a. Let x denote the number of calculators in this package of 10 that will be returned for refund or replacement within a 2-year period. Using the binomial probabilities table, obtain the probability distribution of x and draw a graph of the probability distribution. Determine the mean and standard deviation of x.

b. Using the probability distribution of part a, find the probability that exactly 2 of the 10 calculators will be returned for refund or replacement within a 2-year period.

5.59 A fast food chain store conducted a taste survey before marketing a new hamburger. The results of the survey showed that 70% of the people who tried this hamburger liked it. Encouraged by this result, the company decided to market the new hamburger. Assume that 70% of all people like this hamburger. On a certain day, eight customers bought it for the first time.

a. Let x denote the number of customers in this sample of eight who will like this hamburger. Using the binomial probabilities table, obtain the probability distribution of x and draw a graph of the probability distribution. Determine the mean and standard deviation of x.

b. Using the probability distribution of part a, find the probability that exactly three of the eight customers will like this hamburger.

5.5 The Hypergeometric Probability Distribution

In Section 5.4, we learned that one of the conditions required to apply the binomial probability distribution is that the trials are independent, so that the probabilities of the two outcomes or events (success and failure) remain constant. If the trials are not independent, we cannot apply the binomial probability distribution to find the probability of x successes in n trials. In such cases we replace the binomial by the **hypergeometric probability distribution**. Such a case occurs when a sample is drawn without replacement from a finite population.

As an example, suppose 20% of all auto parts manufactured at a company are defective. Four auto parts are selected at random. What is the probability that three of these four parts are good? Note that we are to find the probability that three of the four auto parts are good and one is defective. In this case, the population is very large and the probability of the first, second, third, and fourth auto parts being defective remains the same at .20. Similarly, the probability of any of the parts being good remains unchanged at .80. Consequently, we will apply the binomial probability distribution to find the probability of three good parts in four.

Now suppose this company shipped 25 auto parts to a dealer. Later, it finds out that 5 of those parts were defective. By the time the company manager contacts the dealer, 4 auto parts from that shipment have already been sold. What is the probability that 3 of those 4 parts were good parts and 1 was defective? Here, because the 4 parts were selected without replacement from a small population, the probability of a part being good changes from the first selection to the second selection, to the third selection, and to the fourth selection. In this case we cannot apply the binomial probability distribution. In such instances, we use the hypergeometric probability distribution to find the required probability.

Hypergeometric Probability Distribution

Let

N = total number of elements in the population

r = number of successes in the population

$N - r$ = number of failures in the population

n = number of trials (sample size)

x = number of successes in n trials

$n - x$ = number of failures in n trials

The probability of x successes in n trials is given by

$$P(x) = \frac{{}_{r}C_{x}\ {}_{N-r}C_{n-x}}{{}_{N}C_{n}}$$

Examples 5–15 and 5–16 provide applications of the hypergeometric probability distribution.

■ EXAMPLE 5–15

Calculating probability by using hypergeometric distribution formula.

Brown Manufacturing makes auto parts that are sold to auto dealers. Last week the company shipped 25 auto parts to a dealer. Later, it found out that 5 of those parts were defective. By the time the company manager contacted the dealer, 4 auto parts from that shipment had already been sold. What is the probability that 3 of those 4 parts were good parts and 1 was defective?

Solution Let a good part be called a success and a defective part be called a failure. From the given information,

N = total number of elements (auto parts) in the population = 25

r = number of successes (good parts) in the population = 20

$N - r$ = number of failures (defective parts) in the population = 5

n = number of trials (sample size) = 4

x = number of successes in four trials = 3

$n - x$ = number of failures in four trials = 1

Using the hypergeometric formula, we calculate the required probability as follows:

$$P(x = 3) = \frac{{}_rC_x \, {}_{N-r}C_{n-x}}{{}_NC_n} = \frac{{}_{20}C_3 \, {}_5C_1}{{}_{25}C_4} = \frac{\dfrac{20!}{3!(20-3)!} \cdot \dfrac{5!}{1!(5-1)!}}{\dfrac{25!}{4!(25-4)!}}$$

$$= \frac{(1140)(5)}{12{,}650} = \mathbf{.4506}$$

Thus, the probability that 3 of the 4 parts sold are good and 1 is defective is .4506.

