PHY331
Magnetism

Lecture 2

Last week…

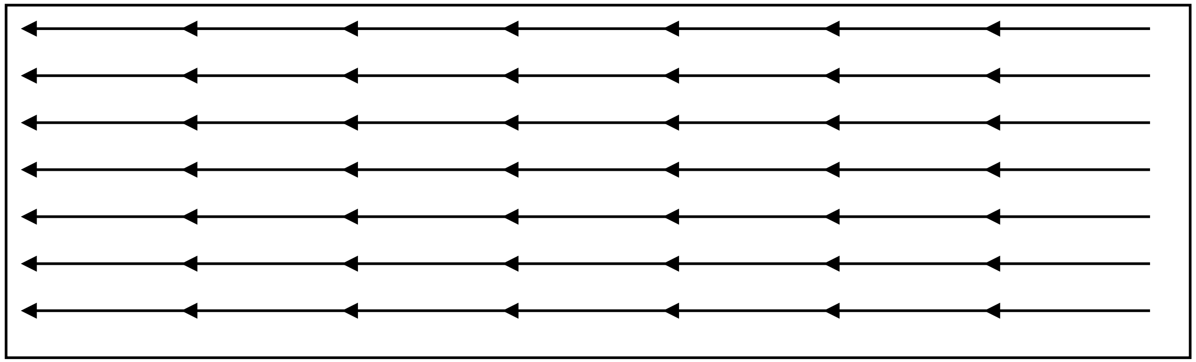
- Revised basic concepts (B and H, energy, torque and force in a magnetic field).
- Talked about different magnetic materials:
  - Diamagnetic
  - Paramagnetic
  - Ferromagnetic

This week…

- Derive magnetic dipole moment of a circulating electron.
- Discuss motion of a magnetic dipole in a constant magnetic field.
- Show that it precesses with a frequency called the Larmor precessional frequency

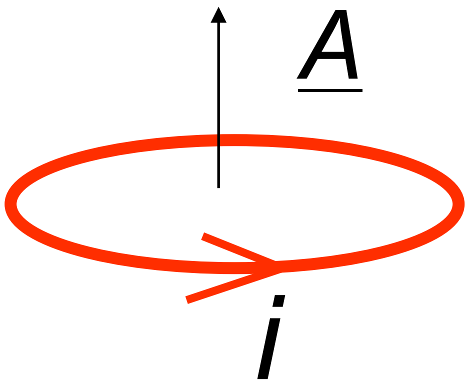
Magnetism on an atomic scale

The old "school picture" of a magnetic material is not so inaccurate if we replace the little arrows by atomic magnetic dipole moments



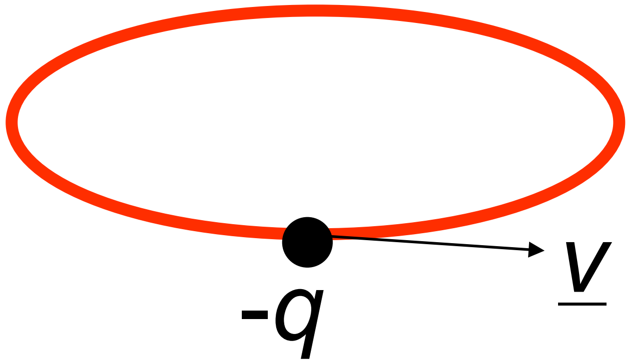
Force on a conductor
in a magnetic field.

