"from page 26 to 29"

"Sampling Distributions"

**Single Mean** if $X \sim N(\mu, \sigma^2) \Rightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N(\mu, \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n})$

① you can find it in "exercises for find the mean, median..."

② $X$: life of a certain battery

$X \sim N(\mu = 5, \sigma^2 = (1)^2)$

① $n = 5 \Rightarrow E(\bar{X}) = \mu_{\bar{X}} = \mu = 5$

② $n = 5 \Rightarrow Var(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{1}{5} = .2$

③ $n = 16 \Rightarrow \bar{X} \sim N(5, \frac{1}{16} = (\frac{1}{4})^2)$

$\therefore P(4.5 < \bar{X} < 5.4) = P(\frac{4.5-5}{1/4} < Z < \frac{5.4-5}{1/4})$

$= P(-2 < Z < 2) = P(Z < 2) - P(Z < -2)$

$= .9772 - .0228$

$= .9544$

.00
-2.0 — .0228

.00
2.0 — .9772

④ $n = 16 \Rightarrow \bar{X} \sim N(5, \frac{\sigma^2}{n} = \frac{1}{16} = (\frac{1}{4})^2)$

$P(\bar{X} < 5.5) = P(Z < \frac{5.5-5}{1/4} = 2) = .9772$

⑤ $n = 16 \Rightarrow \bar{X} \sim N(5, (\frac{1}{4})^2)$

$P(\bar{X} > 4.75) = P(Z > \frac{4.75-5}{1/4} = -1) = 1 - P(Z < -1)$

$= 1 - .1587$

$= .8413$

.00
-1.0 — .1587

⑥ $n = 9 \Rightarrow \bar{X} \sim N(5, \frac{1}{9} = (\frac{1}{3})^2)$

$\therefore P(\bar{X} > a) = .1492$

$\Rightarrow P(Z > \frac{a-5}{1/3}) = .1492$

$\Rightarrow 1 - P(Z < \frac{a-5}{1/3}) = .1492$

$\Rightarrow P(Z < \frac{a-5}{1/3}) = .8508$

$\Rightarrow \frac{a-5}{1/3} = 1.04$

$\Rightarrow a = 5.347$

.04
1.0 — .8508

③ X: lifespan of a certain light bulb.

$X \sim N(\mu = 400, \sigma^2 = (10)^2)$

① $P(X > 380) = P(Z > \frac{380-400}{10} = -2) = 1 - P(Z < -2)$

$= 1 - .0228 = .9772$

② $P(X < 380) = P(Z < \frac{380-400}{10} = -2) = .0228$

③ $n = 4 \Rightarrow \bar{X} \sim N(400, \frac{(10)^2}{4} = (\frac{10}{3})^2)$

$\therefore P(\bar{X} < 405) = P(Z < \frac{405-400}{10/3} = 1.5)$

$= .9332$

| | .00 |
|---|---|
| 1.5 | .9332 |

④ $X \sim N(55, \sigma^2 = (10)^2)$

$n = 64$

ⓐ $\bar{X} \sim N(55, \frac{\sigma^2}{n} = \frac{(10)^2}{64} = (\frac{10}{8})^2)$

ⓑ $\mu_{\bar{X}} = E(\bar{X}) = 55$

ⓒ $\sigma_{\bar{X}} = \frac{10}{8}$

ⓓ exceeds : يتجاوز أو يزيد عن

$P(\bar{X} > 52) = P(Z > \frac{52-55}{10/8} = -2.4)$

$= 1 - P(Z < -2.4)$

$= 1 - .0082$

$= .9918$

| | .00 |
|---|---|
| -2.4 | .0082 |

⑤ X: The amount of time that customers using ATM.

