مذكرة حل أسئلة اختبارات سابقة (١٠٦ريض)

قام بتصوير هذه المذكرة :

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ولا أريد لها مقابل سوى دعوة صادقة في ظهر ؟لغيب

وتذكر أنه ما من عبد مسلم يدعو لأخيه بظهر الغيب

إلا قال الملك : ولك بمثل

أتمنى للجميع ؟لتوفيق و؟لنجاح ...

المملكة العربية السعودي

جامعة الملك سعود

مركز تميم

للطباعة والتصوير

هاتف: ٢٠٣٤٦١٧

١٠٦

الكليــــــة ....

رقم ورمز المادة .... ١٥٦ ريض

اسم المــــادة .... اختبارات الشهر الثاني

اسم الدكـــتور .... ابو عمر

ملاحظـــات ....

المحتـــوى ....

التاريـــخ ....

عدد الصفحـــات ....

SR. 3

طباعة ـ تصوير ـ تغليف

Multiple Choices

Mark {a,b,c,d} for the correct answers in the space given below for Q.1-to-Q.10 [$10\times 1=10$]

Q.No.	1	2	3	4	5	6	7	8	9	10
{a,b,c,d}										

Q. No:1 $\lim_{x\to 0}\dfrac{\int_0^x \sin^2 t\,dt}{x^3}$ is equal to

(a) 0 (b) $\frac{1}{3}$ (c) ∞ (d) none of these

Q. No:2 The value of the integral $\int_0^1 \tan^{-1} x\,dx$ is equal to

(a) $\frac{\pi}{4}-\frac{1}{2}\ln 2$ (b) $\frac{\pi}{4}-\ln 2$ (c) $\frac{\pi}{4}+\frac{1}{2}\ln 2$ (d) none of these

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Q. No:3 To evaluate the integral $\int x^2\sqrt{x^2+25}\,dx$ we use the substitution

(a) $x=\tan\theta$ (b) $x=5\sec\theta$ (c) $u=5\tan x$ (d) none of these

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Q. No:4 The partial fractions for solving $\int \frac{1}{x^2\left(x^2+1\right)}\,dx$ is

(a) $\frac{A}{x^2}+\frac{Bx+C}{x^2+1}$ (b) $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$ (c) $\frac{A}{x}+\frac{Bx+C}{x^2+1}$ (d) none of these

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Q. No:5 The value of the integral $\int \sec^4\theta\,d\theta$ is

(a) $\frac{\sec^5\theta}{5}+c$ (b) $\frac{\tan^3\theta}{3}+c$ (c) $\tan\theta+\frac{\tan^3\theta}{3}+c$ (d) none of these

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Q. No:6 To evaluate the integral $\int \frac{1}{x^2-2x+3}\,dx$ we use

(a) Integration by parts (b) Partial fractions (c) Completing the square (d) none of these

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Q. No:7 To evaluate the integral $\int \frac{\sin x}{\sin x+\cos x}\,dx$ we put

(a) $u=\tan\left(\frac{x}{2}\right)$ (b) $u=\sin x$ (c) $x=\tan\left(\frac{u}{2}\right)$ (d) none of these

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Q. No:8 The value of the integral $\int \sin 3x \sin x\,dx$ is equal to

(a) $\frac{\sin 2x}{2}-\frac{\sin 4x}{4}+c$ (b) $\frac{\sin 2x}{4}-\frac{\sin 4x}{8}+c$ (c) $\frac{\sin 2x}{4}+\frac{\sin 4x}{8}+c$ (d) none of these

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Q. No:9 To evaluate the integral $\int \tan^3 x\sec^5 x\,dx$ we put

(a) $u=\sec x$ (b) $u=\sec^2 x$ (c) $u=\tan x$ (d) none of these

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Q. No:10 To evaluate the integral $\int \frac{x^{2/3}-1}{x^{3/2}+2}\,dx$ we put

(a) $u=x^{1/2}$ (b) $u=x^{1/3}$ (c) $u=x^{1/6}$ (d) none of these

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Full Questions:

Question No: 11 Evaluate $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$ [2]

Question No: 12 Determine whether the following improper integral $\int_0^1 \ln x\, dx$ converges or diverges. [2]

Question No: 13 Evaluate the integral $\int \frac{x-1}{x^4+3x^2+2}\,dx$ [3]

Question No: 14 Evaluate the integral $\int \frac{dx}{\left(9-x^2\right)^{3/2}}$ [3]

