King Saud University | Time: 3 Hours
Department of Mathematics | Full marks: 50
MATH 107 (Dr B. Halouani)

Final Exam
(Summer Semester, 1428-1429)

Question 1[6]:
Use Cramer's Rule to solve the system of linear equations:

$$\begin{cases} x + 2y + 3z &= 1 \\ 2x + 5y + 3z &= -2 \\ x + 8z &= 8 \end{cases} .$$

Question 2[6]:
(a) Find the area of the triangle ABC where $A(1, 2, 0)$, $B(3, 5, 4)$ and $C(3, 2, 3)$.
(b) Find the angle between vectors $a = 6i-4j+2k$ and $b = 12i+15j-6k$.
(c) Find the distance from the point $L(3, 3, 1)$ to the line joining the points $M(1, 1, 1)$ and $N(1, 2, 3)$.

Question 3[12]:
(a) Identify the surface S: $4x^2 + 36y^2 - 9z^2 - 36 = 0$. Find its traces on the coordinate planes and sketch the surface.
(b) Find the tangential and normal components of acceleration for the curve $r(t) = e^t\ i + \sin t\ j + \tan t\ k$ at time t. Also find the curvature κ.
(c) Find the rectangular coordinates of the point given spherical coordinates: $P(3, \frac{\pi}{2}, \pi)$ and $Q(3, \pi, 0)$.

Question 4[8]:
(a) Show that $\lim_{(x,y)\to(0,0)} \dfrac{3x^2 + y^2}{x^2 - 3y^2}$ does not exist.
(b) Use Chain Rule to show that:
i/ $f_{xy} = f_{yx}$ if $f(x, y) = \sin^2 x \cos y$.
ii/ $y\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial y} = 0$, if $w = f\left(x^2 - y^2, y^2 - x^2\right)$.

Question 5[4]: The electric potential V at (x, y, z) is given by

$$V(x, y, z) = x^4yz - xy^3 + z.$$

(a) Find the rate of change of V at $P(1, 1, -3)$ in direction from P to origin.
(b) In what direction does V increases most rapidly ?

(c) What is the maximum rate of change at P ?

Question 6[6]

Find the points on the hyperboloid of two sheets $x^2 - 2y^2 - 4z^2 = 16$ at which the tangent plane is parallel to the plane $4x - 2y + 4z = 5$.

Question 7[8]

(a) Find all the critical points and indicate whether each point gives a local maximum, local minimum or whether it is a saddle point for the function $f(x, y) = 2x^4 - x^2 + y^2$.

(b) Use Lagrange multipliers to find the largest product of real numbers x,y and z, if $x + y + z^2 = 20$.