Compounded interest

(symbols):-

F_{vc}: *Future value of compounded interst*

(Rules):-

$$F_{vc} = P_v(1+i)^t$$
$$t = \log_{1+i}\left(\frac{F_{vc}}{P_v}\right)$$
$$i = \left(\frac{F_{vc}}{P_v}\right)^{\frac{1}{t}} - 1$$

(Examples) :-

1) **How long dose it take to double your capital :-**
   a) **If you put it in an account paying compound interest at rate of 7.5% ?**

$$F_{vc} = 2P_v$$
$$P_v(1+i)^t = 2P_v$$
$$(1+i)^t = 2$$
$$(1.075)^t = 2$$
$$t = \log_{1.075} 2 = 9.6$$

$$\boxed{\frac{\ln\left(\frac{Fv}{Pv}\right)}{\ln(1+i)}}$$

   b) **What if the account pays simple interest ?**

$$F_{vs} = 2P_v$$
$$P_v(1+ti) = 2P_v$$
$$(1+ti) = 2$$
$$t = \frac{1}{i}$$
$$t = \frac{1}{0.075} = 13.\overline{33}$$

(Proofs):-

1) **Prove that $\mathbf{F_{vs} = P_v(1 + ti)}$:-**

$$
\begin{aligned}
F_{vs} &= P_v + I \\
&= P_v + tiP_v \\
&= P_v(1 + ti)
\end{aligned}
$$

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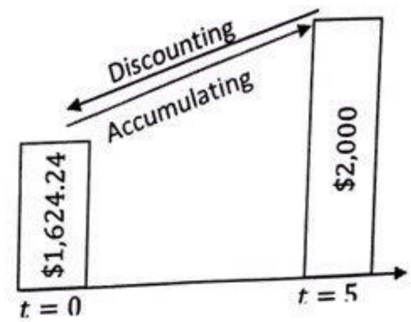
(FV)

2) **How much do you need to invest now to get \$2,000 after 5 years if the rate of interest is 4.25% ?**

$$2{,}000 = \overset{?}{P_v}(1 + 0.0425)^5$$

$$P_v = \frac{2{,}000}{(1.0425)^5} = \$1{,}624.24 \Rightarrow P_v$$



(Proofs):-

1) **Prove that $F_{vc} = P_v(1 + i)^t$:-**

$$\begin{aligned} After\ 1\ year &= P_v + I \\ &= P_v + tiP_v \\ &= P_v(1 + ti) \end{aligned}$$

$$\begin{aligned} After\ 2\ year &= P_v(1 + i) + P_v(1 + i)i \\ &= P_v(1 + i) + P_v(i + i^2) \\ &= P_v(1 + i + i + i^2) \Leftrightarrow P_v(i^2 + 2i + 1) \\ &= P_v(1 + i)(1 + i) \\ &= P_v(1 + i)^2 \end{aligned}$$

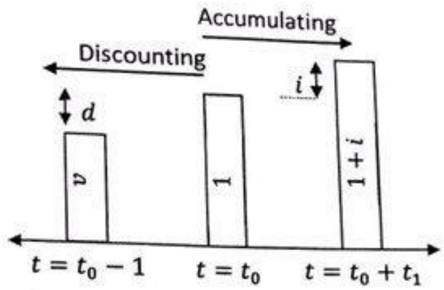
$$\boxed{(a \pm b)^2 = (a^2 \pm 2ab + b^2)}$$

$$\begin{aligned} After\ 3\ year &= P_v(1 + i)^2 + P_v(1 + i)^2 i \\ &= P_v(1 + 2i + i^2) + P_v(1 + 2i + i^2)i \\ &= P_v(1 + 2i + i^2) + P_v(i + 2i^2 + i^3) \\ &= P_v(1 + 2i + i^2 + i + 2i^2 + i^3) \Leftrightarrow P_v(i^3 + 3i^2 + 3i + 1) \\ &= P_v(1 + i)^3 \end{aligned}$$

$$\boxed{(a + b)^3 = (a^3 + 3a^2b + 3ab^2 + b^3)}$$

so after t years will be $F_{vc} = P_v(1 + i)^t$

Discounting



(symbols):-

$1 + i$: *growth factor*
v: *discount factor*
d: *Effective discount rate*

(Rules):-

$$v = \frac{1}{1+i}$$
$$d = 1 - v$$
$$i = \frac{1}{1-d}$$
$$P_v = F_{vc}(1-d)^t$$
$$P_v = F_{vs}(1-td).$$
$$d = iv$$

(Examples) :-

مقابل قرض   المقترض يوافق على أن يعيد

1) In return for a loan of £100 a borrower agrees to repay £110 after seven months :-

a) Find the rate of interest per annum.