الاختبار الشهري 2 (106)

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II

(1) $\lim_{x \to 0} \dfrac{\int_0^x \sin^2 t \, dt}{x^3}$

$= \dfrac{\int_0^0 \sin^2 t \, dt}{0} = \dfrac{0}{0}$ by L'H.R

$= \lim_{x \to 0} \dfrac{\frac{d}{dx}\int_0^x \sin^2 t \, dt}{3x^2}$

$= \lim_{x \to 0} \dfrac{\sin^2 x}{3x^2} = \dfrac{0}{0}$ by L'H.R

$= \lim_{x \to 0} \dfrac{2 \sin x \cos x}{6x}$

$= \lim_{x \to 0} \dfrac{\sin 2x}{6x} = \dfrac{0}{0}$ by L'HR

$= \lim_{x \to 0} \dfrac{\cos 2x \cdot 2}{6} = \dfrac{\cos 0 \cdot 2}{6}$

$= \dfrac{1}{3}$ (b)

(2) $\int_0^1 \tan^{-1} x \, dx$ by parts بالفيديو

$u = \tan^{-1} x \Rightarrow du = \dfrac{1}{1+x^2} dx$

$dv = dx \Rightarrow v = x$

$\int u \, dv = u \cdot v - \int v \cdot du$

$= x \tan^{-1} x - \int \dfrac{x}{1+x^2} dx$

$= \left(x \tan^{-1} x - \frac{1}{2} \ln |1+x^2| \right)_0^1$

$= (1 \tan^{-1}(1) - \frac{1}{2} \ln 2) - (0 - \frac{1}{2} \ln 1)$ $\underbrace{\frac{1}{2}\ln 1}_{=0}$

$= \dfrac{\pi}{4} - \dfrac{1}{2} \ln 2$

(a)

(3) $\int x^2 \sqrt{x^2+25} \, dx$

لحل هذا التكامل نستخدم التعويض

$x = 5 \tan \theta$ (d)

(4) $\int \dfrac{1}{x^2(x^2+1)} dx$

$\Rightarrow \dfrac{1}{(x)(x)(x^2+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{(x^2+1)}$

(5) $\int \sec^4 \theta \, d\theta$

$= \int \sec^2 \theta \cdot \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta) \sec^2 \theta \, d\theta$

$= \int (\sec^2 \theta + \tan^2 \theta \sec^2 \theta) \, d\theta$

$= \tan \theta + \dfrac{\tan^3 \theta}{3} + c$ (c)

(6) $\int \dfrac{1}{x^2 - 2x + 3} dx$

لحل هذا التكامل نستخدم

١- التجزئ أو ٢- الكسور الجزئية أو ٣- اكمال المربع

٤- أو لا شئ مما ذكر

$\therefore \sqrt{b^2 - 4ac} = \sqrt{4 - 4(1)(3)} = \sqrt{-8}$

(c) أى المقام لا يحلل لذا نستخدم اكمال المربع

(7) $\int \dfrac{\sin x}{\sin x + \cos x} dx$

درس (ج) في المنوع

بعد التكامل لكي نرجع المسألة الى x نضع

مكان كل u نضع $\tan \frac{x}{2}$

(a)

(8) $\int \sin 3x \sin x \, dx$

متطابقه في آخر صفحه في درس الفردي والزوجي

$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$$= \frac{1}{2}\int \cos(2x) - \cos(4x) \, dx$$

$$= \frac{1}{2}\left[\frac{\sin(2x)}{2} - \frac{\sin(4x)}{4}\right] + c$$

$$= \frac{\sin(2x)}{4} - \frac{\sin(4x)}{8} + c \qquad (b)$$

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(9) $\int \tan^3 x \sec^3 x \, dx$

درس الفردي والزوجي

$$= \int \tan^2 x \tan x \sec^3 x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \sec^3 x \, dx$$

$$= \int \sec^5 x \tan x - \sec^3 x \tan x \, dx$$

$$= \int \sec^4(x)(\sec x \tan x) - \sec^2(x)(\sec x \tan x)$$

$u = \sec x \Rightarrow du = \sec x \tan x$

(a)

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(10) $\int \frac{x^{2/3} - 1}{x^{3/2} + 2} \, dx$

فقرة (ب) في المتنوع

$\frac{2}{3}, \frac{3}{2}$ المقام المشترك 6

$\therefore u = x^{1/6}$ (c)