$X \sim N(\mu = 3, \sigma^2 = (1.4)^2)$

$n = 49$, $\bar{X} \sim N(3, \frac{(1.4)^2}{49} = (\frac{1.4}{7})^2)$

① $P(\bar{X} \geq 2.8) = P(Z > \frac{2.8-3}{1.4/7} = -1)$

(≥ : at least)

$= 1 - P(Z < -1)$

$= 1 - .1587$

$= .8413$

| | .00 |
|---|---|
| -1.0 | .1587 |

② $P(2.7 < \bar{X} < 3.2) = P(\frac{2.7-3}{1.4/7} < Z < \frac{3.2-3}{1.4/7})$

$= P(-1.5 < Z < 1) = P(Z < 1) - P(Z < -1.5)$

$= .8413 - .0668 = .7745$

| | .00 |
|---|---|
| 1.0 | .8413 |

| | .00 |
|---|---|
| -1.5 | .0668 |

(6) تكرار السؤال الثاني :

(7) X : life of an industrial machine

$X \sim N(\mu = 6, \sigma^2 = (1)^2)$

$n = 4 \Rightarrow \bar{X} \sim N(6, \frac{1}{4} = (\frac{1}{2})^2)$

① $E(\bar{X}) = \mu_{\bar{X}} = 6$

② $\sigma_{\bar{X}}^2 = Var(\bar{X}) = \frac{1}{4} = .25$

③ $P(\bar{X} < 5.5) = P(Z < \frac{5.5-6}{1/2} = -1)$

$= .1587$

| | .00 |
|---|---|
| -1.0 | .1587 |

④ $P(\bar{X} > a) = .1442$

$\Rightarrow P(Z > \frac{a-6}{1/2}) = .1442$

$\Rightarrow 1 - P(Z < \frac{a-6}{1/2}) = .1442$

$\Rightarrow P(Z < \frac{a-6}{1/2}) = .8508$

| | .04 |
|---|---|
| 1.0 | .8508 |

$\Rightarrow \frac{a-6}{1/2} = 1.04 \Rightarrow a = 6.52$

**Two means**

if we have $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ which are indep.

then $\bar{X}_1 \sim N(\mu_1, \frac{\sigma_1^2}{n_1})$ , $\bar{X}_1 \sim N(\mu_2, \frac{\sigma_2^2}{n_2})$ and

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$

① $n_1 = 36$ , $X_1 \sim N(\mu_1 = 70, \sigma_1^2 = (4)^2)$

$n_2 = 49$ , $X_2 \sim N(\mu_2 = 85, \sigma_2^2 = (5)^2)$ which $X_1$ and $X_2$ are indep.

ⓐ

$\bar{X}_1 \sim N(\mu_1 = 70, \sigma_{\bar{X}}^2 = \frac{\sigma_1^2}{n_1} = \frac{(4)^2}{36} = (\frac{4}{6})^2)$

ⓑ $\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2 = 70 - 85 = -15, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = (\frac{4}{6})^2 + (\frac{5}{7})^2 = \frac{421}{441})$

ⓒ $P(70 < \bar{X}_1 < 71) = P(\frac{70-70}{4/6} < Z < \frac{71-70}{4/6})$

$= P(0 < Z < 1.5) = P(Z < 1.5) - P(Z < 0)$

$= .4332 - .5 = .4332$

| | .00 |
|---|---|
| 0.0 | .5000 |

| | .00 |
|---|---|
| 1.5 | .4332 |

(d) $P(\bar{X}_1 - \bar{X}_2 > -16) = P\left(Z > \frac{-16-(-15)}{\sqrt{421/441}} \simeq -1.02\right)$

$= 1 - P(Z < -1.02)$ — .02

$= 1 - .1539 = .8461$ — $-1.0$ .1539

(2) $n_1 = 25$, $X_1 \sim N(\mu_1 = 100, \sigma_1^2 = (6)^2)$

$n_2 = 36$, $X_2 \sim N(\mu_2 = 97, \sigma_2^2 = (5)^2)$ which $X_1$ and $X_2$ are indep.

