" from page 4 to 5 and from 7 to 13 "

Exercises in

Probability, conditional probability and independence

(11) $p(B) = .3$, $p(A|B) = .4$

then $p(A\cap B) = p(A|B)\,p(B)$ $\left[\text{from } p(A|B) = \dfrac{p(A\cap B)}{p(B)}\right]$

$= (.4)(.3) = .12$

(12) A: the computer system has electrical failure

B: the computer has virus

$\therefore p(A) = .15$, $p(B) = .25$, $p(A\cap B) = .1$

then $p(A\cup B) = p(A) + p(B) - p(A\cap B) = .15 + .25 - .1 = .3$

(13) 3 balls (independently and with replaced)

4b
2g

$p(b) = \frac{4}{6} = \frac{2}{3}$

$p(g) = \frac{2}{6} = \frac{1}{3}$

عدد الطرق لترتيب اللعبة هي 3

$p(2g \text{ and } 1b) = p(1^{st}\text{ ball blake and } 2^{nd} \text{ and } 3^{th} \text{ ball are } g)$

$+ p(1^{st}\, g \text{ and } 2^{nd} \text{ ball blake and } 3^{th}\, g)$

$+ p(1^{st} \text{ and } 2^{nd} \text{ ball are } g \text{ and } 3^{th} \text{ ball is blake})$

$= p(1^{st}\, b)\, p(2^{nd}\, g)\, p(3^{th}\, g)$

$+ p(1^{st}\, g)\, p(2^{nd}\, b)\, p(3^{th}\, g)$

$+ p(1^{st}\, g)\, p(2^{nd}\, g)\, p(3^{th}\, b)$

we doing this because the indep.

$= \left[\left(\frac{4}{6}\right)\left(\frac{2}{6}\right)\left(\frac{2}{6}\right)\right](3) = \frac{8}{36} = \frac{2}{9} = \frac{6}{27}$

(14) E: student from engineering college

$E^c = C$: " " computer science "

T: " take 324 stat before

T^c: " not take " "

$n(E) = 60$, $n(S) = 20$

$p(E\cap T) = \frac{10}{100} = .1 = \frac{n(E\cap T)}{n(E)}$

$\Rightarrow n(E\cap T) = (.1)(60) = 6$

$p(E^c\cap T) = \frac{5}{100} = .05 = \frac{n(E^c\cap T)}{n(E^c)}$

$\Rightarrow n(E^c\cap T) = (.05)(20) = 1$

	T	T^c	Σ
E	$6 = n(E\cap T)$	$54 = n(E\cap T^c)$	$60 = n(E)$
E^c	$1 = n(E^c\cap T)$	$19 = n(E^c\cap T^c)$	$20 = n(E^c)$
Σ	$7 = n(T)$	$73 = n(T^c)$	$80 = n(S)$

وهناه بدلا من n(S) لانه ذكر ان 10%
من طلاب الهندسة (أي من عدد طلاب الهندسة)
ولم يذكر ان 10% من الطلاب (أي من عدد الطلاب الكلي)
هم طلاب هندسة واخذوا المقرر سابقا
واخذ n(E^c) بدلا من n(S) لانه ذكر ان 5%
من طلاب علوم الحاسب (أي من عدد طلاب علوم
الحاسب) ولم يذكر ان 5% من الطلاب
(أي من عدد الطلاب الكلي) هم طلاب علوم حاسب
واخذوا المقرر سابقا

[1] $p(T) = \frac{n(T)}{n(S)} = \frac{7}{80} = .0875$

[2] $p(E^c|T) = \frac{p(E^c\cap T)}{p(T)} = \frac{(1/80)}{(7/80)} = \frac{1}{7} = .14284$

(16) $P(A_1)=.4$, $P(A_1\cap A_2)=.2$, $P(A_3|A_1\cap A_2)=.75$

then $P(A_2|A_1)=\frac{P(A_2\cap A_1)}{P(A_1)}=\frac{.2}{.4}=.5$

and as given that

$P(A_3|A_1\cap A_2)=.75=\frac{P(A_3\cap A_1\cap A_2)}{P(A_1\cap A_2)}$ $\Rightarrow P(A_3\cap A_1\cap A_2)=P(A_3|A_1\cap A_2)\,P(A_1\cap A_2)$

$=(.75)(.2)=.15$

(17) $P(A)=.9$, $P(B)=.6$, $P(A\cap B)=.5$

then $\boxed{1}$ $P(A\cap B^c)=P(A)-P(A\cap B)$ $[\text{from } P(A)=P(A\cap B)+P(A\cap B^c)]$

