KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS AND ASTRONOMY

QUANTUM MECHANICS (453 PHYS)
PROBLEM SET 1

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PROBLEM (1)

If Ψ is a wavefunction for a quantum system, if we preformed the transformation $\Psi \to \Psi' = e^{i\phi}\Psi$ $\phi = const.$
Show that this transformation does not affect the physical description.

PROBLEM (2)

Given the wavefunction

$$\psi(x) = Ne^{-\xi x^2/2}$$

Compute the uncertainty in position for this system Δx
Provided that

$$\int_{-\infty}^{+\infty} x^{2n} e^{-\lambda x^2} = \frac{1 \cdot 3 \cdots (2n-1)}{(2\lambda^n)} \times \sqrt{\frac{\pi}{\lambda}}$$

PROBLEM (3)

A quantum system state vector is given by the eigenvector decomposition

$$\Psi = A(\Phi_1 + 2\Phi_2 + \Phi_3)$$

The corresponding eigenvalues are $\lambda_1 = -1, \lambda_2 = 0$ and $\lambda_3 = 1$, respectively.

1. Find the normalisation constant A
2. If $\hat{\Omega}$ is the operator that posses the eigenvectors Φ_i , compute its expected value w.r.t the full state Φ.

PROBLEM (4)

Show - using the position representation- that for any given $\psi(x)$ a real wavefunction , the quantity $\langle \hat{p} \rangle$ always vanishes.

PROBLEM (5)

Show that the function e^{-ikx} and e^{+ikx} are eigenfunctions to the momentum operator.

PROBLEM (6)

Find the eigenfunctions to the operator

$$\hat{O} = \hat{x} - \frac{\hat{p}}{i\hbar}$$

PROBLEM (7)

Show that the functions $f(x) = 2x$ and $x^2 + 1$ are orthogonal over the interval $[-1, +1]$.

PROBLEM (8)

If the state of a quantum particle is decomposed to

$$\Psi = \sum_i \alpha_i \psi_i,$$

find $||\Psi||^2$.