Full Questions:

(11) $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x = (1+0)^\infty$

$= 1^\infty$

Let $y = \left(1 + \frac{2}{x}\right)^x$

$$\ln y = x \ln\left(1 + \frac{2}{x}\right)$$

$$\ln y = \frac{\ln(1 + 2x^{-1})}{x^{-1}}$$

$$\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln(1 + 2x^{-1})}{x^{-1}} \quad \text{by L'H.R}$$

$$" = \lim_{x \to \infty} \frac{\frac{-2x^{-2}}{1 + 2x^{-1}}}{-x^{-2}}$$

$$" = \lim_{x \to \infty} \frac{-2x^{-2}}{-x^{-2}(1 + 2x^{-1})}$$

$$" = \lim_{x \to \infty} \frac{2}{(1 + 2x^{-1})}$$

$$" = \lim_{x \to \infty} \frac{2}{\left(1 + 2\frac{1}{x}\right)} = \frac{2}{1+0} = 2$$

$$\therefore \lim_{x \to \infty} \ln y = 2$$

$$\Rightarrow \lim_{x \to \infty} y = e^2$$

$$\therefore \lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x = e^2$$

(12) $\int_0^1 \ln x \, dx$ C'g or d'g

$\int \ln x \, dx$ by parts

$u = \ln x \rightarrow du = \frac{1}{x} dx$

$dv = dx \rightarrow v = x$

$\int u \, dv = uv - \int v \cdot du$

$= x \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx$

$= x \ln x - x$

$\int_0^1 \ln x \, dx = \lim_{t \to 0^-} \int_t^1 \ln x \, dx$

$= \lim_{t \to 0^-} \left(x \ln x - x \right)_t^1$

$= \lim_{t \to 0^-} \left((1 \underbrace{\ln 1}_{=0} - 1) - (t \ln t - t) \right)$

$= \lim_{t \to 0^-} \left(-1 - t \ln t + t \right)$

$= -1 + 0 \ln 0^- + 0$

$= -1 - \infty = -\infty$

$\therefore$ improper integral diverges

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(13) $\int \frac{x-1}{x^4 + 3x^2 + 2} dx$ by partial fractions

$$\frac{x-1}{(x^2+2)(x^2+1)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+1}$$

$$" = \frac{(Ax+B)(x^2+1) + (Cx+D)(x^2+2)}{(x^2+2)(x^2+1)}$$

$\Rightarrow$

$x - 1 = (Ax+B)(x^2+1) + (Cx+D)(x^2+2)$

بمساواة المعاملات

$0 = A + C$ معامل x^3

$0 = B + D$ " x^2

$1 = A + 2C$ " x

$-1 = B + 2D$ الحد الثابت

$\bar{A} + \bar{C} = \bar{0}$

$A + 2C = 1$

$C = 1$

$\Rightarrow A = -1$

$\bar{B} + \bar{D} = \bar{0}$

$B + 2D = -1$

$D = -1$

$\Rightarrow B = 1$

$\therefore \int \frac{x-1}{(x^2+2)(x^2+1)} dx$

$= \int \frac{-x+1}{x^2+2} dx + \int \frac{x-1}{x^2+1} dx$

$= \int \frac{-x}{x^2+2} dx + \int \frac{1}{x^2+2} dx$

$+ \int \frac{x}{x^2+1} dx - \int \frac{1}{x^2+1} dx$

$= -\frac{1}{2} \ln |x^2+2| + \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$+ \frac{1}{2} \ln |x^2+1| - \tan^{-1} x + C$

(14) $\displaystyle\int \frac{dx}{(9-x^2)^{3/2}}$

Let $x = 3\sin\theta \Rightarrow dx = 3\cos\theta \, d\theta$

$$\int \ldots = \int \frac{3\cos\theta \, d\theta}{(9-9\sin^2\theta)^{3/2}} = \int \frac{3\cos\theta}{(9(1-\sin^2\theta))^{3/2}} \, d\theta$$

