

1. (3 points) Draw the truth table of the following statement: $(p \rightarrow q) \vee (\neg p \rightarrow \neg q)$.

3

p	q	$\neg p$	$\neg q$	$(p \rightarrow q)$	$\neg p \rightarrow \neg q$	$(p \rightarrow q) \vee (\neg p \rightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

So $(p \rightarrow q) \vee (\neg p \rightarrow \neg q)$ is a tautology

2. (4 points) Using logic laws, show that $(p \rightarrow q) \wedge r \equiv \neg(r \rightarrow p) \vee \neg(r \rightarrow \neg q)$.

4

$$\begin{aligned}
 \neg(r \rightarrow p) \vee \neg(r \rightarrow \neg q) &\equiv \neg(\neg r \vee p) \vee \neg(\neg r \vee \neg q) \\
 &\equiv (\neg \neg r \wedge \neg p) \vee (\neg \neg r \wedge \neg \neg q) \\
 &\equiv r \wedge (\neg p \vee q) \\
 &\equiv r \wedge (p \rightarrow q) \\
 &\equiv (p \rightarrow q) \wedge r
 \end{aligned}$$

3. (3 points) State the converse, the inverse and the contrapositive for the following statement: "If $m \cdot n = l$, then $m \geq 0$ or $n \geq 0$ or $l \geq 0$, where $m, n, l \in \mathbb{Z}$."

- 1
- The converse is: If $m \geq 0$ or $n \geq 0$ or $l \geq 0$ then $m \cdot n = l$.
- 1
- The inverse is: If $m \cdot n \neq l$ then $m < 0$ and $n < 0$ and $l < 0$.
- 1
- The contrapositive is: If $m < 0$ and $n < 0$ and $l < 0$ then $m \cdot n \neq l$.

1. (3 points) Draw the truth table of the following statement: $\neg(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$.

3

p	q	$\neg q$	$p \rightarrow q$	$\neg(p \rightarrow q)$	$p \wedge \neg q$	$\neg(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

So $\neg(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$ is a tautology \Leftrightarrow
 $\neg(p \rightarrow q) \equiv p \wedge \neg q$

2. (4 points) Using logic laws, show that $(u \wedge w) \rightarrow [v \rightarrow (u \wedge v \wedge w)]$ is a tautology.

4

$$\begin{aligned}
 (u \wedge w) \rightarrow [v \rightarrow (u \wedge v \wedge w)] &\equiv \\
 \neg(u \wedge w) \vee [v \rightarrow (u \wedge v \wedge w)] &\equiv \\
 \neg(u \wedge w) \vee [\neg v \vee (u \wedge v \wedge w)] &\equiv \\
 [\neg(u \wedge w) \vee \neg v] \vee (u \wedge v \wedge w) &\equiv \\
 \underbrace{\neg(u \wedge w)}_{\neg A} \vee \underbrace{(u \wedge v \wedge w)}_A &\equiv T
 \end{aligned}$$

3. (3 points) State the converse, the inverse and the contrapositive for the following statement: "If the integer $a + b - c$ is an even, then a is even or b is even or c is even, where $a, b, c \in \mathbb{Z}$."

1

• The converse is: If a is even or b is even or c is even then $(a+b-c)$ is even.

1

• The inverse is: If $(a+b-c)$ is odd then a, b, c are all odd.

1

• The contrapositive is: If a odd and b odd and c odd then $(a+b-c)$ odd