KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS

QUANTUM MECHANICS H.W №6

Salwa Alsaleh

PROBLEM (1) THE HYDROGEN ATOM WAVEFUNCTION

1. Let $\Psi(x)$ be the total wavefunction of the electron in the hydrogen atom. Write the explicit expression of it including all the dependencies on quantum numbers.
2. construct a table of all the quantum numbers of the H-atom with their ranges.
3. What is the total number of electrons allowed to have a single principle quantum number ?

PROBLEM (2) CONSERVED QUANTITIES OF SIMPLIFIED H-ATOM

Use the expression for the Hamiltonian of the H-atom

$$\hat{H} = H_{free} + \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r^2}$$

1. Define the quantity $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$, called the fine structure constant. What is the approximate numerical value and physical dimension ? write the Hamiltonian in terms of the fine structure constant
2. List the set of mutually commuting operators of H-atom. Compare this list to the quantities you expect to be conserved in the classical atom.
3. Construct the energy eigenstates, do they carry different quantum numbers? (This is called degeneracy)

PROBLEM (3) ZEEMAN EFFECT

If the atom was under the influence of a magnetic field **B**, a splitting of its spectrum is observed experimentally.(Note that we assume that $\mathbf{B} = B\mathbf{e}_z$)

1. The electron is a negatively charged particle. as it orbits the atom, we expect it to have an ' orbital magnetic moment'
$$\hat{\mu} = g_\ell \frac{e\hbar}{2m_e}\hat{\mathbf{L}}$$
What is the spectrum of this operator ?

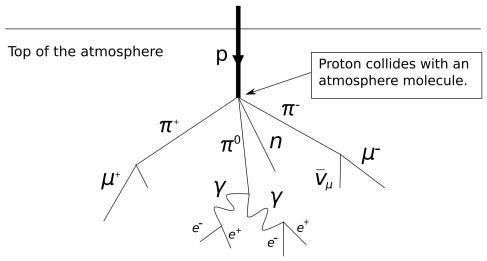
2. The coupling between the external magnetic field and the magnetic moment is equal to $\mathbf{B}\cdot\mu$, write the Hamiltonian for this coupling explicitly, call it $\hat{H}_{int}$

3. Does $\hat{H}_{int}$ commute with $\hat{L}_z$ and $\hat{L}$?, what do you expect t happen with the degeneracy discussed in the previous question ?

4. Compute the energy shift due to $\hat{H}_{int}$.

PROBLEM (4) THE MASS OF THE MUON

The muon is a negatively charged particle identical to the electron, but about 400 times heaver. It was first discovered in the cosmic rays as in the figure In order to measure its mass,



we ' somehow' attach it to a Hydrogen atom instead of the electron forming a *H-Muonium* MuH. Use the relation of the energy levels of the Hydrogen atom.

$$E_n \frac{m_e\alpha^2c^2}{2n^2}$$

To measure the mass of the muon from the change of wavelength of Balmer series transition for example