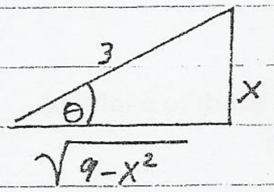
$$= \int \frac{3\cos\theta}{(9\cos^2\theta)^{3/2}} \, d\theta = \int \frac{3\cos\theta}{9^{3/2}(\cos^2\theta)^{3/2}} \, d\theta$$

$$= \int \frac{3\cos\theta}{27\cos^3\theta} \, d\theta = \frac{1}{9}\int \frac{1}{\cos^2\theta} \, d\theta = \frac{1}{9}\int \sec^2\theta \, d\theta$$

$$= \frac{1}{9}\tan\theta + c$$

$3\sin\theta = x$

$\sin\theta = \frac{x}{3}$



$$= \frac{1}{9}\left(\frac{x}{\sqrt{9-x^2}}\right) + c$$

Multiple Choices

Mark {a,b,c,d} for the correct answers in the space given below for **Q.1-to-Q.10** [$10\times 1=10$]

Q.No.	1	2	3	4	5	6	7	8	9	10
{a,b,c,d}										

Q. No:1 The integral $\int \frac{5^{Cos(x)}}{Csc(x)}\, dx$ is equal to

(a) $\frac{5^{Cos(x)}}{\ln(5)}+c$ (b) $-\frac{5^{Cos(x)}}{\ln(5)}+c$ (c) $-5^{Cos(x)}+c$ (d) None of these

Q. No:2 If $\log_2 x=3$ then x is equal to

(a) 9 (b) 6 (c) 8 (d) None of these.

Q. No:3 $\cosh(\ln(2))$ is equal to

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) None of these

Q. No:4 $\lim_{x\to\frac{\pi}{2}} \frac{Cos(x)}{x-\frac{\pi}{2}}$ is equal to

(a) 0 (b) 1 (c) -1 (d) None of these

Q. No:5 To evaluate the integral $\int \frac{x^{1/2}}{x^{1/3}+1}dx$ the best substitution is

(a) $u=x^{\frac{1}{2}}$ (b) $u=x^{\frac{1}{3}}$ (c) $u=x^{\frac{1}{6}}$ (d) None of these

Q. No:6 If $6x-x^2=A-(x-B)^2$, then

(a) $A=9, B=3$ (b) $A=3, B=9$, (c) $A=1, B=2$, (d) None of these.

Q. No:7 If $\frac{2}{x^4+x^2}$ then the correct partial fraction decomposition is

(a) $\frac{A}{x^2}+\frac{B}{x^2+1}$, (b) $\frac{A}{x^2}+\frac{Bx+c}{x^2+1}$, (c) $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$, (d) None of these.

Q. No:8 $\int \cot^2 x\, dx$ is equal to

(a) $-\cot x - x + c$, (b) $-\cot x + x + c$, (c) $-\csc^2 x + c$, (d) None of these

Q. No: 9 If $\sqrt{x^2-4}=2\tan\theta$ then

(a) $x=2Sec\theta$, (b) $x=2\tan\theta$, (c) $x=2Cot\theta$ (d) None of these.

Q. No: 10 $\int \tan^{-1}x\; dx$ is equal to

(a) $x\tan^{-1}x+\ln(x^2+1)+c$,(b) $x\tan^{-1}x+\frac{1}{2}\ln(x^2+1)+c$,(c) $x\tan^{-1}x+x+c$,(d)None of these

Full Questions:

Question No: 11 Evaluate the integral $\int \frac{3^x}{4+9^x}\, dx$

Question No: 12 Evaluate the integral $\int \frac{x}{2x^2+x-1}\, dx$

Question No: 13 Evaluate the integral $\int \tan^3(x) Sec^5(x)\; dx$

Question: 14 Find the $\lim\limits_{x \to 1^+} (x-1)^{\frac{1}{x-1}}$ if it exists.

الاختبار الشهري الثاني (106)

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I

(1) $\int \frac{5^{\cos(x)}}{\csc(x)}\,dx$

$= -1\int 5^{\cos(x)}(-\sin(x))\,dx = -\frac{5^{\cos(x)}}{\ln 5} + C$

(b)

(2) $\log_2 X = 3$

$X = 2^3 = 8$ (c)

(3) $\cosh(\ln(2))$

$= \frac{e^{\ln(2)} + e^{-\ln(2)}}{2} = \frac{2 + e^{\ln(2)^{-1}}}{2}$

$= \frac{2 + 2^{-1}}{2} = \frac{2 + \frac{1}{2}}{2} = \frac{2.5}{2} = \frac{5}{4}$

او بالآلة الحاسبة (c)

(4) $\lim_{x \to \frac{\pi}{2}} \frac{\cos(x)}{x - \frac{\pi}{2}} = \frac{0}{0}$ by L'H.R