$$110 = 100\,(1+i)^{\frac{7}{12}}$$

$$(1+i)^{\frac{7}{12}} = \frac{110}{100}$$ هنا بين الدين

$$1 + i = \left(\frac{110}{100}\right)^{\frac{12}{7}}$$

$$i = \left(\frac{110}{100}\right)^{\frac{12}{7}} - 1$$

$$i = 18\%$$

b) Find the rate of discount per annum.

$$100 = 110(1-d)^{\frac{7}{12}}$$

$$(1-d)^{\frac{7}{12}} = \frac{100}{110}$$

$$1-d = \left(\frac{100}{110}\right)^{\frac{12}{7}}$$

$$d = -\left(\frac{100}{110}\right)^{\frac{12}{7}} + 1$$

$$d = 15\%$$

c) Shortly after receiving the loan the borrower requests that he be allowed to repay the loan by a payment of \$50 on the original settlement date and a second payment six months after this date. Assuming that the lender agrees to the request and that the calculation is made on the original interest basis, find the amount of the second payment under the revised transaction.

$$P_v = F_{vc1} + F_{vc2}$$

$$100 = 50(1+i)^{\frac{-7}{12}} + F_{vc2}(1+i)^{\frac{-13}{12}}$$

$$100 = 50(1.18)^{\frac{-7}{12}} + F_{vc2}(1.18)^{\frac{-13}{12}}$$

$$F_{vc2}(1.18)^{\frac{-13}{12}} = 100 - 50(1.18)^{\frac{-7}{12}}$$

$$F_{vc2} = \frac{100 - 50(1.18)^{\frac{-7}{12}}}{(1.18)^{\frac{-13}{12}}}$$

$$F_{vc2} = 65.32$$

2) What is the present value of \$6,000 due simple discount in the month $d = 8\%$?

$$P_v = 6{,}000\left(1 - \frac{1}{12}0.08\right) = \$5{,}960$$

3) **The commercial rate of discount per annum is 18% (this means that simple discount is applied with a rate of 18%) :-**

   a) **We borrow a certain amount. The loan is settled by a payment of S1,000 after three months. Compute the amount borrowed and the effective annual rate of discount.**

$$P_v = 1{,}000\left(1 - \frac{3}{12}(0.18)\right) = 955$$

   b) **Now the loan is settled by a payment of S1,000 after nine months. Answer the same question**

$$P_v = 1{,}000\left(1 - \frac{9}{12}(0.18)\right) = 865$$

(Proofs):-

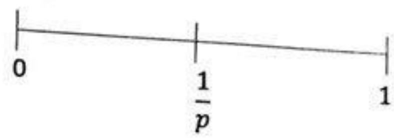
1) **Prove that $d = iv$:-**

$$\begin{aligned} d &= 1 - v \\ &= 1 - \frac{1}{1+i} \\ &= \frac{i}{1+i} \\ &= iv \end{aligned}$$

2) **Prove that $P_v = F_{vc}(1-d)^t$:-**

$$\begin{aligned} F_{vc} &= P_v(1+i)^t \\ P_v &= \frac{F_{vc}}{(1+i)^t} \\ &= F_{vc}v^t \\ &= F_{vc}(1-d)^t \end{aligned}$$

Interest payable



$p = 1$	**Yearly**
$p = 2$	Semi-annually
$p = 4$	Quarterly
$p = 12$	Monthly
$p = 52$	Weakly
$p = 365$	Daily

(symbols):-

i^p: *nominal interest rate* سعر الفائدة الاسمي

i_p: *interest rate per conversion period* سعر الفائدة في فترة التحويل

d^p: *nomial discount rate* سعر الخصم الاسمي

(Rules):-

$$F_{vc} = P_v(1 + i_p)^{pt}$$

$$F_{vc} = P_v\left(1 + \frac{i^p}{p}\right)^{pt}$$

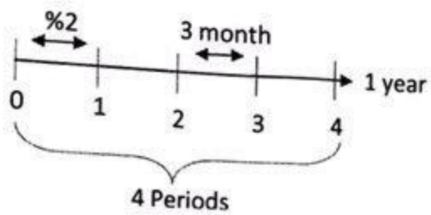
$$i_p = \frac{i^p}{p}$$

$$d = 1 - \left(1 - \frac{d^p}{p}\right)^p$$

$$i = (1 + i_p)^{pt} - 1$$

(Examples) :-

1) **Suppose that you save \$1,000 in a account that pays 2% interest every quarter, How much do you have in one year in this account ?**



$$F_{vc} = 1{,}000(1 + 0.02)^4 = 1{,}082.43$$

2) **What will deliver a high future value after one year deposit of \$1,000 attracting interest at %15 compounded daily or %15.5 compounded semi-annual ?**

$$F_{vc} = 1{,}000\left(1 + \frac{0.15}{365}\right)^{365} = 1612$$
$$F_{vc} = 1{,}000\left(1 + \frac{0.155}{2}\right)^{2} = 1611$$

than the daily > semi-annual

3) **What initial investment subject to annual compounding at %12 is needed to produce \$1,000 after 2 year ?**

$$1{,}000 = P_v(1 + 0.12)^2$$
$$P_v = \frac{1{,}000}{(1.12)^2} = 797.2$$

4) **Compare the following loans :-**

   a) **A loan charging an annual effective rate (AER) 9%?**

$$i = 9\%$$

/

b) 8.75% compounded quarterly?