$=.4$

$\boxed{2}$ $P(A^c\cap B^c)=P(A\cup B)^c=1-P(A\cup B)=1-[P(A)+P(B)-P(A\cap B)]$

$=1-[.9+.6-.5]=1-1=0$

$\boxed{3}$ $P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{.5}{.9}=.556$

also we can use the table to solution the last :

	A	A^c	$\mathcal{E}$
B	.5	.1	.6
B^c	.4	0	.4
$\mathcal{E}$	.9	.1	1

$\boxed{4}$ and $\boxed{5}$

$P(A\cap B)=.5$

$P(A)P(B)=(.9)(.6)=.54$

so as $P(A\cap B)\neq P(A)P(B)$ $\therefore$ A and B are dep.

and as $P(A\cap B)\neq 0$ $\therefore$ A and B are joint

(18)

| choose →
|---|
| 6 girls |
| 2 boys |
| $n = 8$ = total |

2 children with replacement (this given to me that are indep.)

نفس فكرة السؤال 13 تمارين

$\therefore p(G) = \frac{6}{8} = .75$ (G : denote for girl)

$p(B) = \frac{2}{8} = 1 - .75 = .25$ (B : denote for boy)

$\boxed{1}$

$S = \{G, B\} \times \{G, B\} = \{GG, GB, BG, BB\}$

$n(S) = 4 = 2 \times 2$

$\boxed{2}$ $A = \{$at most one boy$\} = \{GB, BG, GG\}$

$\boxed{3}$ $C = \{$no girls$\} = \{BB\}$

$\Rightarrow p(C) = p(\{BB\}) = p(\{B\})\, p(\{B\})$ [because the indep.]

$= (.25)(.25) = .0625$

$\boxed{4}$ $D = \{$exactly one boy$\} = \{GB, BG\}$

$\Rightarrow p(D) = p(\{\overset{\text{or} \cong \cup \cong +}{G\underbrace{B, B}_{\text{and} \cong \cap \cong \times}G}\}) = p(\{GB\}) + p(\{BG\})$

$= p(\{G\})\, p(\{B\}) + p(\{B\})\, p(\{G\})$

$= (.75)(.25) + (.25)(.75) = (2)(.25)(.75) = .375$

$\boxed{5}$

$\rightarrow p(A) = p(\{GB, BG, GG\}) = p(\{GB\}) + p(\{BG\}) + p(\{GG\})$

$= p(\{G\})\, p(\{B\}) + p(\{B\})\, p(\{G\}) + p(\{G\})\, p(\{G\})$

$= (.75)(.25) + (.75)(.25) + (.75)(.75)$

$= 2(.75)(.25) + (.75)^2 = .4375$

Random Variables, Distributions, Expectations and Chebyshiev's Theorem

Discrete Distributions : ① ذكر في نص السؤال أن القطعة المعدنية متزنة أي أن $P(H) = P(T) = \frac{1}{2}$ إذن يمكن تطبيق تعريف قبلي = $\frac{\text{العدد}}{\text{العدد الكلي}}$

a b $S = \{H,T\} \times \{H,T\} \times \{H,T\} = \{\overset{3-0=3}{HHH}, \overset{2-1=1}{HHT}, \overset{2-1=1}{HTH}, \overset{2-1=1}{THH}, \overset{1-2=-1}{TTH}, \overset{1-2=-1}{THT}, \overset{1-2=-1}{HTT}, \overset{0-3=-3}{TTT}\}$

X = number of heads - number of tails , $n(S) = 2 \times 2 \times 2 = 8$

$= -3, -1, 1, 3$

c

$P(X=-3) = P(\{TTT\}) = \frac{1}{8}$

$P(X=-1) = P(\{HTT, THT, TTH\}) = \frac{3}{8}$

$P(X=1) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X=3) = P(\{HHH\}) = \frac{1}{8}$

X	-3	-1	1	3	Σ
$P(X=x) = f(x)$	$1/8$	$3/8$	$3/8$	$1/8$	1
$x f(x)$	$-3/8$	$-3/8$	$3/8$	$3/8$	$E(X) = \Sigma x f(x) = 0 = \mu$
$x^2 f(x)$	$9/8$	$3/8$	$3/8$	$9/8$	$\Sigma x^2 f(x) = E(X^2) = 3$

d

$P(X \le 1) = f(1) + f(-1) + f(-3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$

e

$P(X < 1) = f(-1) + f(-3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

f

$E(X) = \mu = 0$

g $Var(X) = \sigma^2 = E(X^2) - [E(X)]^2 = 3 - 0 = 3$

② [1][2] $S = \{M,F\} \times \{M,F\} = \{\overset{0}{MM}, \overset{1}{MF}, \overset{1}{FM}, \overset{2}{FF}\}$, $P(F) = .2$, $P(M) = 1 - .2 = .8$

X = number of females in the committee.

