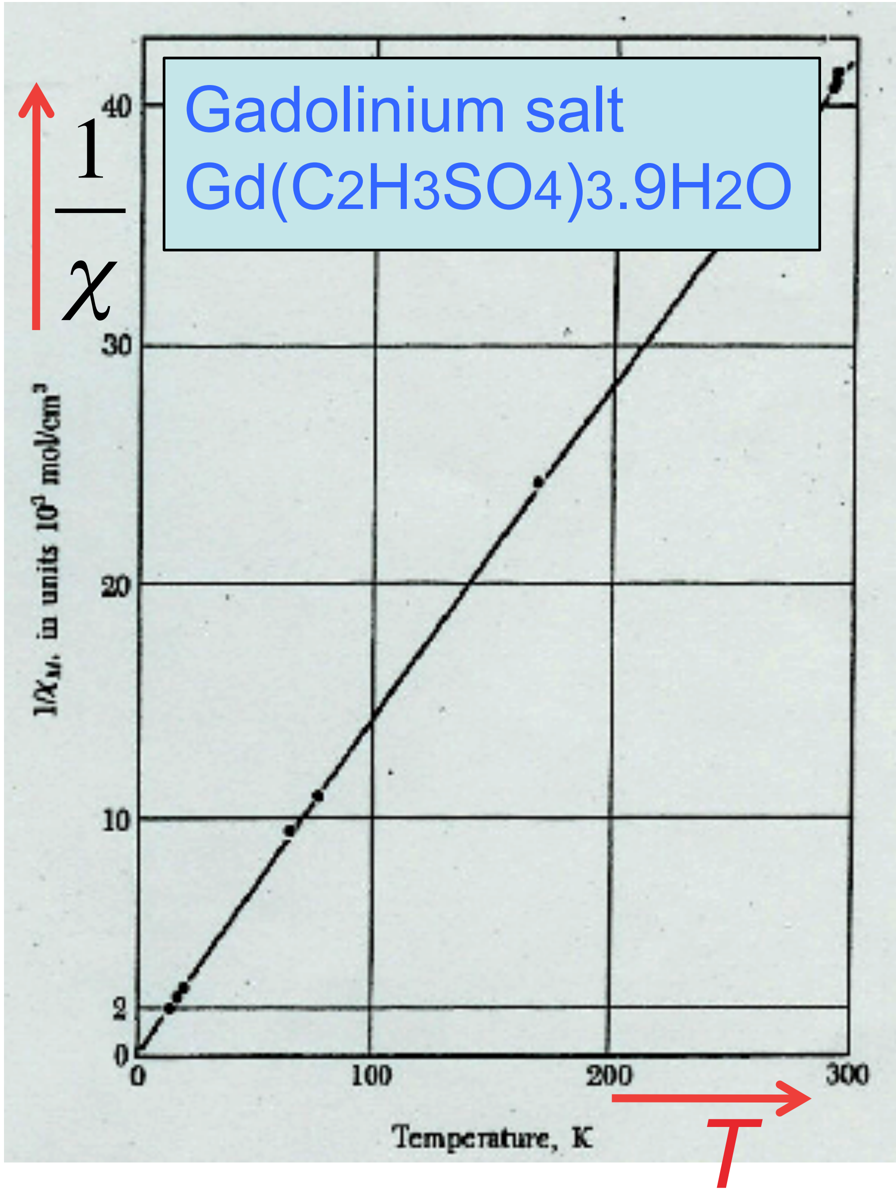
which indicates that, $J_{effective} = g\sqrt{J(J+1)}$

which is consistent with the earlier definitions,

$$\mu_L = -\sqrt{L(L+1)}\,\mu_B$$

$$\mu_S = -2\sqrt{S(S+1)}\,\mu_B$$

and our understanding of what the “Landé *g* factor” is meant to do.



Experimental evidence of Curie’s Law behaviour

$$\frac{1}{\chi} \propto T$$

Paramagnetism..

- Calculated paramagetic susceptability via both quantum and classical approaches similar.
- Both confirm statistical competition between magnetic alignment and thermal effects.
- Now consider systems where have a strong, spontaneous alignment between dipoles even in the absence of a magnetising field.