$$i = \left(1 + \frac{0.0875}{4}\right)^4 = 9.04\%$$

c) 8.5% Payable in convertible monthly?

$$d = 1 - \left(1 - \frac{0.085}{12}\right)^{12} = 8.2\%$$

$$i = \frac{1}{1 - 0.082} - 1 = 8.9\%$$

5) What is the interest rate if deposit subject to annual compounding is doubled after 10 years?

$$F_{vc} = 2P_v$$
$$P_v(1+i)^t = 2P_v$$
$$(1+i)^t = 2$$
$$(1+i)^{10} = 2$$
$$\sqrt[10]{(1+i)^{10}} = \sqrt[10]{2}$$
$$1 + i = \sqrt[10]{2}$$
$$i = \sqrt[10]{2} - 1 = 7.2\%$$

(Proofs):-

1) **Prove that $F_{vc} = P_v(1 + i_p)^{pt}$**

$$After\ 1\ quarter = P_v i_p P_v$$

$$= P_v(1 + i_p)$$

$$After\ 2\ quarter = P_v(1 + i_p) + P_v(1 + i_p)i_p$$

$$= P_v(1 + i_p) + P_v(i_p + i_p{}^2)$$

$$= P_v(1 + i_p + i_p + i_p{}^2) \Leftrightarrow P_v(i_p{}^2 + 2i_p + 2)$$

$$\boxed{(a + b)^2 = (a^2 + 2ab + b^2)}$$

$$= P(1 + i_q)^2$$

so after t years will be $F_{vc} = P(1 + i_q)^{pt}$

Force of interest

(symbols):-

δ: *Force of interest*

i_e: *Effective interest rate of force of interest*

(Rules):-

$$\delta = \ln(1 + i)$$

$$i_e = e^{\delta} - 1$$

$$F_{vc} = P_v e^{it}$$

$\in(e^{(i \times n)})$

Mathematical of finance (1)

(Main objective):-

Introduction to mathematical modelling of financial markets with particular emphasis to the time value of money procedures.

(Syllabus):-

Chapter 1 Interest rate

- Simple interest
- Compound interest
- Discounting
- Interest payable
- Force of interest
- Equation of value

Chapter 2 Annuity

- Annuity immediate - due Annuity immediate
- Perpetuity
- Continuity annuity
- Arithmetic and Geometric annuity

Chapter 3 Loans

- Interest rate
- Time of loans
- Payment period
- Outstanding balance
- Amortization methods
- Sampling and methods

Chapter 4 Bonds

- Price
- Book value
- Fact value
- Redemption value
- Yield rate
- Common rate

Chapter 1

Simple interest

(symbols):-

P_v: *Present value*
i: *Effective interest rate* معدل الفائدة الحقيقي
t: *Time*
F_{vs}: *Future value of simple interst*
I: *Intersest earned* $\Rightarrow$ العوائد المحققة
$P.a$: *Per annum*

(Rules):-

$$I = t \cdot i \cdot P_v$$
$$F_{vs} = P_v(1 + ti)$$

(Examples):-

P_v

1) **Consider deposit of $4,000 for two years the interest rate is 5.5% P.A what will be the deposit of the 2 years ?**

$$F_{vs} = 4{,}000\big(1 + 2(0.055)\big) = \$4{,}440$$

2) **How much interest do you get if you put $1,000 for two years in a savings account that pays simple interest at a rate of 9% :-**

   a) **per annum.**

   $$I = (2)(1{,}000)(0.09) = \$180$$

   b) **And if you leave it in the account for only half a year.**

   $$I = \left(\frac{1}{2}\right)(1{,}000)(0.09) = \$45$$

?

3) **Suppose you put $1,000 in a savings account paying simple interest at 9% per annum for one year. Then, you withdraw the money with interest and put it for one year in another account paying simple interest at 9%. How much do you have in the end?**

$$F_{vs} = 1{,}000\big(1 + 1(0.09)\big) = \$1{,}090$$
$$I = (1)(0.09)(1{,}090) = \$98.1$$
$$Total = F_{vs} + I = \$1188.1$$