PHY331 Magnetism

Lecture 10

Last week…

- We saw that if we assume that the internal magnetic field is proportional to the magnetisation of the paramagnet, we can get a spontaneous magnetization for temperatures less than the Curie Temperature.
- We also found that the larger the field constant (relating internal field to magnetization), the higher the Curie Temperature.

This week….

- A quick ‘revision’ of the concept of the ‘density of states’ of a free electron in a metal / semiconductor.
- Calculation of the paramagnetic susceptability of free electrons (Pauli paramagnetism).
- Will show that paramagnetic suceptability of free electrons is very small and comparable to their diamagnetic susceptability.

Free electrons in a metal.

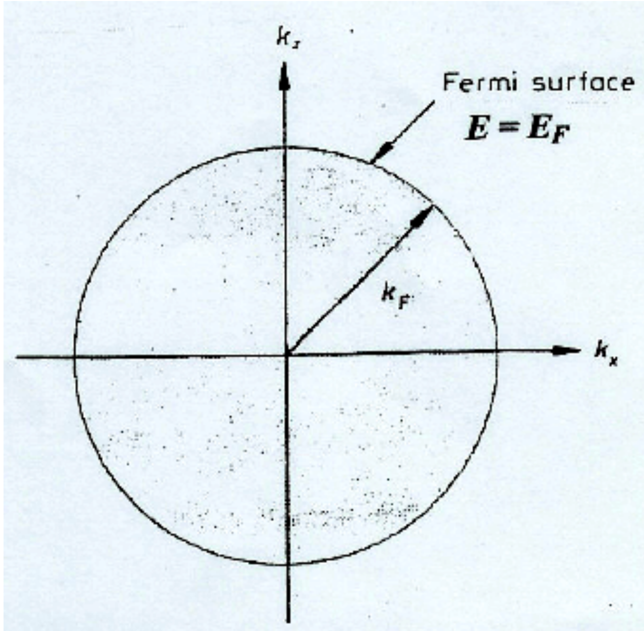
We have distribution of electrons have different energy (E) and wavevectors (k_x, k_y, k_z)

Energy doesn't depend on the individual k values but on the sum of the squares,

$$E_k = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right)$$

values which correspond to an energy less than E_{max}, are bounded by the ***surface of a sphere***

$$E_F = \frac{\hbar^2}{2m}k_F^2$$



E_{max} is the Fermi energy E_F and k_F the Fermi wave vector

The Fermi wave vector $k_F = \left(\frac{3\pi^2 N}{V} \right)^{\frac{1}{3}}$

depends only on the concentration of the electrons,

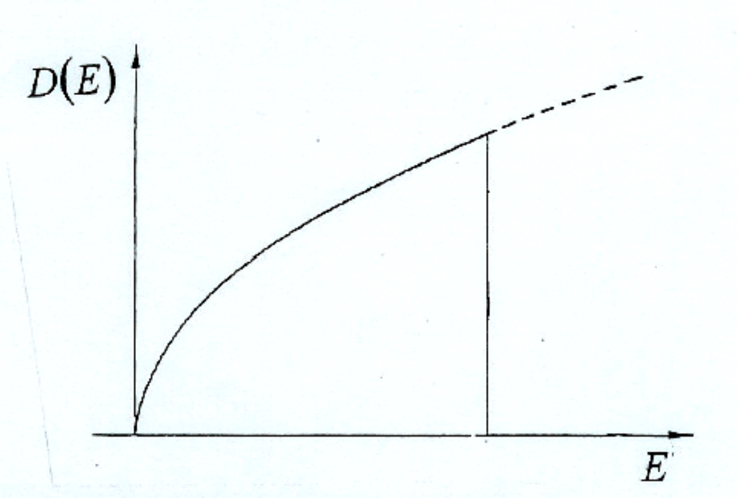
as does $E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{\frac{2}{3}}$ (use deBroglie)

re-arranging gives the number of states $N = \frac{V}{3\pi^2} \left(\frac{2mE}{\hbar^2} \right)^{\frac{3}{2}}$

hence the density of states per unit energy range *D(E)* is,

$$D(E) = \frac{dN}{dE} = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}}$$

a ***parabolic*** density of states is predicted.