$= \lim_{x \to \frac{\pi}{2}} \frac{-\sin(x)}{1} = \frac{-\sin(\frac{\pi}{2})}{1} = \frac{-1}{1} = -1$

(c)

(5) $\int \frac{x^{1/2}}{x^{1/3}+1}\,dx$

المقام المشترك لـ $\frac{1}{2}, \frac{1}{3}$

$u = x^{1/6}$ نستخدم ∴ هو 6 (c)

(6) $6x - x^2 = A - (x - B)^2$

نعمل اكمال مربع الجزء الايسر

$-(x^2 - 6x) = -(x^2 - 6x + 9 - 9)$

$= -((x-3)^2 - 9) = 9 - (x-3)^2$

$= A - (x - B)^2$

$\therefore A = 9 \;،\; B = 3$ (a)

(7) $\frac{2}{x^4 + x^2} = \frac{2}{x^2(x^2+1)}$

$= \frac{2}{x\,x\,(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(x^2+1)}$

(c)

(8) $\int \cot^2 x\,dx$

$= \int (\csc^2(x) - 1)\,dx$

$= -\cot x - x + C$ (a)

(9) $\sqrt{x^2 - 4} = 2\tan\theta$

$x = \frac{2}{1}\sec\theta$ نفرض ▲ مع رسم

أو بحل المعادلة اولاً بالتربيع

$x^2 - 4 = 4\tan^2\theta$

$x^2 = 4 + 4\tan^2\theta$

$x^2 = 4(1 + \tan^2\theta)$ (a)

$x^2 = 4\sec^2\theta \Rightarrow x = 2\sec\theta$

(10) $\int \tan^{-1}(x)\,dx$ by parts فيديو

$u = \tan^{-1}x \rightarrow du = \frac{1}{1+x^2}\,dx$

$dv = dx \rightarrow v = x$

$\int u\,dv = u.v - \int v.du$

$= x\tan^{-1}x - \int \frac{x}{1+x^2}\,dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + C$

(d)

Full Questions

(11) $\int \frac{3^x}{4+9^x}\,dx$ كتله

$$= \int \frac{3^x}{4\left(1+\frac{9^x}{4}\right)}\,dx$$

$$= \int \frac{3^x}{4\left(1+\left(\frac{3^x}{2}\right)^2\right)}\,dx$$

let $u = \frac{3^x}{2} \Rightarrow du = \frac{3^x \ln 3}{2}\,dx$

$$\frac{2}{\ln 3}\,du = 3^x\,dx$$

$$= \int \frac{\frac{2}{\ln 3}\,du}{4\,(1+u^2)} = \frac{1}{2\ln 3}\int \frac{du}{1+u^2}$$

$$= \frac{1}{2\ln 3}\tan^{-1}(u) + c$$

$$= \frac{1}{2\ln 3}\tan^{-1}\left(\frac{3^x}{2}\right) + c$$

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(12) $\int \frac{x}{2x^2+x-1}\,dx$

$$2x^2+x-1 = (2x-1)(x+1)$$

$$\frac{x}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$$

$$\text{"} \quad = \frac{A(x+1)+B(2x-1)}{(2x-1)(x+1)}$$

$$x = A(x+1) + B(2x-1)$$

let $x = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2}A \Rightarrow A = \frac{1}{3}$

" $x = -1 \Rightarrow -1 = -3B \Rightarrow B = \frac{1}{3}$

$$\therefore \int \frac{x}{(2x-1)(x+1)}\,dx = \int \frac{\frac{1}{3}}{2x-1}\,dx + \int \frac{\frac{1}{3}}{x+1}\,dx$$

$$= \frac{1}{3}\int \frac{1}{2x-1}\,dx + \frac{1}{3}\int \frac{1}{x+1}\,dx$$

$$= \frac{1}{3}\cdot\frac{1}{2}\int \frac{2}{2x-1}\,dx + \frac{1}{3}\int \frac{1}{x+1}\,dx$$

$$= \frac{1}{6}\ln|2x-1| + \frac{1}{3}\ln|x+1| + c$$

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(13) $\int \tan^3(x)\sec^5(x)\,dx$

$$= \int \tan^2(x)\tan(x)\sec^5(x)\,dx$$

$$= \int (\sec^2(x)-1)\tan(x)\sec^5(x)\,dx$$

$$= \int (\sec^7(x)\tan x - \sec^5(x)\tan(x))\,dx$$

$$= \int [\sec^6(x)\sec(x)\tan x - \sec^4(x)\sec(x)\tan(x)]\,dx$$

let $u = \sec x \Rightarrow du = \sec x \tan x\,dx$

$$= \int (u^6 - u^4)\,du = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$= \frac{\sec^7(x)}{7} - \frac{\sec^5(x)}{5} + c$$