Ferromagnetism

The bulk properties of a ferromagnet are:

The magnetisation M is large and positive

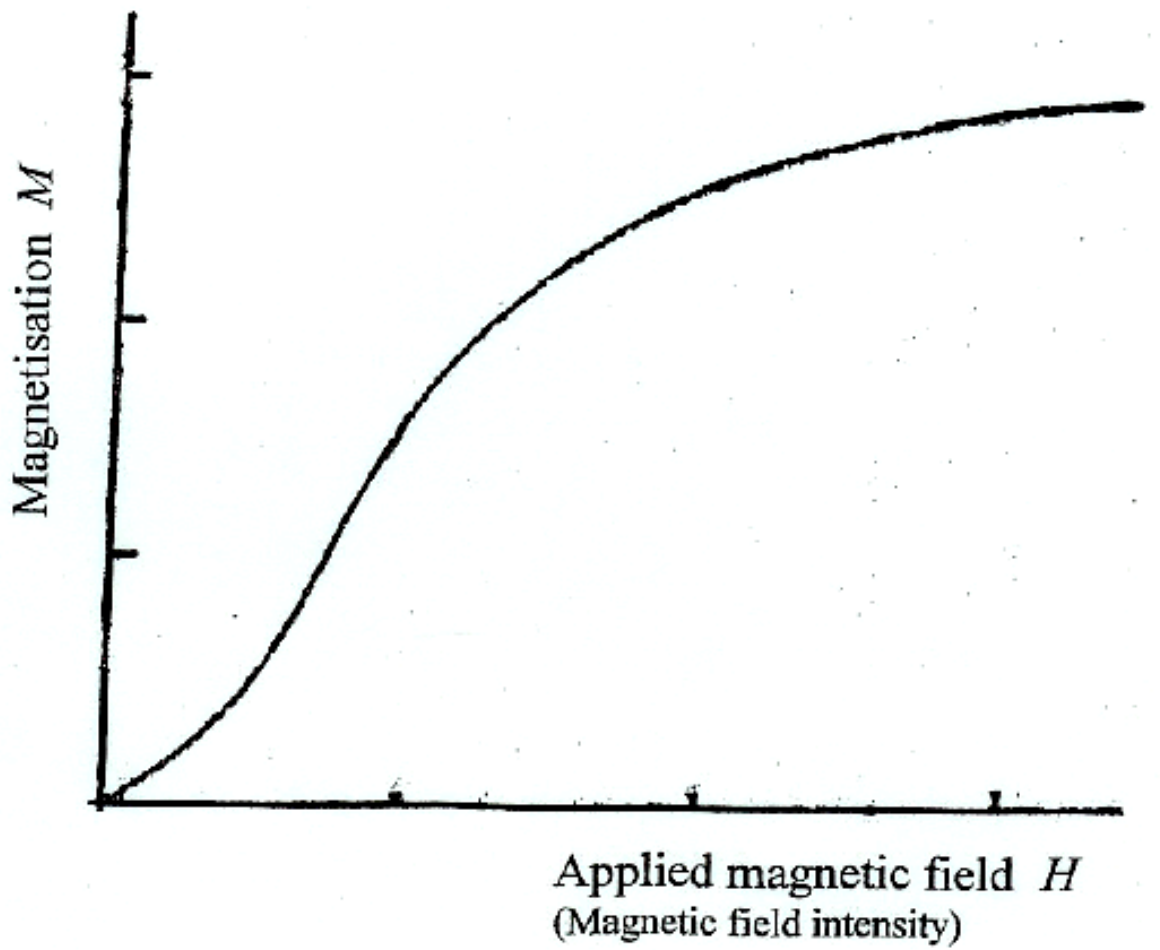
It is contribution to the total B field is significant

$$\chi >> 0 \qquad \text{and} \qquad \mu_r >> 1$$

The magnetisation M is a complex function of the applied magnetic field H

The magnetisation M also depends on the past history of the sample

A graph of M against the applied magnetic field H gives the characteristic magnetisation curve of the ferromagnet



The curve divides naturally into three regions:

A linear region at small values of H in which the ***initial susceptibility*** $\chi = M/H$ can be defined

An intermediate region, in which the slope dM/dH ***rises to a maximum***

A region where dM/dH decreases

to show ***saturation of the magnetisation***

The **magnetisation** ***M*** is a function of ***the past history of the sample***

A permanent magnet and an **iron nail** can both lie in the same **Earth's magnetic field** but, the magnetisation *M* **inside each is quite different**

The permanent magnet has has already been exposed to a much larger **"magnetising field"**

On demagnetising *M* **lags behind** *H*. It requires a reverse field to demagnetise the sample

Important definitions:

