PHYS 551-505
HANDOUT 1 – On the physics of *the hydrogen atom*

1. Construct the states $Y_1^0(\theta,\phi)$ and $Y_1^{-1}(\theta,\phi)$ and show that they are normalized.

2. The electron in the hydrogen atom is in the ground state. Calculate the probability to find it at distances:
   (a) Smaller or equal to one Bohr radius.
   (b) Smaller or equal to two Bohr radii.

3. Calculate the average kinetic and potential energy of the hydrogen electron at its ground state.

4. Calculate the average kinetic and potential energy of the hydrogen electron at the state ψ_{200}.

5. The electron in the hydrogen atom is in its ground state. What is the probability to find the electron at distances larger than those allowed classically?

6. The electron in the hydrogen atom is in the state ψ_{210}. Calculate the probability to find it in a double cone of angle 60^0 around the z-axis.

7. The electron and its anti-particle the *positron* (a particle that has the same mass with the electron but opposite charge) can make a bound system known as *positronium.* Calculate:
   (a) The energy of the ground state of the positronium.
   (b) The average distance between electron and positron at the ground state.

8. The operators for the square angular momentum and the projection of the angular momentum along z direction are given by:

$$\mathbf{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$L_z = -i\hbar\frac{\partial}{\partial\phi}$$

   Show that $Y_1^{-1}(\theta,\phi)$ is an eigenfunction of both operators.

9. The electron in a hydrogen atom is in a state given by:

$$\psi = A\psi_{100} + 2A\psi_{211} + A\psi_{32,-1}$$

(a) Find A.
(b) Find $\langle \mathbf{l}^2 \rangle$, $\langle l_z \rangle$, $\langle E \rangle$, Δl_z the average energy of the electron
(c) Find the time evolution of the state ψ of the atom.

10. For any classical vector the maximum projection along an axis is equal to the magnitude of the vector (when the vector is aligned with the axis). Is this property true for a quantum vector like the angular momentum? If no, then calculate the angle that the angular momentum vector with z axis at the state of maximum projection. Could you give a physical explanation for this? What happens at the classical limit $l \to \infty$?

11. A particle of mass M is forced to move on a sphere of radius *a.* Calculate the allowed values of the energy and make an energy diagram of them.

12. Which of the following statements are true?
(a) Angular momentum is a conserved quantity for the motion in a central potential.
(b) Angular momentum is a quantized quantity only for the motion in a central field.
(c) The three components p_x, p_y, p_z of the momentum can be measured simultaneously with perfect accuracy.
(d) The three components l_x, l_y, l_z of the momentum can be measured simultaneously with perfect accuracy.
(e) The magnitude of the angular momentum could be simultaneously measured with one of its components l_x, l_y, l_z.

13. Prove the following relations for the angular momentum operators: a) $\left[l^2,\, l_z \right] = 0$, b) $\mathbf{l} \times \mathbf{l} = i\hbar \mathbf{l}$.