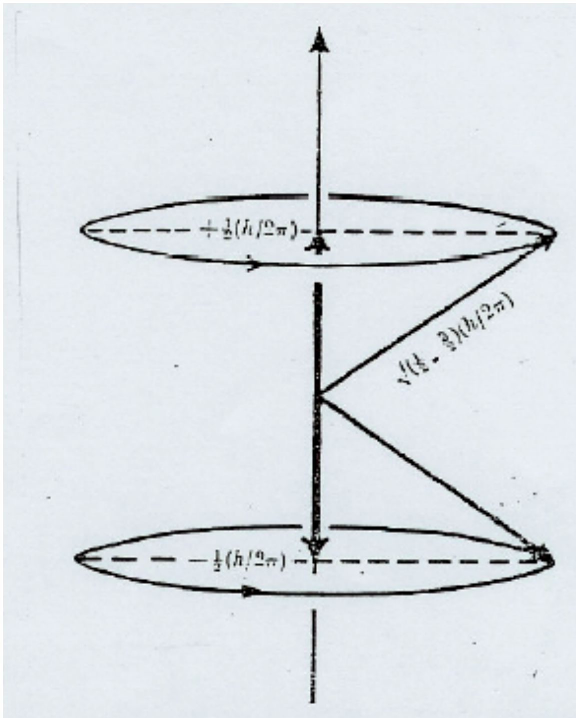


The paramagnetic susceptibility of free electrons - *Pauli paramagnetism*

The magnetic moment per atom is given by, $\mu_J = J g \mu_B$

For an electron with spin only, $L = 0$, $J = S$, $S = ½$, $g = 2$

$$\mu_{electron} = \frac{1}{2} 2\, \mu_B = 1 \mu_B$$



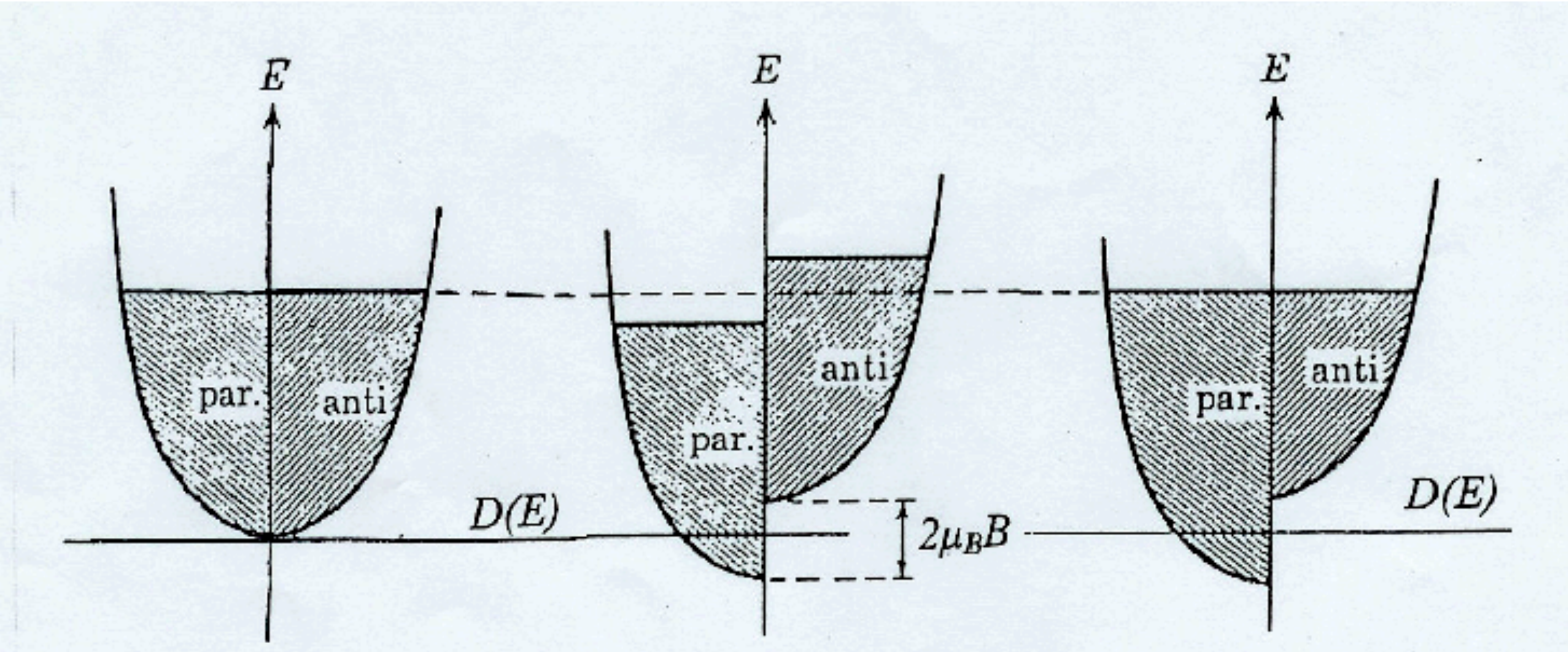
The magnetic energy of the electron in a field B is,

