We need to use the following rules :

1.Demorgan rules:

$$P(A \cup B)^C = P(A^C \cap B^C)$$

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2. Addition Rule :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

We can write it as $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

3.Complement rule :

$$P(A)^C = 1 - P(A)$$

4. Probability of an event

$$P(C) = P(C \cap A) + P(C \cap A^C)$$

5.Conditional rule :

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

6. Disjoint : A , B are disjoined if and only if $P(A \cap B) = 0$

7.Independent : A , B are disjoined if and only if

$$P(A \cap B) = P(A)xP(B)$$

or $P(A/B) = P(A)$

Or $P(A/B) = P(A)$

2. PROBABILITY, CONDITIONAL PROBABILITY, AND INDEPENDENCE

Q1. Let A, B, and C be three events such that: $P(A)=0.5$, $P(B)=0.4$, $P(C\cap A^c)=0.6$, $P(C\cap A)=0.2$, and $P(A\cup B)=0.9$. Then

(a) P(C) =

$$P(C) = P(C\cap A) + P(C\cap A^C)$$
$$= 0.2 + 0.6 = 0.8$$

(b) $P(B\cap A)$ =

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$
$$= 0.5 + 0.4 - 0.9 = 0$$

(c) $P(C/A) =$

$$P(C/A) = \frac{P(A\cap C)}{P(A)} = \frac{0.2}{0.5} = 0.4$$

(d) $P(A^C \cap B^C) =$

$$P(A^C\cap B^C) = P(A\cup B)^C = 1 - P(AUB) = 1 - 0.9 = 0.1$$

Q2. Consider the experiment of flipping a balanced coin three times independently.

(a) The number of points in the sample space is

S = {H,T} x {H,T} x {H,T}

S ={HHH,HHT,HTH,THH,TTH,HTT,THT,TTT}

n(S) = 2x2x2 =8 (Answer :C)

(b) The probability of getting exactly two heads is

A= { HHT,HTH,THH}

P(A) = 3/8 =0.375 (Answer :B)

(c) The events 'exactly two heads' and 'exactly three heads' are

A = exactly two head = { HHT,HTH,THH} → P(A) = 3/8

B = Exactly three head ={ HHH} → P(B) = 1/8

$A\cap B = \emptyset$ → $P(A\cap B) = 0$

A,B are disjoint since $P(A \cap B) = 0$ (Answer B)

(d) The events 'the first coin is head' and 'the second and the third coins are tails' are

A = First coin head = { HHT,HTH,HHH,HTT } → P(A) = 4/8 =1/2

B = Second and third coin tail = { TTT,HTT } → P(A) = 2/8 =1/4

$A \cap B$ = { HTT } → $P(A \cap B) = 1/8$

A,B are not disjoint (Disjoint) since $P(A \cap B) \neq 0$

A,B are independent since (Answer A)

$P(A \cap B) = P(A) \times P(B)$

$1/8 = 1/2 \times 1/4$

H.W

Q3. Suppose that a fair die is thrown twice independently, then

1. the probability that the sum of numbers of the two dice is less than or equal to 4 is;

S = {1,2,3,4,5,6} x {1,2,3,4,5,6}

n(S) = 6 x 6= 36 outcome

A = {sum ≤ 4} = {(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)}

P(A) = 6/36 = 1/6 =0.1667

2. the probability that at least one of the die shows 4 is;

B= { (1,4),(2,4),(3,4),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,4),(6,4)}

P(A) = 11/36 =0.3056

3. the probability that one die shows one and the sum of the two dice is four is;

C ={(1,3),(3,1)}

P (C) = 2/36 =0.0556

4. the event A={the sum of two dice is 4} and the event B={exactly one die shows two} are,

A ={(1,3),(3,1),(2,2)} --------- P(A) =3/36 = 1/12

B={(1,2),(2,1),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)}----P(B) =10/36=5/18

$A \cap B = \phi$ --------------- $P(A \cap B) = 0$

A , B disjoint

Q13. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is: (Let G:Green , B:Black)

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$P(G \cap G \cap B) + P(G \cap B \cap G) + P(B \cap G \cap G) =$

2/6x 2/6x 4/6 + 2/6x 4/6x 2/6 + 4/6x 2/6x 2/6 =3x(2/6x 2/6x 4/6) =3x2/27 =6/27

Q18. Suppose that the experiment is to randomly select with replacement 2 children and register their gender (B=boy, G=girl) from a family having 2 boys and 6 girls.

(1) The number of outcomes (elements of the sample space) of this experiment equals to

S ={B,G} x{B,G} = 2 x2 =4 outcomes (Answer = (A) 4)

S ={ BB ,BG,GB,GG} ----- P(B) =2/8=1/4 , P(G) =6/8 = 3/4

(2) The event that represents registering at most one boy is

{ BG,GB,GG} (Answer = (A)

(3) The probability of registering no girls equals to

P({ BB}) = P(B) x P(B) = 1/4 x 1/4=1/16 =0.0625 (B)

(4) The probability of registering exactly one boy equals to

P({ BG,GB}) = P (BG)+ P(GB) = 1/4x3/4 + 3/4x1/4 = 2x(1/4x3/4) =6/16=0.375 (B)

(5) The probability of registering at most one boy equals to

P({{ BG,GB,GG}) = P(BG)+P(GB)+P(GG) = 1/4x3/4+3/4x1/4+3/4x3/4= 0.9375