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14) $\lim_{x\to 1^+} (x-1)^{\frac{1}{x-1}} = (1-1)^{\frac{1}{1-1}} = 0^{\infty}$

Let $y = (x-1)^{\frac{1}{x-1}} \Rightarrow \ln y = \frac{1}{x-1}\ln(x-1)$

$$\lim_{x\to 1^+} \ln y = \lim_{x\to 1^+} \frac{\ln(x-1)}{(x-1)}$$

$$\text{"} = \lim_{x\to 1^+} \frac{\left(\frac{1}{x-1}\right)}{1} = \lim_{x\to 1^+} \frac{1}{x-1}$$

$$\text{"} = \frac{1}{0} = +\infty$$

$$\therefore \lim_{x\to 1^+} \ln y = +\infty \Rightarrow \lim_{x\to 1^+} y = e^{\infty} = \infty$$

$$\therefore \lim_{x\to 1^+} (x-1)^{\frac{1}{x-1}} = \infty$$

Multiple choices

Q.No.	1	2	3	4	5	6	7	8	9	10
{a,b,c,d}										

Q.No: 1 The value of $\lim_{x\to 0}\frac{x-\tan^{-1}(x)}{Sin(x)}$ is equal to

(a) ∞ (b) $-\infty$ (c) 0 (d) None of these

Q.No: 2 The integral $\int_1^e \ln(x)dx$ is equal to

(a) - 1 (b) 1 (c) ∞ (d) None of these

Q.No: 3 The value of $\lim_{x\to 0}\left(\frac{1}{x}\right)^x$ is

(a) 1 (b) -1 (c) 0 (d) none of these

Q.No: 4 If $\int \cos^3 x \sin^2 x dx = g(x)+c$. Then g(x) is equal to

(a) $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}$+c, (b) $\frac{\sin^3 x}{3}+\frac{\cos^5 x}{5}$+c (c) $\frac{(\sin x)^3}{3}+\frac{(\cos x)^5}{5}$ (d) none of these

Q.No: 5 To evaluate the integral $\int \frac{x^3}{\sqrt{9x^2+49}}dx$, we choose

(a) $x=\frac{7}{3}\tan\theta$, (b) $3x=\cos\theta$, (c) $x=7\tan\theta$ (d) $3x=\sin\theta$

Q.No: 6 The partial fractions for solving $\int \frac{x^3-1}{(x^2-1)(x^2+1)}dx$ are

(a) $\frac{A}{x+1}+\frac{Bx+c}{x^2+1}$, (b) $\frac{Ax+B}{x^2-1}+\frac{Cx+D}{x^2+1}$, (c) $\frac{A}{x+1}+\frac{B}{x-1}+\frac{Dx+c}{x^2+1}$, (d) None of these

Q.No: 7 The substitution $u=\sqrt{1+\sqrt{x}}$ transforms the integral $\int\sqrt{1+\sqrt{x}}\ dx$ into

(a) $\int 4(u^4-u^2)du$, (b) $\int 2(u^2-u^4)du$, (c) $\int 2(u^2-u\ )du$, (d) None of these.

Q.No: 8 The value of the integral $\int_0^{\pi}\frac{2}{x}e^{\ln x}Sin(2x)dx$ is

(a) 1, (b) 2, (c) 0, (d)None of these

Q.No: 9 The substitution $u=\tan(\frac{x}{2})$ transforms the integral $\int\frac{1}{1-Cosx}dx$ into

(a) $\int\frac{2}{(u+1)^2}du$, (b) $\int\frac{2}{1+u^2}du$, (c) $\int\frac{2}{u^2}du$ (d) None of these

Q.No: 10 The integral $\int_1^{\infty}x^{\alpha}dx$ converges if

(a) $-1<\alpha<0$, (b) $0<\alpha<1$, (c) $\alpha<-1$ (d) None of these

Full Questions

Question No: 11 Find the $\lim\limits_{x\to 0}\dfrac{\int_0^x Sin(t^2)\,dt}{Sin(x^2)}$ if it exist.