$= 0, 1, 2$

[3] $P(X=0) = P(\{MM\}) = P(\{M\})\,P(\{M\}) = (.8)(.8) = .64$

$P(X=1) = P(\{MF, FM\}) = (.8)(.2)(.2) = .32$

$P(X=2) = P(\{FF\}) = (.2)(.2) = .04$

$\therefore$

x	0	1	2	Σ
$P(X=x)=f(x)$	.64	.32	.04	1
$x\,f(x)$	0	.32	.08	$E(X) = \Sigma x f(x) = .4$
$x^2 f(x)$	0	.32	.16	$E(X^2) = \Sigma x^2 f(x) = .48$

[4] P(at least one female in the committee)

$= P(X \geq 1) = P(X=1) + P(X=2) = f(1) + f(2) = .32 + .04 = .36$

[5] P(at most one female in the committee)

$= P(X \leq 1) = P(X=0) + P(X=1) = .64 + .32 = .96$

[6] $\mu = E(X) = \Sigma x f(x) = .4$

[7] $Var(X) = \sigma^2 = \Sigma x^2 f(x) = E(X^2) - [E(X)]^2 = .48 - (.4)^2 = .48 - (.16) = .32$

③

[i][ii] 40 of 5, 60 of 10, total = 100 → 2 card with replacement (indep.), $S = \{5,10\} \times \{5,10\} = \{\overset{10}{(5,5)}, \overset{15}{(5,10)}, \overset{15}{(10,5)}, \overset{20}{(10,10)}\}$

$P(5) = \frac{40}{100} = .4$, $P(10) = \frac{60}{100} = .6$

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X = the total sum of the two cards.

$= 10, 15, 20$

[iii] $P(X=10) = P(\{(5,5)\}) = (.4)(.4) = .16$

$P(X=15) = P(\{(5,10),(10,5)\}) = P(\{(5,10)\}) + P(\{(10,5)\})$

$= (.4)(.6) + (.4)(.6) = (2)(.4)(.6) = .48$

$P(X=20) = P(\{(10,10)\}) = (.6)(.6) = .36$

$\therefore$

x	10	15	20	Σ
$P(X=x)=f(x)$	.16	.48	.36	1
$x\,f(x)$	1.6	7.2	7.2	$E(X) = 16$
$x^2 f(x)$	16	108	144	$E(X^2) = 268$

[iv] $p(X=0)=0$

[v] $p(X>10)=p(X=15)+p(X=20)=.48+.36=.84$

[vi] $\mu=E(X)=16$

[vii] $\sigma^2=Var(X)=E(X^2)-[E(X)]^2=268-(16)^2=12$

(4)

[1] $\mu=E(X)=(-3)(.1)+(6)(.5)+(9)(.4)=6.3$

[2] $E(X^2)=(-3)^2(.1)+(6)^2(.5)+(9)^2(.4)=51.3$

[3] $Var(X)=\sigma_X^2=E(X^2)-[E(X)]^2=(51.3)-(6.3)^2=11.61$

[4] $\mu_{2X+1}=E(2X+1)=E(2X)+E(1)=2E(X)+1=2(6.3)+1=13.6$

[5]

$\sigma_{2X+1}^2=Var(2X+1)=Var(2X)+Var(1)=4Var(X)+0=4(11.61)=46.44$

(5)

[A] $f(x)=\frac{x+1}{10} \Rightarrow f(0)=\frac{1}{10}<1$

$f(1)=\frac{2}{10}<1$

$f(2)=\frac{3}{10}<1$

$f(3)=\frac{4}{10}<1$

$f(4)=\frac{5}{10}<1$

and $\sum f(x)=\frac{1+2+3+4+5}{10}=\frac{15}{10}\neq 1$

$\therefore$ it is not probability

[B] $f(x)=\frac{x-1}{10} \Rightarrow f(0)=\frac{-1}{5}<0$

$\therefore$ it is not probability

[C] $f(x)=\frac{1}{5} \Rightarrow f(0)=f(1)=f(2)=f(3)=f(4)=\frac{1}{5}<1$

and $\sum f(x)=\frac{1+1+1+1+1}{5}=\frac{5}{5}=1$

$\therefore$ it is probability

(6) [1] $E(X)=\mu=\sum x f(x)=(0)(\frac{1}{3})+1(\frac{1}{3})+2(\frac{1}{3})=\frac{1+2}{3}=\frac{3}{3}=1$