$$\underline{m} = i\,\underline{A}$$



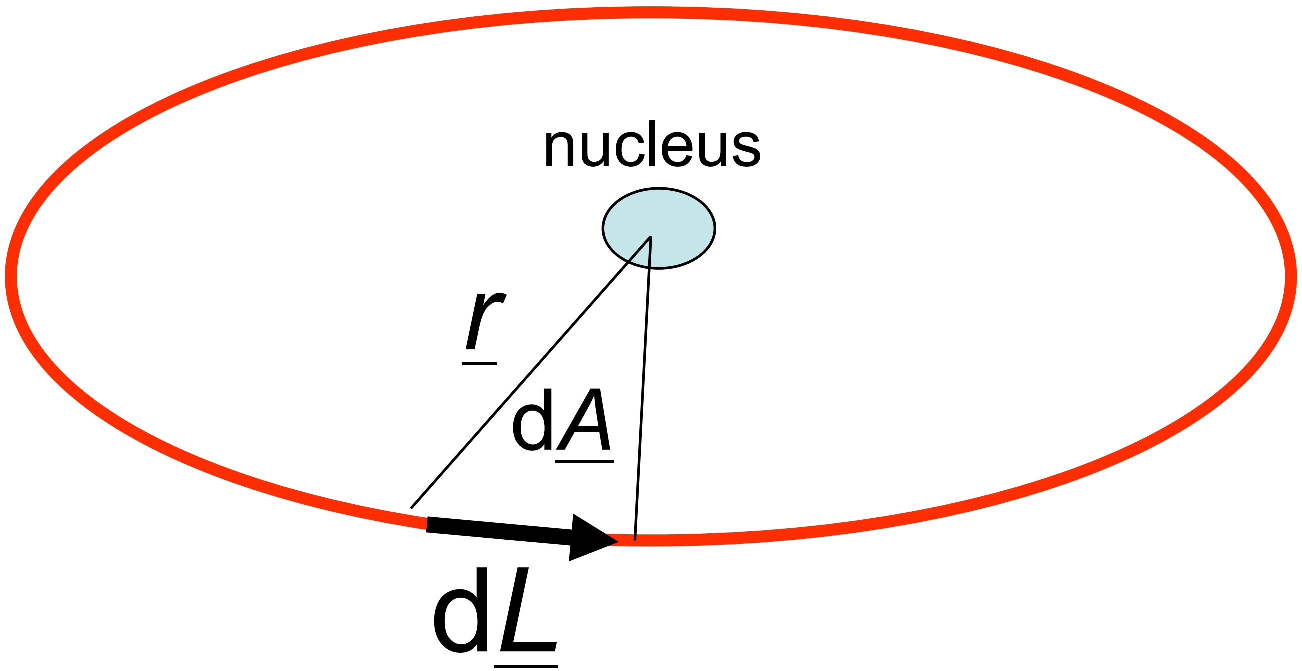
$$\underline{F} = i\,d\underline{l} \times \underline{B}$$

circulating electron



$$\underline{F} = q\,\underline{v} \times \underline{B}$$

The magnetic dipole moment of a circulating electron



Triangle area = 1/2 x base x height

$$d\underline{A} = \frac{1}{2}\underline{r} \times d\underline{L}$$

- start with $\underline{m} = i\underline{A}$

  and re-write as, $\underline{m} = \int i d\underline{A}$

  from the diagram, $\underline{m} = \int i \frac{\underline{r} \times d\underline{l}}{2}$

- since $i = -dq/dt$ and $\underline{v} = d\underline{l}/dt$

  then $i d\underline{l} = -\underline{v} dq$ so that,

$$\underline{m} = -\int \frac{\underline{r} \times \underline{v} dq}{2} = -\int \frac{\underline{r} \times \underline{v}}{2} dq$$

Remember that the angular momentum L of the circulating electron is

$$\underline{L} = m\,(\underline{r} \times \underline{v})$$

so that the magnetic dipole moment $\underline{\mathbf{m}}$ of the circulating electron is,

$$\underline{m} = -\frac{1}{2}\frac{\underline{L}}{m}\int dq \qquad \text{or} \qquad \underline{m} = -\frac{\underline{L}}{2m}\,q \qquad (1)$$

where q is the total charge circulating. Note the negative sign.

Magnetic dipole moment (*m*) dependent on *L*

- This is a really important result! It demonstrates that an electron in a stable atomic orbital having an angular momentum *L* has a magnetic dipole moment *m*.

- This is will be particularly important in the quantum theory of paramagnetism that we will cover later. Here, we will use the fact that both *L* (and *S*) are quantised

Atomic theory of Diamagnetism

Diamagnetic frog

- Diamagnetic materials have small *negative* values of the susceptibility χ. Magnetisation will oppose an applied magnetic field.
- ***ALL*** materials are diamagnetic.
- The small diamagnetic contribution to the magnetisation M is overwhelmed if the material under investigation is also a paramagnet or a ferromagnet.
- If all materials are diamagnetic……..it must involve some fundamental mechanism.

A very simplified picture… (Weber 1854)

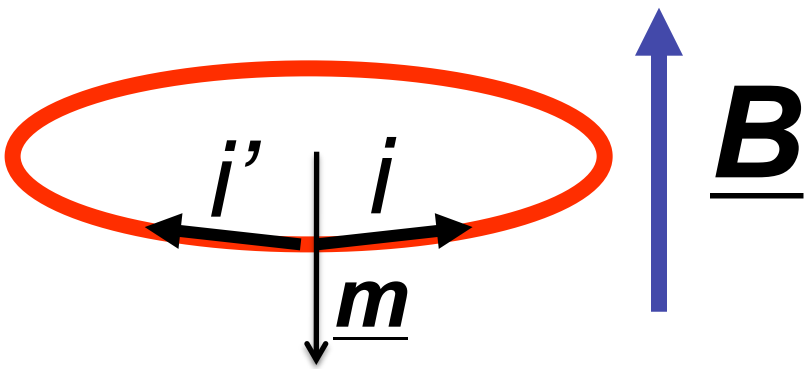
Apply a field *B* to a current loop.

The flux through the loop changes a ***back*** E.M.F. is induced.

The ***back*** E.M.F. and its associated current (*i*') oppose the applied *B* field (Lenz's Law).