Question No: 12 Evaluate the integral $\int \dfrac{1}{x^2\sqrt{9-x^2}}\,dx$

Question No: 13 Evaluate the integral $\int \dfrac{x^2+12x+12}{x^3-4x}\,dx$

Question No: 14 Discuss whether the integral $\int_0^1 x^{-(\frac{7}{3})}\,dx$ converges or diverges, if it Converges find its value.

الاختبار الشهري الثاني (٢)

28/29

1) $\lim_{x\to 0} \frac{x-\tan^{-1}(x)}{\sin x} = \frac{0}{0}$

ⓒ

by L'HR $\lim_{x\to 0} \frac{1-\left(\frac{1}{1+x^2}\right)}{\cos x} = \frac{1-1}{1} = 0$

2) $\int_1^e \ln(x)\,dx$

فيديو

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = dx \rightarrow v = x$

$\int u\,dv = u\cdot v - \int v\cdot du$

$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}dx$

$= x\ln x - x + c$

$\therefore \int_1^e \ln(x)\,dx = (x\ln x - x)\Big|_1^e$

$= (e\ln e - e) - (1\ln 1 - 1)$

ⓑ

$= (\cancel{e-e}) - (0-1) = 1$

3) $\lim_{x\to 0}\left(\frac{1}{x}\right)^x = \infty^0$

Let $y = \left(\frac{1}{x}\right)^x \Rightarrow \ln y = x\ln\frac{1}{x}$

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{\ln(x^{-1})}{x^{-1}} = \lim_{x\to 0}\frac{-\frac{1}{x}}{-x^{-2}}$

$= \lim_{x\to 0}\frac{x^2}{x} = \lim_{x\to 0} x = 0$

ⓐ

$\therefore \lim_{x\to 0} y = \lim_{x\to 0}\left(\frac{1}{x}\right)^x = e^0 = 1$

4) $\int \cos^3 x\,\sin^2 x\,dx = g(x) + c$

$= \int \cos^2 x\,\cos x\,\sin^2 x\,dx$

$= \int (1-\sin^2 x)\cos x\,\sin^2 x\,dx$

$= \int (\cos x\sin^2 x - \cos x\sin^4 x)\,dx$

$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

ⓒ

$\therefore g(x) = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5}$

5) $\int \frac{x^3}{\sqrt{9x^2+49}}\,dx$

ⓐ

لحساب هذا التكامل

نفرض $x = \frac{7}{3}\tan\theta$

6) $\int \frac{x^3-1}{(x^2-1)(x^2+1)}\,dx$

تكسير هذا المقدار يصبح

ⓒ

$(x^2-1) = (x-1)(x+1)$

$\frac{x^3-1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Dx+C}{x^2+1}$

7) التعويض بـ $u = \sqrt{1+\sqrt{x}}$

يحول التكامل $\int\sqrt{1+\sqrt{x}}\,dx$ إلى

$u = \sqrt{1+\sqrt{x}} \rightarrow du = \frac{\frac{1}{2\sqrt{x}}}{2\sqrt{1+\sqrt{x}}}\,dx$

$\Rightarrow u^2 = 1+\sqrt{x}$

$u^2 - 1 = \sqrt{x}$

$du = \frac{1}{4\sqrt{x}\,\sqrt{1+\sqrt{x}}}\,dx$

$4\sqrt{x}\,\sqrt{1+\sqrt{x}}\,du = dx$

ⓐ

$4(u^2-1)\cdot u\,du = dx$

$= \int u\cdot 4u(u^2-1)\,du$

$= \int 4u^2(u^2-1)\,du = \int 4(u^4-u^2)\,du$

8) $\int_0^{\pi} \frac{2}{x} \cancel{e}^{\cancel{\ln} x} \sin(2x)\,dx$

$= \int_0^{\pi} \frac{2}{\cancel{x}} \cdot \cancel{x} \cdot \sin(2x)\,dx$

$= \int_0^{\pi} \cancel{2} \sin(2x)\,dx = \Big[-\cos(2x)\Big]_0^{\pi}$

$= -\left(\cos 2\pi - \cos 0\right) = 0$ (c)

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9) $u = \tan\left(\frac{x}{2}\right)$ التعويض