[2] $E(X^2)=\sum x^2 f(x)=(0)^2(\frac{1}{3})+(1)^2(\frac{1}{3})+(2)^2(\frac{1}{3})=\frac{1+4}{3}=\frac{5}{3}=1.67$

$\therefore Var(X)=E(X^2)-[E(X)]^2=1.67-1=0.67$

(7) $f(x)=kx,\ x=1,2,3$

[1] we know that $\sum f(x)=1 \Rightarrow k+2k+3k=1 \Rightarrow 6k=1 \Rightarrow k=\frac{1}{6}$

$\therefore f(x)=\frac{x}{6},\ x=1,2,3$

[2] $F(1)=P(X\le 1)=P(X=1)=f(1)=\frac{1}{6}$

$F(2)=P(X\le 2)=P(X=2)+P(X=1)=\frac{3}{6}$

 $F(3)=P(X\le 3)=P(X=3)+P(X=2)+P(X=1)=1$

$$\therefore F(x) = P(X \le x) = \begin{cases} 0 & x < 1 \\ 1/6 & 1 \le x < 2 \\ 3/6 = 1/2 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

$\boxed{3}$ $P(.5 < X \le 2.5) = P(X \le 2.5) - P(X < .5) = F(2.5) - F(.5)$

$= \frac{1}{2} - 0 = \frac{1}{2}$

because in discrete random we have

$P(a < X \le b) = F(b) - F(a)$

$P(a < X < b) = F(b-1) - F(a)$

$P(a \le X \le b) = F(b) - F(a-1)$ and

$P(X = x) = F(x) - F(x-1)$

⑧

$\boxed{1}$ we have $F(x) = \begin{cases} 0 & , x < 0 \\ .25 & , 0 \le x < 1 \\ .6 & , 1 \le x < 2 \\ 1 & , 2 \le x \end{cases}$

$f(0) = .25 - 0 = .25$

$f(1) = .6 - .25 = .35$

$f(2) = 1 - .6 = .4$

$$\therefore f(x) = \begin{cases} .25 & , x = 0 \\ .35 & , x = 1 \\ .4 & , x = 2 \end{cases}$$

$\boxed{2}$

$P(1 \le X < 2) = \begin{cases} P(X=1) = .35 = f(1) & \text{, by used } f(x) \\ F(2-1) - F(1-1) = F(1) - F(0) = .6 - .25 = .35 & \text{, by used } F(x) \end{cases}$

$\boxed{3}$ $P(X > 2) = 1 - P(X \le 2) = \begin{cases} 1 - (f(0) + f(1) + f(2)) = 0 & \text{, by used } f(x) \\ 1 - F(2) = 1 - 1 = 0 & \text{, by used } F(x) \end{cases}$

⑨ we know that $\sum f(x) = 1$

$\Rightarrow (.4) + C + (.3) + (.1) = 1 \quad \Rightarrow \quad .8 + C = 1 \quad \Rightarrow C = 1 - .8 = .2$

⑩ $\boxed{a}$ $\sum f(x) = 1 \Rightarrow .2 + .3 + .2 + C = 1 \Rightarrow C = .3$

$\boxed{b}$ $P(0 < X \le 2) = f(1) + f(2) = .2 + .3 = .5$

$\boxed{c}$ $\mu = E(X) = (-1)(.2) + (0)(.3) + (1)(.2) + (2)(.3) = .6$

$\boxed{d}$ $E(X^2) = (-1)^2(.2) + (0)^2(.3) + (1)^2(.2) + (2)^2(.3) = 1.6$

$\boxed{e}$ $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2 = 1.6 - (.6)^2 = 1.6 - .36 = 1.34$

(11) we know that $\sum f(x) = 1$

$$\Rightarrow k\binom{2}{0}\binom{3}{3} + k\binom{2}{1}\binom{3}{2} + k\binom{2}{2}\binom{3}{1} = 1 \Rightarrow k + 6k + 3k = 1 \Rightarrow 10k = 1 \Rightarrow k = \frac{1}{10}$$

$$\therefore f(x) = \frac{1}{10}\binom{2}{x}\binom{3}{3-x}, \quad x = 0, 1, 2$$

(12)

$\boxed{a}$ $p(X=2) = F(2) - F(2-1) = F(2) - F(1) = \frac{11}{16} - \frac{5}{16} = \frac{6}{16} = \frac{3}{8}$

