KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS AND ASTRONOMY

ADVANCED CLASSICAL MECHANICS (508 PHYS)
PROBLEM SET 1

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PROBLEM (1)

Given a vector field $\mathbf{F} = y\mathbf{e}_x - x\mathbf{e}_y$.

1. Plot this field using MATHMATICA.
2. Show that this field is non-conservative.

PROBLEM (2)

Prove the following identity (you may use MATHMATICA)

$$rot(rot(\mathbf{A})) = \mathbf{A}\Delta + div(\mathbf{A})\nabla \tag{0.1}$$

PROBLEM (3)

The vector potential for a magnetic dipole $\mathbf{m}_D$ is given by

$$\mathbf{A} = \frac{\mu_0}{2\pi}\frac{\mathbf{m}_D \wedge \mathbf{r}}{r^3} \tag{0.2}$$

Find the magnetic flux field $\mathbf{B} = rot(\mathbf{A})$.

PROBLEM (4)

Given the equation of motion $\dot{\mathbf{v}} = \omega \wedge \mathbf{v}$ for a charged particle rotating in a magnetic field with cyclotron frequency $|\omega|$, show that the equation can be rewritten in any of the following equivalent forms

$$\dot{v}^i = \Omega^i_j v^j$$
$$\frac{\partial |v\rangle}{\partial t} = \hat{\Omega}|v\rangle$$

with $\Omega = \epsilon_{ijk}\omega^k$

PROBLEM (5)

A particle of mass m confined to the surface of the two-sphere has the two-dimensional coordinate map, valid in the northern hemisphere, given by

$$x = x \quad , y = y \quad , z = \sqrt{R^2 - x^2 - y^2} \tag{0.3}$$

Where R is radius of the sphere. Find the kinetic energy of the particle in terms of these coordinates maps.

PROBLEM (6)

Calculate the two-dimensional metric tensor for the torus given by

$$x(\theta,\phi) = (a + b\cos\phi)\cos\theta,$$
$$y(\theta,\phi) = (a + b\cos\phi)\sin\theta,$$
$$z(\theta,\phi) = b\sin\phi.$$

PROBLEM (7)

A point particle of mass m is constrained to move on a wire that is in the shape of an ellipse

$$x = a\cos\phi,$$
$$y = b\sin\phi,$$
$$z = 0$$

1. Plot the elliptic surface, for any value of a and b.
2. Find the kinetic energy in terms of parameter ϕ.
3. Find an integral expression for its path length for one complete cycle around an ellipse. (*Hint*: the arc-length of an ellipse can be written in terms of a complete elliptic integral of the second kind.)

PROBLEM (8)

Consider a point in a two-dimensional plane $P(x, y) = x\mathbf{e}_1 + y\mathbf{e}_2$ that is mapped to a skewed coordinate system given by

$$u = (x/2\cos\alpha + y/2\sin\alpha),$$
$$v = (-x/2\cos\alpha + y/2\sin\alpha)$$

Where $\alpha = const$

1. Find the covariant basis λ_i for the new coordinates.
2. Compute the metric for this surface.
3. Find the contravariant basis

PROBLEM (9)

We have $S^{\mu\nu}$ a symmetric tensor, and $A^{\mu\nu}$ an anti-symmetric tensor. Show that $S^{\mu\nu}A_{\mu\nu} = 0$.