

PHY331

Magnetism

Lecture 3

Last week...

- Derived magnetic dipole moment of a circulating electron.
- Discussed motion of a magnetic dipole in a constant magnetic field.
- Showed that it precesses with a frequency called the Larmor precessional frequency

This week...

- Discuss Langevin's theory of diamagnetism.
- Use angular momentum of precessing electron in magnetic field to derive the magnetization of a sample and thus diamagnetic susceptibility.
- Will set the scene for the calculation of paramagnetic susceptibility.

Langevin's theory of diamagnetism

- We want to calculate the sample magnetisation M and the diamagnetic susceptibility χ
(recall $\mathbf{M} = \chi\mathbf{H}$)
- Every circulating electron on every atom has a *magnetic dipole moment*
- *The sum* of the magnetic dipole moments on any atom is zero (*equal numbers circulating clockwise and anticlockwise ?*)
- *The magnetisation* M arises from the reaction to the torque Γ due to the applied magnetic field B which creates the Larmor precessional motion at frequency ω_L .

- i) the angular momentum L of the circulating electron is,
$$L = mvr = m\omega r^2$$
- The angular momentum L_p of the precessional motion is,
$$L_p = m\omega_L \langle r^2 \rangle$$

where ω_L is the Larmor frequency and $\langle r^2 \rangle$ is the mean square distance from an axis through the nucleus which is parallel to B

- ii) We showed last week that the magnetic moment \underline{m}_p per electron associated with L_p is

$$\underline{m}_p = -\frac{e}{2m} L_p$$

- so the total magnetisation M of the sample must be,

$$M = N_{atoms} \times Z_{electrons/atom} \times \underline{m}_p$$

$$M = N \times Z \times \underline{m}_p$$

So substitute for each quantity, in turn,

$$M = - N Z \frac{e}{2m} L_p$$

$$M = - N Z \frac{e}{2m} m \omega_L \langle r^2 \rangle$$

$$M = - N Z \frac{e}{2m} m \left(\frac{eB}{2m} \right) \langle r^2 \rangle$$

so that,

$$\chi = \frac{M}{H} = - \mu_0 N Z \frac{e^2}{4m} \langle r^2 \rangle$$

Now express $\langle r^2 \rangle$ in terms of the mean square radius of the orbit $\langle \rho^2 \rangle$, where

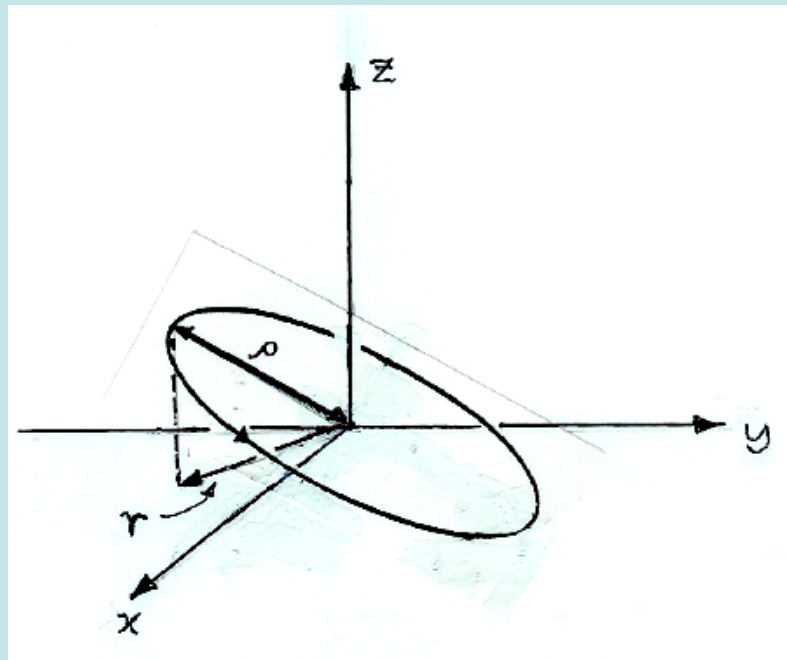
$$\langle r^2 \rangle = \frac{2}{3} \langle \rho^2 \rangle$$

will give

$$\chi = -\mu_0 N Z \frac{e^2}{6m} \langle \rho^2 \rangle$$

The result we wanted!!

$$\begin{aligned}\rho^2 &= x^2 + y^2 + z^2 \\ x^2 &= y^2 = z^2 = \rho^2 / 3 \\ r^2 &= x^2 + y^2\end{aligned}$$



What does this mean???

- The atom as a whole does not possess a permanent magnetic dipole moment.
- The magnetisation is produced by the influence of the external magnetic field on the electron orbits and particularly by the precessional motion.
- The diamagnetic susceptibility χ is small and negative, because $\langle \rho^2 \rangle$ is small.
- Negative susceptibility means that diamagnetism opposes applied magnetic field. (prefix: *dia* - in opposite or different directions)
- Note that measurements of χ were once used to get estimates of atomic size through the values of $\langle \rho^2 \rangle$.

Remember - all materials are diamagnetic –

but this small effect is swamped if also paramagnetic or ferromagnetic.

Paramagnetism

- Paramagnets have a small positive magnetisation M (directed parallel the applied field \underline{B}).
- Each atom has a permanent magnetic dipole moment.
- Langevin (classical) theory
- The paramagnet consists of an array of permanent magnetic dipoles \underline{m}
- In a uniform field \underline{B} they have

$$\text{Potential Energy} = - \underline{m} \cdot \underline{B}$$

A dipole *parallel* to the field has the lowest energy

- ***BUT***, the B field causes *precession* of m about B .
- ***However*** it can't alter the *angle* between m and B (as the L_z component is *constant* in the precession equations).

- ***For the dipole to lower its energy (and become parallel to the field)*** we need a *second mechanism*. This is provided by the thermal vibrations
- The magnetic field "would like" the dipoles aligned to lower their energy
- The thermal vibrations "would like" to randomise and disorder the magnetic dipoles

- We can therefore expect a statistical theory, based on a “competition” between these two mechanisms,

$$k_B T \approx -\underline{m} \cdot \underline{B}$$

to calculate the magnetisation M of a paramagnet

Calculate the magnetisation M of a paramagnet

- M must be the vector sum of all the magnetic dipole moments

$$M = \sum \begin{matrix} \text{[1] Resolved component} \\ \text{of a dipole in field direction} \end{matrix} \times \begin{matrix} \text{[2] Number of dipoles with} \\ \text{this orientation} \end{matrix}$$

To do this, use Boltzmann statistics to obtain the number dn of dipoles with energy between E and $E + dE$

$$dn = c \exp(-E/kT) dE$$

where $k =$ Boltzmann's constant, $c =$ is a constant of the system and $T = T$ measured in Kelvin

How to find c ?

Integrate over all the energies, which must give N the total number of dipoles

$$N = \int_0^{\infty} c \exp(-E/kT) dE$$

Next week we will do this to calculate the susceptibility of a paramagnet. The important result we will get will be That susceptibility is temperature dependent with

$$\chi = C/T$$

This is Curie's law.

Summary

- Calculated the susceptibility for a diamagnet

$$\chi = -\mu_0 N Z \frac{e^2}{6m} \langle \rho^2 \rangle$$

- Argued that all materials have a small diamagnetic susceptibility resulting from the precession of electrons in a magnetic field.
- This precession results in a magnetic moment, and thus a susceptibility.
- Then discussed paramagnetism, resulting from atoms having a permanent magnetic moment.
- Sketched out how we will calculate paramagnetic susceptibility.