If the resistance of the loop is ***small****, the new induced current continues for as long as the *B* field is applied (**e.g.* like a circulating electron?).

This circulating current results causes a magnetic dipole which opposes applied magnetic field.



'Atomic version' of Lenz's law

Can very simplistically think of diamagnetism as 'atomic-version' of Lenz's law (i.e. a response to a magnetic field that creates a current that 'flows' around each atom).

DO NOT consider this as a bulk phenomena - diamagnetism **ONLY** works at the scale of an individual atom!

Actually, the 'best' way to think of diamagnetism is a magnetic field caused by the <u>precession</u> of electrons in an applied magnetic field.

Precession: the motion of a magnetic dipole in a constant magnetic field

- Every circulating electron on every atom has a ***magnetic dipole moment***

- The magnetic field $\underline{B}$ produces a torque $\underline{\Gamma}$ on each dipole

- by Newton's Law, $$\frac{d\underline{L}}{dt} = \underline{\Gamma} = \underline{m} \times \underline{B} \quad (2)$$

rate of change of (angular) momentum = applied torque we already know $\underline{m} = \underline{m}(\underline{L})$ so that,

$$\frac{d\underline{L}}{dt} = -\frac{q}{2m}\underline{L} \times \underline{B}$$

(substitute (1) into (2))

must be the vector equation of motion *(it will be precessional motion)*

so for a B field in the vertical z direction only,

$$\frac{dL_x}{dt}\hat{i} + \frac{dL_y}{dt}\hat{j} + \frac{dL_z}{dt}\hat{k} = -\frac{q}{2m}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ L_x & L_y & L_z \\ 0 & 0 & B \end{vmatrix}$$

this gives,

$$\frac{dL_x}{dt} = -\frac{q}{2m} L_y B \qquad \frac{dL_y}{dt} = +\frac{q}{2m} L_x B \qquad \frac{dL_z}{dt} = 0$$

the component of L along z is constant,

say $$L_z = L\cos\alpha$$

so solving the x and y components gives,

$$\frac{d^2 L_x}{dt^2} = -\frac{qB}{2m}\frac{dL_y}{dt} = -\left(\frac{qB}{2m}\right)^2 L_x$$

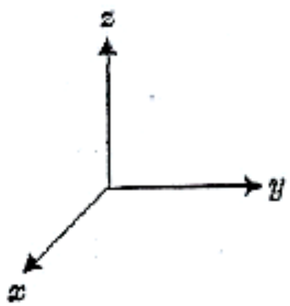
and a similar equation for L_y,

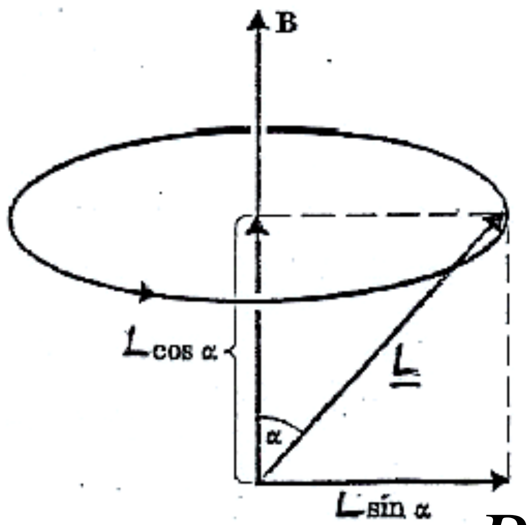
$$\frac{d^2L_y}{dt^2} = -\left(\frac{qB}{2m}\right)^2 L_y$$

(recall $\ddot{x} + \omega^2 x = 0$ for SHM)

i.e. the equations of two simple harmonic motions at 90º, so we have circular motion with a constant L_z component.

$$L_x = L\,sin(\alpha)\,cos(\omega t)$$
$$L_y = L\,sin(\alpha)\,sin(\omega t)$$
$$L_z = L\,cos(\alpha)$$





This is precessional motion with

$$\omega_L = \frac{qB}{2m}$$

where ω_L is known as the Larmor precessional frequency when $q = e$.

Summary

Saw that the magnetic dipole moment $\underline{m}$ of the circulating electron in a loop is

$$\underline{m} = -\frac{\underline{L}}{2m} q$$

Applied Newton's Law, to get SHM solutions of form

$$\frac{d\underline{L}}{dt} = \underline{\Gamma} = \underline{m} \times \underline{B}$$

$$\frac{d^2 L_x}{dt^2} = -\left(\frac{qB}{2m}\right)^2 L_x \qquad \frac{d^2 L_y}{dt^2} = -\left(\frac{qB}{2m}\right)^2 L_y$$

where $L_y = L \sin(\alpha) \sin(\omega t)$ *etc.. Precession in a field with a frequency* $\omega = qB/2m$