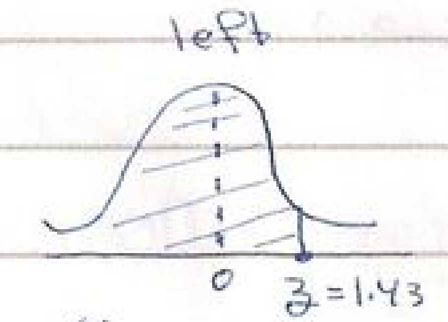
"From page 21 to 23"

"Normal Distribution"

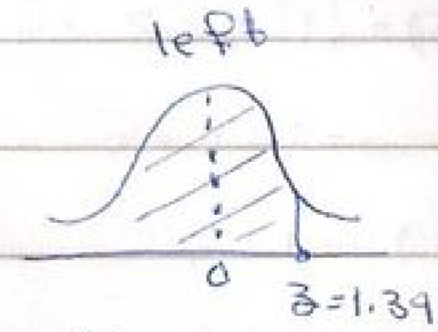
1) A) $Z \sim N(0,1)$

1) $p(Z<1.43) = .9236$



.03
1.4 —— .9236

2) $p(Z<1.39) = .9177$

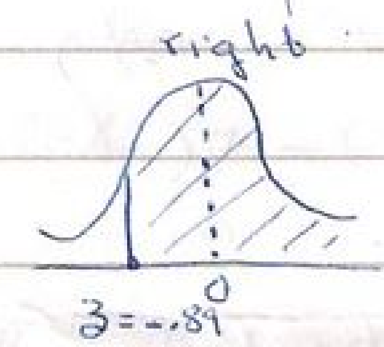


.09
1.3 —— .9177

3) $p(Z>-.89)$

$= 1 - p(Z<-.89)$

$= 1 - .1867 = .8133$



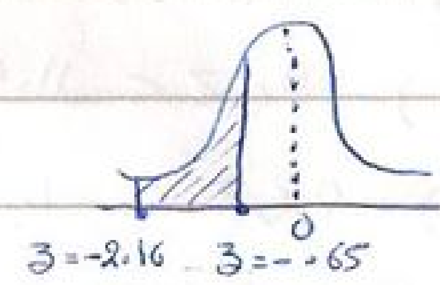
.09
-0.8 —— .1867

4) $p(-2.16<Z<-.65)$

$= p(Z<-.65) - p(Z<-2.16)$

$= .2578 - .0154$

$= .2424$



	.05	.06
-2.1	.0158	.0154
-0.6	.2578	

5) $p(.43<Z<k) = .0427$

$\Rightarrow p(Z<k) - p(Z<.43) = .0427$

$\Rightarrow p(Z<k) - .8238 = .0427$

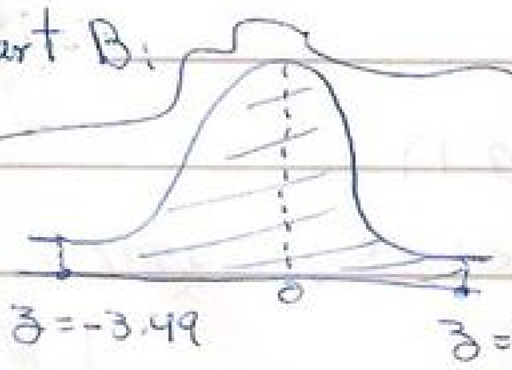
$\Rightarrow p(Z<k) = .0427 + .8238 = .8665$

$\therefore k = 1.11$

.03
.4 —— .8238

.01
1.1 —— .8665

(B) $Z \sim N(0,1)$, Note for part B:



From this curve we can see that

$P(Z<-3.49)=0$ and

$P(Z<3.49)=1$

so, $\forall\, a \le -3.49$, $b \ge 3.49$ we have:

$P(Z<a)=0$

$P(Z<b)=1$

① $P(Z<-3.9)=0$

② $P(Z>4.5)=1-P(Z<4.5)$

$=1-1=0$

③ $P(Z<3.7)=1$

④ $P(Z>-4.1)=1-P(Z<-4.1)$

$=1-0=1$

② X : The finished diameter of a piston ring

$X \sim N(\mu=12, \sigma^2=(.03)^2)$

① $P(X<12.05)=P\left(\frac{X-\mu}{\sigma}<\frac{12.05-12}{.03}\right)=P(Z<1.67)=.9525$

② exceeding : يتجاوز أي أكبر

$P(X>11.97)=P\left(\frac{X-\mu}{\sigma}\ \frac{11.97-12}{.03}\right)=P(Z>-1)=1-P(Z<-1)$