حل: $\int \frac{1}{1-\cos x}\,dx$ يحول التكامل ←

$\cos x = \frac{1-u^2}{1+u^2}$, $dx = \frac{2}{1+u^2}\,du$

$= \int \frac{1}{1-\left(\frac{1-u^2}{1+u^2}\right)} \frac{2}{1+u^2}\,du$

$= \int \frac{2}{\frac{\cancel{1}+u^2-\cancel{1}+u^2}{1+u^2}} \cdot \frac{1}{1+u^2}\,du$

$= \int \frac{\cancel{2}(1+u^2)}{\cancel{2}u^2} \cdot \frac{1}{\cancel{(1+u^2)}}\,du$

$= \int \frac{1}{u^2}\,du$ (d)

---

10) $\int_1^{\infty} x^a\,dx$ c/g if

$\lim_{t\to\infty} \int_1^t x^a\,dx = \lim_{t\to\infty} \left(\frac{x^{a+1}}{a+1}\right)_1^t$

$= \lim_{t\to\infty} \left(\frac{t^{a+1}}{a+1} - \frac{1}{a+1}\right)$

هذا الجزء معرف فيما عدا عند $a=-1$ ومعرف لكل قيم $a<-1$

هذا الجزء معرف فيما عدا عند $a=-1$

(c)

Full Questions

11) $\lim_{x\to 0} \frac{\int_0^x \sin(t^2)\,dt}{\sin(x^2)} = \frac{\int_0^0 \cdots}{\sin(0)} = \frac{0}{0}$

by L'H.R $\lim_{x\to 0} \frac{\frac{d}{dx}\int_0^x \sin(t^2)\,dt}{2x\cos(x^2)}$

$= \lim_{x\to 0} \frac{\sin(x^2)}{2x\cos(x^2)} = \lim_{x\to 0} \frac{\tan(x^2)}{2x}$

by L'HR again $\lim_{x\to 0} \frac{\cancel{2}x\sec^2(x^2)}{\cancel{2}}$

$= \lim_{x\to 0} \frac{x}{\cos^2(x^2)} = \frac{0}{1} = 0$

---

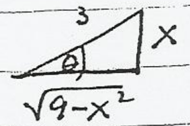
12) $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

Let $x = 3\sin\theta \rightarrow dx = 3\cos\theta\,d\theta$

$= \int \frac{3\cos\theta}{9\sin^2\theta\sqrt{9-9\sin^2\theta}}\,d\theta$

$= \int \frac{\cancel{3\cos\theta}}{9\sin^2\theta \cdot \cancel{3\cos\theta}}\,d\theta = \frac{1}{9}\int \csc^2\theta\,d\theta$

$= \frac{-1}{9}\cot\theta + C$



$= \frac{-1}{9} \frac{\sqrt{9-x^2}}{x} + C$

13) $\int \frac{x^2+12x+12}{x^3-4x}\,dx$

$x(x^2-4) = x(x-2)(x+2)$

$$\frac{x^2+12x+12}{(x)(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$= \frac{A(x-2)(x+2) + B(x)(x+2) + C(x)(x-2)}{x(x-2)(x+2)}$$

$$\therefore x^2+12x+12 = A(x-2)(x+2) + B(x)(x+2) + C(x)(x-2)$$

at $x=0 \Rightarrow 12 = -4A \Rightarrow A=-3$

" $x=2 \Rightarrow 40 = 8B \Rightarrow B=5$

" $x=-2 \Rightarrow -8 = 8C \Rightarrow C=-1$

$$\Rightarrow \int \frac{x^2+12x+12}{x^3-4x}\,dx = \int \frac{-3}{x}dx + \int \frac{5}{x-2}dx + \int \frac{-1}{x+2}$$

$$= -3\ln|x| + 5\ln|x-2| - \ln|x+2| + c$$

14) $\int_0^1 x^{-\left(\frac{2}{3}\right)}\,dx = \int_0^1 \frac{1}{x^{\frac{2}{3}}}\,dx = \int_0^1 \frac{1}{\sqrt[3]{x^2}}\,dx$

at $x=0$ it will be improper

$$= \lim_{t\to 0^+} \int_t^1 x^{-\frac{2}{3}}\,dx = \lim_{t\to 0^+} \left(\frac{x^{\frac{1}{3}}}{\frac{1}{3}}\right)_t^1$$

$$3 \lim_{t\to 0^+} \left(1 - t^{\frac{1}{3}}\right) = 3(1) = 3$$

$\therefore \int_0^1 \ldots$ C and its value 3