(1) $P(\bar{X}_1 \underset{\text{exceed}}{>} \bar{X}_2 + 6) = P(\bar{X}_1 - \bar{X}_2 > 6)$

$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2 = 100 - 97 = 3, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \left(\frac{6}{5}\right)^2 + \left(\frac{5}{6}\right)^2 = \frac{1921}{900}\right)$

$\therefore P\left(Z > \frac{6-3}{\sqrt{1921/900}} = 2.05\right)$ — .05

$= 1 - P(Z < 2.05)$ — 2.0 .9798

$= .0202$

(2) $P(\bar{X}_1 - \bar{X}_2 < 2) = P\left(Z < \frac{2-3}{\sqrt{1921/900}} \simeq -.68\right) = .2483$

.08

$-0.6$ .2483

**Single proportion** $\hat{p} \sim N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}}^2 = \frac{pq}{n}\right)$, $q = 1 - p$, $n \geq 30$

(1) (1) $E(\hat{p}) = 20 \div 100 = .2$, (2) $var(\hat{p}) = \frac{p(1-p)}{n} = \frac{.2(.8)}{5} = .032$, (3) and (4) we can't do it because $n = 5 < 30$

(2) $n = 100$, $p = .25$

(a) as $n = 100 \geq 30$ then $\hat{p} \sim N\left(.25, \frac{.25(1-.25)}{100} = \frac{3}{1600}\right)$

(b) $\mu_{\hat{p}} = E(\hat{p}) = .25$

(c) $\sigma_{\hat{p}} = \sqrt{\frac{3}{1600}} = .0433$

(d) $P(\hat{p} < .2) = P\left(Z < \frac{.2 - .25}{.0433} = .12\right) = .1251$

.02

0.1 .1251

**Two proportion**

ملغي

**t-distribution**

التمارين المطروحة مخصصة لجدول t الذي يحسب المساحة من اليمين

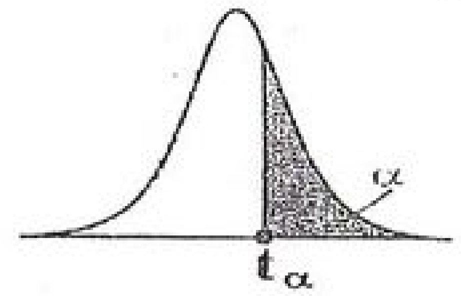
لذا يكتفى بالتمارين الموجودة في المحاضرة حيث هي مخصصة لجدول t الذي يحسب

المساحة من اليسار

Table B:—Quantiles of Student's t-distributions with given degrees of freedom for selected cumulative probabilities.