$\boxed{b}$ $p(2 \le X < 4) = F(4-1) - F(2-1)$

$= F(3) - F(1) = \frac{15}{16} - \frac{5}{16} = \frac{10}{16}$

(13) given that $E(X) = 10$ and $Var(X) = 4$

then $\boxed{a}$

$E(2X-2) = E(2X) - E(2) = 2E(X) - 2 = 2(10) - 2 = 20 - 2 = 18$

$\boxed{b}$ $Var(2X-2) = Var(2X) + Var(-2) = 4Var(X) = 4(4) = 16 = \sigma^2$

$\Rightarrow \sigma = +\sqrt{\sigma^2} = +\sqrt{16} = 4$

(14)

$\boxed{1}$ $\sum f(x) = 1 \Rightarrow 3k + 3k + 2k + k + k = 1 \Rightarrow 10k = 1 \Rightarrow k = \frac{1}{10}$

$\therefore$

x	0	1	2	3	4	$\sum$
$f(x)$	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	1

$\boxed{2}$

$E(X) = \sum x f(x) = (0)(\frac{3}{10}) + (1)(\frac{3}{10}) + (2)(\frac{2}{10}) + (3)(\frac{1}{10}) + (4)(\frac{1}{10}) = \frac{3+4+3+4}{10} = \frac{14}{10} = 1.4$

$\boxed{3}$

$E(X) = \sum x^2 f(x) = (0)^2(\frac{3}{10}) + (1)^2(\frac{3}{10}) + (2)^2(\frac{2}{10}) + (3^2)(\frac{1}{10}) + (4)^2(\frac{1}{10}) = \frac{3+8+9+16}{10} = \frac{36}{16} = 3.6$

$\therefore Var(X) = 3.6 - (1.4)^2 = 1.64$

$\boxed{4}$

x	0	1	2	3	4	$\sum$
$F(x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{8}{10}$	$\frac{4}{10}$	1	

$$\therefore F(x) = p(X \le x) = \begin{cases} 0 & x < 0 \\ \frac{3}{10} & 0 \le x < 1 \\ \frac{6}{10} & 1 \le x < 2 \\ \frac{8}{10} & 2 \le x < 3 \\ \frac{4}{10} & 3 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

$\boxed{5}$ $p(\text{at least 2 errors}) = p(X \ge 2) = 1 - p(X < 2) = \begin{cases} 1 - F(2-1) = 1 - F(1) = 1 - \frac{6}{10} = \frac{4}{10}, & \text{by used } F(x) \\ 1 - (f(0) + f(1)) = 1 - \frac{6}{10} = \frac{4}{10}, & \text{by used } f(x) \end{cases}$

(15) given $f(-1)=.05,\ f(0)=.25,\ f(1)=.25,\ f(2)=.45$

$\boxed{1}$ $P(X<1)=P(X=0)+P(X=-1)=.25+.05=.3$

$\boxed{2}$ $P(X\leq 1)=P(X<1)+P(X=1)=.3+.25=.55$

$\boxed{3}$ $E(X)=(-1)(.05)+0+(.25)+2(.45)=-.05+.25+.9=1.1$

$\boxed{4}$ $E(X^2)=1(.05)+0+(.25)+4(.45)=.05+.25+1.8=2.1$

$\boxed{5}$ $Var(X)=2.1-(1.1)^2=.89$

$\boxed{6}$ $F(1)=P(X\leq 1)=P(X=1)+P(X=0)+P(X=-1)=.05+.25+.25=.55$

Continuous distributions:

(1) $P(X=16)=0$ (because X is continuous random)

(2) $\boxed{1}$ we know $\int_{-\infty}^{\infty} f(x)\,dx=1$

$\Rightarrow \int_0^1 k\sqrt{x}\,dx=1 \Rightarrow k\left[\frac{x^{3/2}}{3/2}\right]_0^1=1 \Rightarrow \frac{2}{3}k=1 \Rightarrow k=\frac{3}{2}=1.5$

$\boxed{2}$ $P(.3<X\leq .6)=\int_{.3}^{.6} 1.5\sqrt{x}\,dx=1.5\left[\frac{x^{3/2}}{3/2}\right]_0^1=.3004$

$\boxed{3}$ $E(X)=\int_0^1 x\,f(x)\,dx=\int_0^1 x(1.5\sqrt{x})\,dx=1.5\int_0^1 x^{3/2}\,dx=1.5\left[\frac{x^{5/2}}{5/2}\right]_0^1=.6$