In the above calculations, the values of combinations can either be calculated using the formula learned in Section 4.6.3 (as done here) or by using a calculator. ■

■ EXAMPLE 5–16

Calculating probability by using hypergeometric distribution formula.

Dawn Corporation has 12 employees who hold managerial positions. Of them, 7 are female and 5 are male. The company is planning to send 3 of these 12 managers to a conference. If 3 managers are randomly selected out of 12,

(a) find the probability that all 3 of them are female

(b) find the probability that at most 1 of them is a female

Solution Let the selection of a female be called a success and the selection of a male be called a failure.

(a) From the given information,

N = total number of managers in the population = 12

r = number of successes (females) in the population = 7

$N - r$ = number of failures (males) in the population = 5

n = number of selections (sample size) = 3

x = number of successes (females) in three selections = 3

$n - x$ = number of failures (males) in three selections = 0

Using the hypergeometric formula, we calculate the required probability as follows:

$$P(x = 3) = \frac{{}_rC_x \, {}_{N-r}C_{n-x}}{{}_NC_n} = \frac{{}_7C_3 \, {}_5C_0}{{}_{12}C_3} = \frac{(35)(1)}{220} = \mathbf{.1591}$$

Thus, the probability that all 3 of the managers selected are female is .1591.

(b) The probability that at most 1 of them is a female is given by the sum of the probabilities that either none or 1 of the selected managers is a female.

To find the probability that none of the selected managers is a female, we use

N = total number of managers in the population = 12
r = number of successes (females) in the population = 7
$N - r$ = number of failures (males) in the population = 5
n = number of selections (sample size) = 3
x = number of successes (females) in three selections = 0
$n - x$ = number of failures (males) in three selections = 3

Using the hypergeometric formula, we calculate the required probability as follows:

$$P(x = 0) = \frac{{}_rC_x \, {}_{N-r}C_{n-x}}{{}_NC_n} = \frac{{}_7C_0 \, {}_5C_3}{{}_{12}C_3} = \frac{(1)(10)}{220} = .0455$$

To find the probability that 1 of the selected managers is a female, we use

N = total number of managers in the population = 12
r = number of successes (females) in the population = 7
$N - r$ = number of failures (males) in the population = 5
n = number of selections (sample size) = 3
x = number of successes (females) in three selections = 1
$n - x$ = number of failures (males) in three selections = 2

Using the hypergeometric formula, we obtain the required probability as follows:

$$P(x = 1) = \frac{{}_rC_x \, {}_{N-r}C_{n-x}}{{}_NC_n} = \frac{{}_7C_1 \, {}_5C_2}{{}_{12}C_3} = \frac{(7)(10)}{220} = .3182$$

The probability that at most 1 of the 3 managers selected is a female is

$$P(x \leq 1) = P(x = 0) + P(x = 1) = .0455 + .3182 = \mathbf{.3637}$$ ■

EXERCISES

■ CONCEPTS AND PROCEDURES

5.60 Explain the hypergeometric probability distribution. Under what conditions is this probability distribution applied to find the probability of a discrete random variable x? Give one example of the application of the hypergeometric probability distribution.

5.61 Let $N = 8$, $r = 3$, and $n = 4$. Using the hypergeometric probability distribution formula, find
a. $P(x = 2)$ **b.** $P(x = 0)$ **c.** $P(x \leq 1)$

5.62 Let $N = 14$, $r = 6$, and $n = 5$. Using the hypergeometric probability distribution formula, find
a. $P(x = 4)$ **b.** $P(x = 5)$ **c.** $P(x \leq 1)$

5.63 Let $N = 11$, $r = 4$, and $n = 4$. Using the hypergeometric probability distribution formula, find
a. $P(x = 2)$ **b.** $P(x = 4)$ **c.** $P(x \leq 1)$

5.64 Let $N = 16$, $r = 10$, and $n = 5$. Using the hypergeometric probability distribution formula, find
a. $P(x = 5)$ **b.** $P(x = 0)$ **c.** $P(x \leq 1)$

■ APPLICATIONS

5.65 An Internal Revenue Service inspector is to select 3 corporations from a list of 15 for tax audit purposes. Of the 15 corporations, 6 earned profits and 9 incurred losses during the year for which the tax returns are to be audited. If the IRS inspector decides to select 3 corporations randomly, find the probability that the number of corporations in these 3 that incurred losses during the year for which the tax returns are to be audited is
a. exactly 2 **b.** none **c.** at most 1