| df | Cumulative probability 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
|---|---|---|---|---|---|
| 1 | 3.078 | 6.3138 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.9200 | 4.3027 | 6.965 | 9.9248 |
| 3 | 1.638 | 2.3534 | 3.1825 | 4.541 | 5.8409 |
| 4 | 1.533 | 2.1318 | 2.7764 | 3.747 | 4.6041 |
| 5 | 1.476 | 2.0150 | 2.5706 | 3.365 | 4.0321 |
| 6 | 1.440 | 1.9432 | 2.4469 | 3.143 | 3.7074 |
| 7 | 1.415 | 1.8946 | 2.3646 | 2.998 | 3.4995 |
| 8 | 1.397 | 1.8595 | 2.3060 | 2.896 | 3.3554 |
| 9 | 1.383 | 1.8331 | 2.2622 | 2.821 | 3.2498 |
| 10 | 1.372 | 1.8125 | 2.2281 | 2.764 | 3.1693 |
| 11 | 1.363 | 1.7959 | 2.2010 | 2.718 | 3.1058 |
| 12 | 1.356 | 1.7823 | 2.1788 | 2.681 | 3.0545 |
| 13 | 1.350 | 1.7709 | 2.1604 | 2.650 | 3.0123 |
| 14 | 1.345 | 1.7613 | 2.1448 | 2.624 | 2.9768 |
| 15 | 1.341 | 1.7530 | 2.1315 | 2.602 | 2.9467 |
| 16 | 1.337 | 1.7459 | 2.1199 | 2.583 | 2.9208 |
| 17 | 1.333 | 1.7396 | 2.1098 | 2.567 | 2.8982 |
| 18 | 1.330 | 1.7341 | 2.1009 | 2.552 | 2.8784 |
| 19 | 1.328 | 1.7291 | 2.0930 | 2.539 | 2.8609 |
| 20 | 1.325 | 1.7247 | 2.0860 | 2.528 | 2.8453 |
| 21 | 1.323 | 1.7207 | 2.0796 | 2.518 | 2.8314 |
| 22 | 1.321 | 1.7171 | 2.0739 | 2.508 | 2.8188 |
| 23 | 1.319 | 1.7139 | 2.0687 | 2.500 | 2.8073 |
| 24 | 1.318 | 1.7109 | 2.0639 | 2.492 | 2.7969 |
| 25 | 1.316 | 1.7081 | 2.0595 | 2.485 | 2.7874 |
| 26 | 1.315 | 1.7056 | 2.0555 | 2.479 | 2.7787 |
| 27 | 1.314 | 1.7033 | 2.0518 | 2.473 | 2.7707 |
| 28 | 1.313 | 1.7011 | 2.0484 | 2.467 | 2.7633 |
| 29 | 1.311 | 1.6991 | 2.0452 | 2.462 | 2.7564 |
| 30 | 1.310 | 1.6973 | 2.0423 | 2.457 | 2.7500 |
| 35 | 1.3062 | 1.6896 | 2.0301 | 2.438 | 2.7239 |
| 40 | 1.3031 | 1.6839 | 2.0211 | 2.423 | 2.7045 |
| 45 | 1.3007 | 1.6794 | 2.0141 | 2.412 | 2.6896 |
| 50 | 1.2987 | 1.6759 | 2.0086 | 2.403 | 2.6778 |
| 60 | 1.2959 | 1.6707 | 2.0003 | 2.390 | 2.6603 |
| 70 | 1.2938 | 1.6669 | 1.9945 | 2.381 | 2.6480 |
| 90 | 1.2910 | 1.6620 | 1.9867 | 2.368 | 2.6316 |
| 120 | 1.2887 | 1.6577 | 1.9799 | 2.358 | 2.6175 |
| 160 | 1.2869 | 1.6545 | 1.9749 | 2.350 | 2.6070 |
| 200 | 1.2858 | 1.6525 | 1.9719 | 2.345 | 2.6006 |
| ∞ | 1.282 | 1.645 | 1.96 | 2.326 | 2.576 |

Percentage Points of the $t$ Distribution; $t_{\nu,\alpha}$ $\{P(T > t_{\nu,\alpha}) = \alpha\}$