(3) $\boxed{i}$ $\int_0^2 k(x+1)\,dx=1 \Rightarrow k\left[\frac{x^2}{2}+x\right]_0^2=1 \Rightarrow k\left[\frac{4}{2}+2\right]=1 \Rightarrow k\left[\frac{8}{2}\right]=1$

$\Rightarrow 4k=1 \Rightarrow k=\frac{1}{4} \quad \therefore f(x)=\frac{1}{4}(x+1),\ 0<x<2$

$\boxed{ii}$ $P(0<X\leq 1)=\frac{1}{4}\int_0^1 (x+1)\,dx=\frac{1}{4}\left[\frac{x^2}{2}+x\right]_0^1=\frac{1}{4}\left[\frac{1}{2}+1\right]=\frac{1}{4}\left[\frac{3}{2}\right]=\frac{3}{8}=.375$

$\boxed{iii}$ $F(x)=P(X\leq x)=\begin{cases} 0 & ,x<0 \\ \int_0^x f(t)\,dt=\frac{1}{4}\int_0^x (t+1)\,dt=\frac{1}{4}\left(\frac{x^2}{2}+x\right) & ,0\leq x<2 \\ 1 & ,2\leq x \end{cases}$

(4) given $f(x)=\frac{3}{2}x^2,\ -1<x<1$

$\boxed{1}$ $P(0<X<1)=\int_0^1 \frac{3}{2}x^2\,dx=\frac{3}{2}\left[\frac{x^3}{3}\right]_0^1=\frac{1}{2}$

$\boxed{2}$ $E(X)=\int_{-1}^1 x\,f(x)\,dx=\frac{3}{2}\int_{-1}^1 x^3\,dx=\frac{3}{2}\left[\frac{x^4}{4}\right]_{-1}^1=\frac{3}{2}\left[\frac{1}{4}-\frac{1}{4}\right]=0$

$\boxed{3}$ $E(X^2)=\int_{-1}^1 x^2 f(x)\,dx=\frac{3}{2}\int_{-1}^1 x^4 \quad =\frac{3}{2}\left[\frac{x^5}{5}\right]_{-1}^1$

$\Rightarrow Var(X)=E(X^2)-[E(X)]^2=.6$

$\boxed{4}$ $E(2X+3)=2E(X)+3=3$

$\boxed{5}$ $Var(2X+3)=4\,Var(X)=4(.6)=2.4$

⑤ $\boxed{1}$ we know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$\Rightarrow \int_0^2 kx\,dx = 1 \Rightarrow k\left[\frac{x^2}{2}\right]_0^2 = 1 \Rightarrow k\left[\frac{4}{2}\right] = 1 \Rightarrow k = \frac{1}{2} \quad \therefore f(x) = \frac{1}{2}x \; , 0 < x < 2$

$\boxed{2}$ $F(x) = P(X \leq x) = \begin{cases} 0 & , \; x < 0 \\ \int_0^x \frac{1}{2} t\,dt = \frac{x^2}{4} & , \; 0 \leq x < 2 \\ 1 & , \; 2 \leq x \end{cases}$

$\boxed{3}$ $P(0 < X < 1) = \begin{cases} \frac{1}{2}\int_0^1 x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 = \frac{1}{4} & , \text{by used } f(x) \\ F(1) - F(0) = \frac{1}{4} - \frac{0}{4} = \frac{1}{4} & , \text{by used } F(x) \end{cases}$

$\boxed{4}$ $P(X = 1) = 0$ (because X is continuous random)

$P(2 < X < 3) = \int_2^3 f(x)\,dx = 0$ (because X defind in $0 < X < 2$)

⑥ $f(x) = \begin{cases} 6x(1-x) & ; \; 0 < x < 1 \\ 0 & ; \; o.w \end{cases}$

$\boxed{1}$ $\mu = E(X) = \int_0^1 x f(x)\,dx = 6\int_0^1 x^2(1-x)\,dx = 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 6\left[\frac{1}{3} - \frac{1}{4}\right] = 6\frac{1}{12} = \frac{1}{2}$

$\boxed{2}$ $E(X^2) = \int_0^1 x^2 f(x)\,dx = 6\int_0^1 x^3(1-x)\,dx = 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = 6\left[\frac{1}{4} - \frac{1}{5}\right] = 6\frac{1}{20} = \frac{3}{10}$

$\therefore var(X) = E(X^2) - [E(X)]^2 = \frac{3}{10} - \left(\frac{1}{4}\right) = \frac{12-10}{40} = \frac{2}{40} = \frac{1}{20}$

