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QUANTUM MECHANICS H.W №4

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PROBLEM (1) THE ALGEBRA OF ANGULAR MOMENTUM

Construct a table that contains the set of operators of angular momentum L^2, L_z, L_y, L_x with the operation of commutation between each possible pair of them. And an other one containing the set $L_z, L_+, L-$.
What are the main observations from these two tables ?

PROBLEM (2) PARITY AND CENTRAL-BODY PROBLEM

We define the **Parity** transformation as an operator Π. That transforms the position vector ' operator'

$$\vec{r} \longrightarrow -\vec{r}$$

Recall that to transform an operator A by the parity we preform $\Pi A \Pi^{-1}$

1. Show that the linear momentum operator $\vec{p}$ transform under parity as

$$\vec{p} \longrightarrow -\vec{p}$$

   What so called the radial vector

2. Show that the angular momentum operator $\vec{L}$ transform under parity as

$$\vec{L} \longrightarrow \vec{L}$$

   What so called ' axial-vector' .

3. The eigenstates of the parity operator satisfy the relations:

$$\Pi|+\rangle = +\,|+\rangle$$
$$\Pi|-\rangle = -\,|-\rangle$$

   Does L commute with Π ? *Hint :* Make use of the relation $\Pi L \Pi^{-1} = L$

4. Since in a Central body problem, we have seen that the set of mutually operators are H, L^2, L_z Is parity conserved observable in this problem ?

Problem (3) Building the angular momentum states

Use the ladder operators L_- and/or L_+ to construct the states of $\ell = 2$, then compute the angle of the angular momentum vector with respect to the $z-$ direction.

Problem (4) Orthogonality of spherical harmonics

Show that the spherical harmonics $Y_0^1(\theta,\phi)$ and Y_1^1 are orthogonal

$$Y_0^1 = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\vartheta$$
$$Y_1^1 = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\vartheta\, e^{i\varphi}$$