$=1-.1587=.8913$

	.00
-1.0	.1587

③ $P(11.95<X<12.05)$

$=P(X<12.05)-P(X<11.95)$

$=P\left(Z<\frac{12.05-12}{.03}\right)-P\left(Z<\frac{11.95-12}{.03}\right)$

$=P(Z<1.67)-P(Z<-1.67)$

$=.9525-.0475$

$=.905$

	.07
1.6	.9525

	.07
-1.6	.0475

③ X: a life of a certain type of small motor

$X \sim N(\mu=10, \sigma^2=(2)^2)$

The manufacturer replaces free all motors that fail while under guarantee $=k$

if he is willing to replace only 15% of motors that fail

$\Rightarrow P(X<k)=.015$

$\Rightarrow P\left(\frac{X-\mu}{\sigma}<\frac{k-10}{2}\right)=.015$

	.07
-2.1	.0150

$\Rightarrow \frac{k-10}{2}=-2.17$

$\Rightarrow k=5.66$

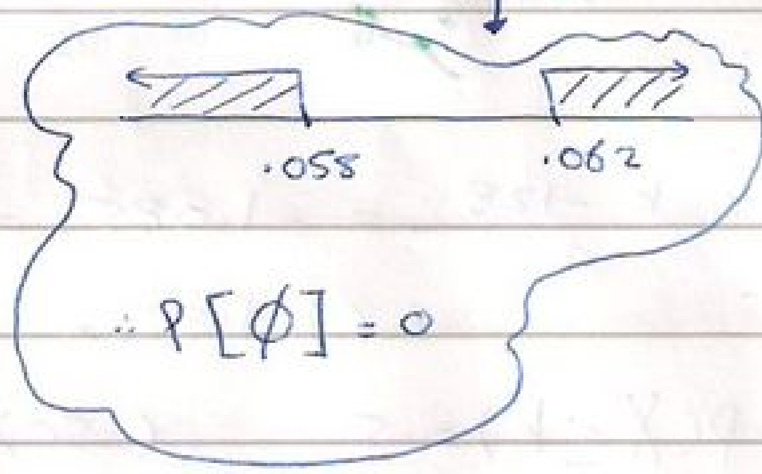
④ X: Produced bolts with diameters

$X \sim N(\mu = .06, \sigma^2 = (.001)^2)$

① $P[X < .058 \overset{\cup}{\text{or}} X > .062]$

$= P(X < .058) + P(X > .062) - P[(X < .058) \cap (X > .062)]$

$= P(Z < \frac{.058 - .06}{.001}) + P(Z > \frac{.062 - .06}{.001})$



$= P(Z < -2) + P(Z > 2)$

$= P(Z < -2) + [1 - P(Z < 2)]$

$= .0228 + [1 - .9772]$

$= .0456$

	.00
-2.0	.0228

	.00
2.0	.9772

② $P(X > a) = .1949$

$\Rightarrow 1 - P(X < a) = .1949$

$\Rightarrow P(X < a) = 1 - .1949$

$\Rightarrow P(X < a) = .8051$

$\Rightarrow P(Z < \frac{a - .06}{.001}) = .8051$

	.06
.8	.8051

$\Rightarrow \frac{a - .06}{.001} = .86$

$\Rightarrow a = .06086 \simeq .0609$

⑤ Skip

⑥ X: The weight of a large number of fat persons is nicely modeled

$X \sim N(\mu = 128, \sigma^2 = (9)^2)$

① $P(\underset{\text{at most}}{X \leq 110}) = P(Z < \frac{110 - 128}{9} = -2) = .0228$

	.00
-2.0	.0228

$\Rightarrow .0228 \times 100 = 2.28\%$

② $P(X > 149) = P(Z > \frac{149 - 128}{9} = 2.33) = 1 - P(Z < 2.33)$

$= 1 - .9901 = .0099$

$\Rightarrow .0099 \times 100 = .99\%$

	.03
2.3	.9901

③ $P(\underset{\text{above}}{X > k}) = .86$ $(86\% \div 100 = .86)$

$\Rightarrow P(Z > \frac{k - 128}{9}) = .86 \Rightarrow 1 - P(Z < \frac{k - 128}{9}) = .86 \Rightarrow P(Z < \frac{k - 128}{9}) = 1 - .86 = .14$

	.08		.09
-1.0	.1401	.1400	.1379

∴ so we take -1.08 which .1401 is closest from .1400

$\Rightarrow \frac{k-128}{4} = -1.08 \Rightarrow k = 118.28$

④ $P(X \leq k) = .5$ $(50\% \div 100 = .5)$
below

$\Rightarrow P(Z < \frac{k-128}{4}) = .5$

	.00
0.0	.5000

$\Rightarrow \frac{k-128}{4} = 0$

$\Rightarrow k = 128$

⑦ X: The lifespan of a certain electronic device

$X \sim N(\mu = 40, \sigma^2 = (2)^2)$

① $P(X < 38) = P(Z < \frac{38-40}{2} = -1) = .1587$

	.00
-1.0	.1587

② $P(38 < X < 40) = P(\frac{38-40}{2} < Z < \frac{40-40}{2})$

$= P(-1 < Z < 0) = P(Z < 0) - P(Z < -1)$

$= .5 - .1587 = .3413$

	.0
0.0	.5

③ as $X \sim N(40, (2)^2)$ which is a continuous random variables so $P(X = 38) = 0$