| ν | α: 0.40 | 0.30 | 0.20 | 0.15 | 0.10 | 0.05 | 0.025 | 0.02 | 0.015 | 0.01 | 0.0075 | 0.005 | 0.0025 | 0.0005 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.325 | 0.727 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 15.895 | 21.205 | 31.821 | 42.434 | 63.657 | 127.322 | 636.590 |
| 2 | 0.289 | 0.617 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 5.643 | 6.965 | 8.073 | 9.925 | 14.089 | 31.598 |
| 3 | 0.277 | 0.584 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 3.896 | 4.541 | 5.047 | 5.841 | 7.453 | 12.924 |
| 4 | 0.271 | 0.569 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.298 | 3.747 | 4.088 | 4.604 | 5.598 | 8.610 |
| 5 | 0.267 | 0.559 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.003 | 3.365 | 3.634 | 4.032 | 4.773 | 6.869 |
| 6 | 0.265 | 0.553 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 2.829 | 3.143 | 3.372 | 3.707 | 4.317 | 5.959 |
| 7 | 0.263 | 0.549 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.715 | 2.998 | 3.203 | 3.499 | 4.029 | 5.408 |
| 8 | 0.262 | 0.546 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.634 | 2.896 | 3.085 | 3.355 | 3.833 | 5.041 |
| 9 | 0.261 | 0.543 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.574 | 2.821 | 2.998 | 3.250 | 3.690 | 4.781 |
| 10 | 0.260 | 0.542 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.527 | 2.764 | 2.932 | 3.169 | 3.581 | 4.587 |
| 11 | 0.260 | 0.540 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.491 | 2.718 | 2.879 | 3.106 | 3.497 | 4.437 |
| 12 | 0.259 | 0.539 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.461 | 2.681 | 2.836 | 3.055 | 3.428 | 4.318 |
| 13 | 0.259 | 0.538 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.436 | 2.650 | 2.801 | 3.012 | 3.372 | 4.221 |
| 14 | 0.258 | 0.537 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.415 | 2.624 | 2.771 | 2.977 | 3.326 | 4.140 |
| 15 | 0.258 | 0.536 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.397 | 2.602 | 2.746 | 2.947 | 3.286 | 4.073 |
| 16 | 0.258 | 0.535 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.382 | 2.583 | 2.724 | 2.921 | 3.252 | 4.015 |
| 17 | 0.257 | 0.534 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.368 | 2.567 | 2.706 | 2.898 | 3.222 | 3.965 |
| 18 | 0.257 | 0.534 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.356 | 2.552 | 2.689 | 2.878 | 3.197 | 3.922 |
| 19 | 0.257 | 0.533 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.346 | 2.539 | 2.674 | 2.861 | 3.174 | 3.883 |
| 20 | 0.257 | 0.533 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.336 | 2.528 | 2.661 | 2.845 | 3.153 | 3.850 |
| 21 | 0.257 | 0.532 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.328 | 2.518 | 2.649 | 2.831 | 3.135 | 3.819 |
| 22 | 0.256 | 0.532 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.320 | 2.508 | 2.639 | 2.819 | 3.119 | 3.792 |
| 23 | 0.256 | 0.532 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.313 | 2.500 | 2.629 | 2.807 | 3.104 | 3.768 |
| 24 | 0.256 | 0.531 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.307 | 2.492 | 2.620 | 2.797 | 3.091 | 3.745 |
| 25 | 0.256 | 0.531 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.301 | 2.485 | 2.612 | 2.787 | 3.078 | 3.725 |
| 26 | 0.256 | 0.531 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.296 | 2.479 | 2.605 | 2.779 | 3.067 | 3.707 |
| 27 | 0.256 | 0.531 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.291 | 2.473 | 2.598 | 2.771 | 3.057 | 3.690 |
| 28 | 0.256 | 0.530 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.286 | 2.467 | 2.592 | 2.763 | 3.047 | 3.674 |
| 29 | 0.256 | 0.530 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.282 | 2.462 | 2.586 | 2.756 | 3.038 | 3.659 |
| 30 | 0.256 | 0.530 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.278 | 2.457 | 2.581 | 2.750 | 3.030 | 3.646 |
| 40 | 0.255 | 0.529 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.250 | 2.423 | 2.542 | 2.704 | 2.971 | 3.551 |
| 60 | 0.254 | 0.527 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.223 | 2.390 | 2.504 | 2.660 | 2.915 | 3.460 |
| 120 | 0.254 | 0.526 | 0.845 | 1.041 | 1.289 | 1.658 | 1.980 | 2.076 | 2.196 | 2.358 | 2.468 | 2.617 | 2.860 | 3.373 |
| ∞ | 0.253 | 0.524 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.170 | 2.326 | 2.432 | 2.576 | 2.807 | 3.291 |