$\boxed{3}$ $E(4X+5) = 4E(X) + 5 = 4\left(\frac{1}{2}\right) + 5 = 2 + 5 = 7$

$\boxed{4}$ $Var(4X+5) = 16\,Var(X) = 16\left(\frac{1}{20}\right) = \frac{4}{5}$

used $f(x)$

(7) [1] $p(X<6)=\int_0^6 \frac{1}{10}\,dx=\frac{1}{10}[x]_0^6=\frac{1}{10}(6)=\frac{3}{5}$

[2] $E(X)=\int_0^{10} x f(x)\,dx=\frac{1}{10}\int_0^{10} x\,dx=\frac{1}{10}\left[\frac{x^2}{2}\right]_0^{10}=\frac{1}{10}\left[\frac{100}{2}\right]=\frac{10}{2}=5$

[3] $E(X^2)=\int_0^{10} x^2 f(x)\,dx=\frac{1}{10}\int_0^{10} x^2\,dx=\frac{1}{10}\left[\frac{x^3}{3}\right]_0^{10}=\frac{1}{10}\left[\frac{10^3}{3}\right]=\frac{100}{3}$

[4] $Var(X)=E(X^2)-[E(X)]^2=\frac{100}{3}-(5)^2=\ldots\ldots$

[5]

$$F(x)=p(X\le x)=\begin{cases}0 & , x<0\\ \int_0^x f(t)\,dt=\int_0^x \frac{1}{10}\,dx=\frac{x}{10} & , 0\le x<10\\ 1 & , 10\le x\end{cases}$$

[6]

$$p(1<X\le 5)=\begin{cases}F(5)-F(1)=\frac{5}{10}-\frac{1}{10}=\frac{4}{10} & \text{, by used } F(x)\\ \frac{1}{10}\int_1^5 dx=\frac{1}{10}[x]_1^5=\frac{1}{10}[5-1]=\frac{4}{10} & \text{, by used } f(x)\end{cases}$$

(8) . [a] $E(X)=\int_0^{10} x f(x)\,dx=\frac{1}{50}\int_0^{10} x^2\,dx=\frac{1}{50}\left[\frac{x^3}{3}\right]_0^{10}=\frac{1}{50}\left[\frac{10^3}{3}\right]=6.667$

[b] $E(X^2)=\int_0^{10} x^2 f(x)\,dx=\frac{1}{50}\int_0^{10} x^3\,dx=\frac{1}{50}\left[\frac{x^4}{4}\right]_0^{10}=\frac{1}{50}\left[\frac{10^4}{4}\right]$

$\therefore Var(X)=E(X^2)-[E(X)]^2=5.55$

[c] $p(X>5)=\int_5^{10}\frac{1}{50}x\,dx=\frac{1}{50}\left[\frac{x^2}{2}\right]_5^{10}=.75$

(9) [1] $P(0<X<2)=F(2)-F(0)=\frac{2}{2+1}-\frac{0}{0+1}=\frac{2}{3}=.667$

[2] $P(X\le k)=.5 \Rightarrow F(k)=.5 \Rightarrow \frac{k}{k+1}=.5 \Rightarrow k=.5k+.5 \Rightarrow .5k=.5 \Rightarrow k=1$

(10) [1] $P(X > .5) = \int_{.5}^{1} 20x^3(1-x)\,dx = .8125$

[2] $P(.25 < X < 1.75) = P(.25 < X < 1)$ [because X defind in $0 < x < 1$]

$$= \int_{.25}^{1} 20x^3(1-x)\,dx = 20\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_{.25}^{1} = .9844$$

[3] $E(X) = \int_0^1 20x^4(1-x)\,dx = 20\left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1 = .667 = \mu$

[4] $E(X^2) = 20\int_0^1 x^5(1-x)\,dx = 20\left[\frac{x^6}{6} - \frac{x^7}{7}\right]_0^1 = \ldots$

$\therefore Var(X) = E(X^2) - [E(X)]^2 = .0317 \Rightarrow \sigma = \sqrt{\sigma^2} = \ldots$

[5] $P(\mu - 2\sigma \le X \le \mu + 2\sigma) = P(.6036 \le X \le .7304)$.

$$= \int_{.6036}^{.7304} f(x)\,dx = .965 \quad \text{[exact value]}$$

[6] the lower bound value by using chebyshev's thory is will be

$P(\mu - 2\sigma \le X \le \mu + 2\sigma) = P(-2\sigma \le X - \mu \le 2\sigma)$.