$$E = -\underline{\mu}_e \cdot \underline{B}$$

or, $E = -\mu_B B$ ***parallel*** to the field

and $E = +\mu_B B$ ***antiparallel*** to the field

Add and subtract these energies from the existing electron energies in the parabolic bands



Pauli paramagnetism - the approximate method, at $T = 0$ K

$\mu_B B$ is typically ***very small*** in comparison with $k\, T_F$

$$\mu_B B \ll k\, T_F$$

The ***number*** of electrons $\Delta n_\downarrow$ transferred from antiparallel states to parallel states is,

$$\Delta n_\downarrow = D_\downarrow(E_F)\, \mu_B\, B$$

The ***magnetisation*** M they produce is,

$$M = 2\, \Delta n_\downarrow\, \mu_B$$

since each electron has $1\mu_B$, so each ***transfer*** is worth $2\mu_B$

therefore, $$M = 2D_{\downarrow}(E_F)\,\mu_B^2\,B$$

and since obviously, $D_{\downarrow}(E_F) = \dfrac{D(E_F)}{2}$ and $B = \mu_0 H$

we have, $$\chi = \frac{M}{H} = \mu_0\,\mu_B^2\,D(E_F)$$

or,

$$\chi_{Pauli} = \mu_0\,\mu_B^2\,D(E_F)$$

we can express $D(E_F)$ as, $$D(E_F) \approx \frac{3}{2}\frac{N}{E_F}$$

so that, $$\chi_{Pauli} = \frac{3N\mu_0\,\mu_B^2}{2E_F}$$

and using the fictitious Fermi temperature, $E_F = k\,T_F$

then, $$\chi_{Pauli} = \frac{3N\mu_0\,\mu_B^2}{2kT_F} = \frac{\text{constant}}{T_F}$$

Let us compare with the Curie's law behaviour

(from Brillouin's treatment of the paramagnet)

$$\chi_{Curie} = \frac{\mu_0\,N\,g^2 J(J+1)\mu_B^2}{3kT}$$

When $J \Rightarrow S$, $S \Rightarrow ½$, $g \Rightarrow 2$

$$\chi_{Curie} = \frac{\mu_0 \, N \, \mu_B^2}{kT}$$

1) the ***paramagnetic susceptibility*** of the ***free electrons*** is ***smaller by a factor*** $\approx T_F/T$ than the atomic moment model, with,

$T_F \approx 6 \times 10^4$ K and $T_{room} \approx 3 \times 10^2$ K

2) the susceptibility is ***reduced*** by such a large factor, that it becomes comparable to the much smaller ***diamagnetic susceptibility*** of the free electrons,

$$\chi_{diamag} = -\frac{1}{3}\chi_{Pauli}$$

is Landau's result

Summary

- We saw how the application of a magnetic field resulted in an energy difference between electrons parallel and antiparallel to the magnetic field.
- This resulted in a transfer of electrons from antiparallel to parallel states, causing a net magnetisation.
- We could then derive an expression for the Pauli paramagnetic susceptibility.
- This predicted a susceptibility similar to the diamagnetic susceptibility of the free electrons.