R_M = Remanent Magnetisation

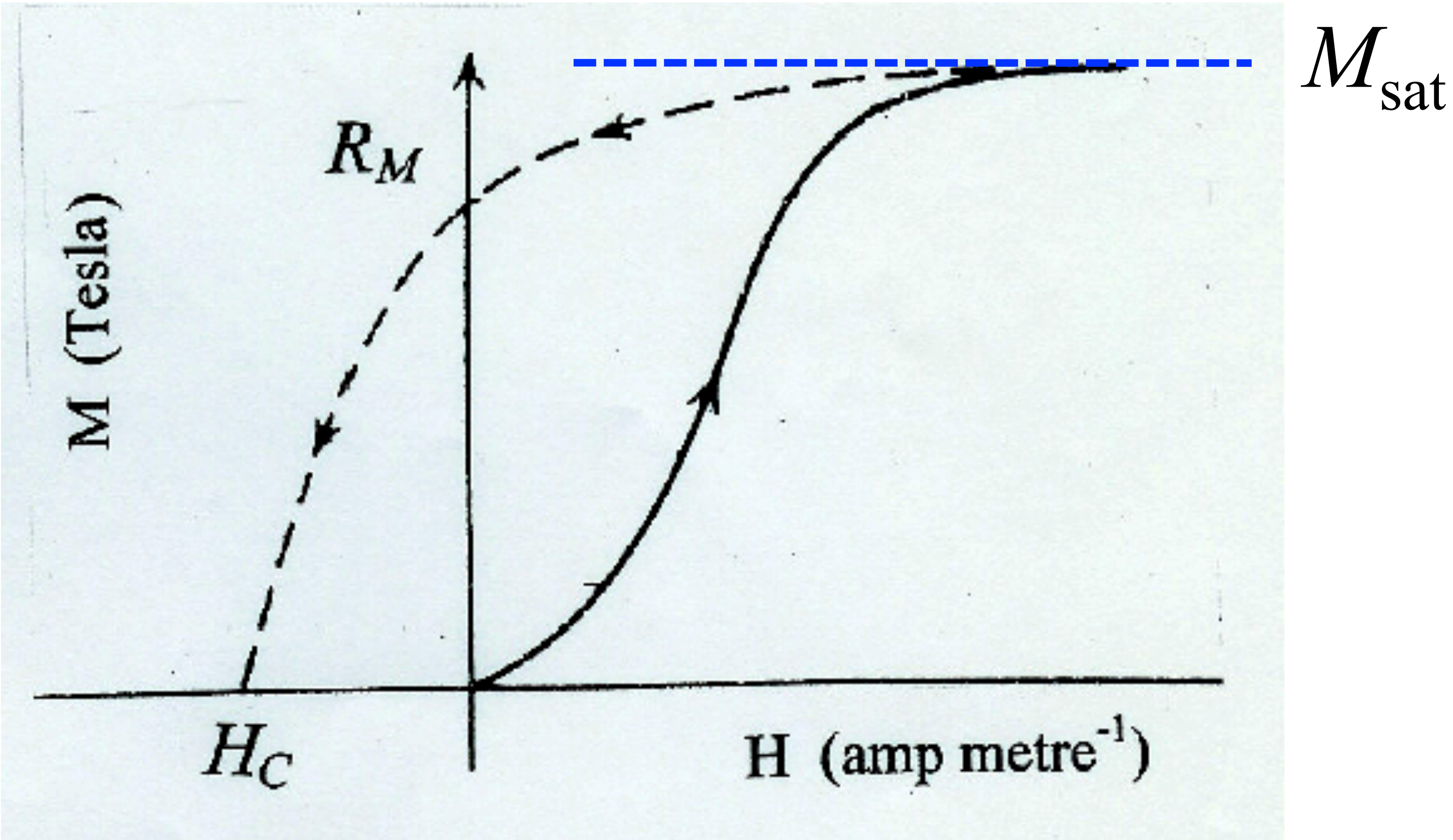
H_C = Coercive Field

M_{sat} = Saturation magnetisation

$$M_{sat} = Np\mu_B$$

N: atoms m^{-3}

$p\mu_B$: Magnetic dipole per atom



Summary

- Derived a quantum theory of Paramagnetism.

$$\chi = \frac{\mu_0 N g^2 J(J+1)\mu_B^2}{3kT} = \frac{C}{T}$$

- Introduced ferromagentism. Saw magnetization was a function of previous history.

- Next week will look at this phenomena in more detail using the domain theory of ferromagnetism.