$= P(|X - \mu| \le 2\sigma) \ge 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} = .75$

[7]

$E(Y) = E(3X - 1.5) = 3E(X) - 1.5 = .5$

$Var(Y) = Var(3X - 1.5) = 9\,Var(X) = .2853$

chebyshev's Theorem: $P(|X - \mu| < k\sigma) = P(\mu - k\sigma < X < \mu + k\sigma) \ge 1 - \frac{1}{k^2}$

(1) we solve it in the last exercise part [6]

(2) $\mu = 12$, $\sigma^2 = 9 \Rightarrow \sigma = \sqrt{9} = 3$

$P(3 < X < 21)$

$\mu - k\sigma = 3$	$\mu + k\sigma = 21$
$\Rightarrow 12 - k(3) = 3$	$\Rightarrow 12 + 3k = 21$
$\Rightarrow 3k = 12 - 3$	$\Rightarrow 3k = 21 - 12$
$\Rightarrow 3k = 9$	$\Rightarrow 3k = 9$
$\Rightarrow k = 3$	$\Rightarrow k = 3$

$\Rightarrow P(3 < X < 21) \ge 1 - \frac{1}{3^2}$

$= \frac{8}{9}$

(3) $E(X)=5$, $Var(X)=4 \Rightarrow \sigma=2$

$\boxed{i}$ $P(1<X<9)$

$\mu+k\sigma=9$

$\mu-k\sigma=1$ $\Rightarrow 5+2k=9$

$\Rightarrow 5-k2=1$ $\Rightarrow 2k=9-5$

$\Rightarrow 2k=4$ $\Rightarrow 2k=4$

$\Rightarrow k=2$ $\Rightarrow k=2$

$\therefore P(1<X<9) \geq 1-\frac{1}{2^2} = \frac{3}{4} \Rightarrow P(1<X<9) \simeq \frac{3}{4}$

$\boxed{ii}$

$P(a<X<b) \geq 1-\frac{1}{k^2} \Rightarrow P(a<X<b) \simeq \frac{15}{16}$

$\Rightarrow 1-\frac{1}{k^2} \simeq \frac{15}{16} \Rightarrow \frac{1}{k^2} \simeq \frac{1}{16} \Rightarrow k^2 \simeq 16 \Rightarrow k \simeq 4$

$\therefore \mu+k\sigma=b \Rightarrow 5+4(2)=b \Rightarrow b=13$

$\mu-k\sigma=a \Rightarrow 5-4(2)=a \Rightarrow a=-3$

(4) $\mu=5$, $\sigma=2.89$

$\boxed{1}$ $P(\mu-1.5\sigma<X<\mu+1.5\sigma) = P(5-1.5(2.89)<X<5+1.5(2.89))$

$= P(5-4.335<X<5+4.335)$

$$= \frac{1}{10}\int_{5-4.335}^{5+4.335} dx = \frac{1}{10}\left[5+4.335-(5-4.335)\right] = \frac{1}{5}\left[4.335\right] = .867$$

$\boxed{2}$ $P(\mu-1.5\sigma<X<\mu+1.5\sigma) > 1-\frac{1}{(1.5)^2} = .556$

$\therefore P(\mu-1.5\sigma<X<\mu+1.5\sigma) \simeq .556$

$\boxed{3}$ we can see that part $\boxed{1}$ > part $\boxed{2}$

⑤ $E(X)=30$, $Var(X)=4$, $E(Y)=10$, $Var(Y)=2$

$\boxed{1}$ $E(2X-3Y-10) = 2E(X) - 3E(Y) - 10 = 2(30) - 3(10) - 10 = 60 - 30 - 10 = 20$

$\boxed{2}$

$Var(2X-3Y-10) = 4\,Var(X) + 4\,Var(Y) + 2(2)(-3)\,Cov(X,Y)$

as X and Y are indep. so $Cov(X,Y)=0$

$\therefore Var(2X-3Y-10) = 4\,Var(X) + 9\,Var(Y) = 4(4) + 9(2) = 16 + 18 = 34$

$\boxed{3}$ $P(24 < X < 36)$

$\mu_x - k\sigma_x = 24$	$\mu_x + k\sigma_x = 36$
$\Rightarrow 30 - 2k = 24$	$\Rightarrow 30 + 2k = 36$
$\Rightarrow 2k = 30 - 24$	$\Rightarrow 2k = 36 - 30$
$\Rightarrow 2k = 6$	$\Rightarrow 2k = 6$
$\Rightarrow k = 3$	$\Rightarrow k = 3$

$\therefore P(24 < X < 36) \geq 1 - \frac{1}{9} = \frac{8}{9} = .8889$

$\therefore P(24 < X < 36) \simeq .8889$