KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS

QUANTUM MECHANICS H.W №4

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PROBLEM (1)

We can represent the angular momentum operators $\hat{L}_i$ as matrices as follows

$$\hat{L}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{L}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \hat{L}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Using this matrix representation Show that

1. $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$
2. $\hat{L}^2 = 2\hbar^2\hat{I}$, where $\hat{I}$ is the identity matrix.

PROBLEM (2)

Given the operator $\hat{L}_+ = \hat{L}_x + i\hat{L}_y$

1. Is it hermitian ?
2. Express it in the matrix representation, and find its eigenvalues.
3. Express it in the x representation.
4. let $\Psi = \hat{L}_+\Phi_{\ell,m}$, find Ψ in terms of the eigenstates $\Phi_{\ell,m}$.

PROBLEM (3)

Show that the spherical harmonics Y_1^0 and Y_1^1 are orthogonal.

PROBLEM (4)

An electron having $\ell = 2$, write and draw all the L_z eigenstates m_ℓ for this electron, indicating the angles.

PROBLEM (5)

Show that

$$\sigma_x\sigma_y + \sigma_y\sigma_x = 0$$

PROBLEM (6)

Given the spin state $\chi = \begin{pmatrix} i\sqrt{2} \\ \sqrt{2} \end{pmatrix}$

1. Normalise it
2. write it in terms of the eigenstates α and β.
3. Calculate ΔS_x w.r.t χ.