*HW assignment date: **01 April 2020***

Homework - I
KSU / Math-244 / Semester-II (1440-1441)

Note 1: The students must submit the completed homework through email to the respective class teachers within 3 days from its assignment date.

Note 2: Every student must submit PDF file of the homework done; in his own handwriting using blue ink and with his signature.

Problem1:

a) If $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 0 & -3 \\ -6 & 7 & -3 \\ -3 & -9 & -2 \end{bmatrix}$, then **find** the values of

   a. x, y and z such that $x\mathbf{A}^2 + y\,\mathbf{AB} + z\,\mathbf{I} = \mathbf{O}$.

b) **Find** $adj\,(\mathbf{A})$ and $\mathbf{A}^{-1}$ for the matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & a \\ 2 & b & c \\ -1 & 1 & 1 \end{bmatrix},$$

where $ab + b + 2a - c \neq 0$

c) Let $\mathbf{A} \in \mathbf{M}_{3\times3}(\mathbb{R})$ with determinant $|\mathbf{A}| = 2$. **Find** $|2(adj(\mathbf{A}))^{-1} + \mathbf{A}|$.

Problem 2:

a) Let $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{4} & \mathbf{5} \\ \mathbf{-2} & \mathbf{1} & \mathbf{-11} & \mathbf{-8} \\ \mathbf{-1} & \mathbf{2} & \mathbf{2} & \mathbf{2} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{3} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{4} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{8} & \mathbf{6} \end{bmatrix}$. **Show** that the matrices **A** and **B** are row equivalent to each other.

b) **Determine t**he value/s of α such that the following linear system:

$$\begin{aligned} x + 2y - z &= 2 \\ x - 2y + 3z &= 1 \\ x + 2y - (\alpha^2 - 3)\,z &= \alpha \end{aligned}$$

has: **(i).** no solution; **(ii).** unique solution; **(iii).** infinitely many solutions.

Problem 3:

a) **Show** that any homogeneous system of linear equations either has only the trivial solution or infinitely many solutions and so every homogeneous linear system is consistent.

b) **Give** example of a homogeneous linear system with only the trivial solution.

c) **Give** example of a homogeneous linear system having infinitely many non-trivial solutions.

d) By using the Cramer's rule, **solve** the following system:

$$\begin{aligned} x + 2y - z &= 2 \\ x + 3y + 3z &= 2 \\ x + 3y + 5z &= 4. \end{aligned}$$

Problem 4:

a) Let $\mathbf{W} = \{\mathbf{A} \in \mathbf{M}_{2\times2}(\mathbb{R}) : \mathbf{AB} = \mathbf{BA}\}$, where $\mathbf{B} = \begin{bmatrix} \mathbf{-1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$. Then:

   (i). Show that **W** is a vector subspace of the vector space $\mathbf{M}_{2\times2}(\mathbb{R})$.

   (ii). Find a basis and dimension of **W**.

b) Find a basis of the vector space $\mathbb{R}^3$ which contains the set $\{(1, 1, 0), (1, -1, 0)\}$.

Problem 5:

a) **Show** that **A** ={ $\begin{bmatrix} \boldsymbol{\alpha} & \boldsymbol{\beta} \\ \gamma & \delta \end{bmatrix}$ ∈ $\mathbf{M}_{2\times2}(\mathbb{R})$: **α+β =** γ **-** δ}is a vector subspace of $\mathbf{M}_{2\times2}(\mathbb{R})$. Also **find** a basis and dimension of the vector space **A**.

b) **Show** that **S**={**X**∈$\mathbf{M}_{2\times2}(\mathbb{R})$: **X** = **-** $\mathbf{X}^{\mathbf{T}}$} is a subspace of the vector space $\mathbf{M}_{2\times2}(\mathbb{R})$ and **show further** that the set {$\begin{bmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$} is a basis for **S**.

c) **Determine** whether {**X**∈$\mathbf{M}_{2\times2}(\mathbb{R})$: **X** = $\mathbf{X}^{\mathbf{T}}$} is a proper subspace of the vector space $\mathbf{M}_{2\times2}(\mathbb{R})$?

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