④ $P(X < k) = .7329$

$\Rightarrow P(Z < \frac{k-40}{2}) = .7329$

$\Rightarrow \frac{k-40}{2} = .62$

$\Rightarrow k = 41.24$

	.02		.03
.6	.7324	.7329	.7357

.0005 .0028

so we can see that .7325 is near from .7324 than .7357, so we take .62 of .7324.

(8) $X \sim N(\mu, \sigma^2)$

$\Rightarrow P(X < \mu + 2\sigma) = P(Z < \frac{(\mu+2\sigma)-\mu}{\sigma} = 2) = .9772$

	.00
2.0	.9772

(9) $X \sim N(\mu, 1)$

$P(X < 3) = .877$

$\Rightarrow P(Z < \frac{3-\mu}{1} = 3-\mu) = .877$

$\Rightarrow 3-\mu = .8770$

$\Rightarrow \mu = 1.84$

	.06
1.1	.8770

(10) X: marks of students in a certain course

$X \sim N(\mu = 70, \sigma^2 = 25 = (5)^2)$

33% of the student failed the exam.

$\Rightarrow P(X < k) = .33$

$\Rightarrow P(Z < \frac{k-70}{5}) = .33$

$\Rightarrow \frac{k-70}{5} = -.44$

$\Rightarrow k = 67.8$

	.04
-.4	.3300

(11) $X \sim N(\mu = 10, \sigma^2 = (36) = (6)^2)$

(1) $P(X \underset{\text{above}}{>} k) = .2296$

$\Rightarrow P(Z > \frac{k-10}{6}) = .2296$

$\Rightarrow \frac{k-10}{6} = -.74$

$\Rightarrow k = 14.44$

	.04
-.7	.2296

(2) $P(X \underset{\text{greater than}}{>} 16) = P(Z > \frac{16-10}{6} = 1) = 1 - P(Z<1) = 1 - .8413 = .1587$

	.00
1.0	.8413

(12) X: The marks of students in a certain course

$X \sim N(\mu = 65, \sigma^2 = 16 = (4)^2)$

A student fails the exam if he obtains a mark less than 60,

$P(X < 60) = P(Z < \frac{60-65}{4} = -1.25) = .1056$

	.05
-1.2	.1056

$\Rightarrow .1056 \times 100 = 10.56\%$

(13) X: The rainfall in a certain city for the month of march

$X \sim N(\mu = 9.22, \sigma^2 = (2.83)^2)$

① $P(X < 11.84) = P(Z < \frac{11.84 - 9.22}{2.83} = .9257 = .93)$

$= .8238$

	.03
.9	.8238

② $P(5 < X < 7) = P(\frac{5 - 9.22}{2.83} < Z < \frac{7 - 9.22}{2.83})$

$= P(-1.49 < Z < -.78)$

$= P(Z < -.78) - P(Z < -1.49)$

$= .2177 - .0681$

$= .1496$

	.08
-.7	.2177

	.09
-1.4	.0681

③ $P(X > 13.8)$

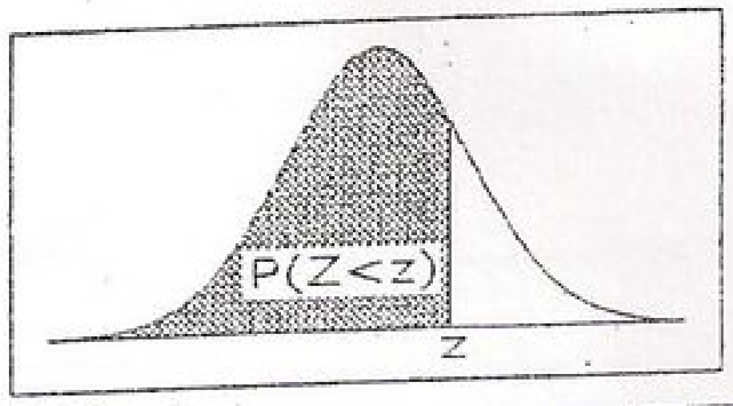
$= P(Z > \frac{13.8 - 9.22}{2.83} = 1.62)$

$= 1 - P(Z < 1.62)$

$= 1 - .9474$

$= .0526$

	.02
1.6	.9474



Areas Under The Standard Normal Curve

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998