Math 106Final exam Solutions (Sem2-36/37)

Exercise1

a) $F'(x) = \frac{4x^3}{\sqrt{x^4+9}} - \frac{8x}{\sqrt{4x^2+9}}$, $F'(2) = \frac{16}{5}$ $\quad(\mathbf{1}) + (\mathbf{1})$

b) $x_0 = 1, x_1 = \frac{5}{4}, x_2 = \frac{3}{2}, \ x_3 = \frac{7}{4}, \ x_4 = 2$

$$S_4 = \frac{1}{12}\left(f(1) + 4f\left(\frac{5}{4}\right) + 2f\left(\frac{3}{2}\right) + 4f\left(\frac{7}{4}\right) + f(2)\right) \ (\mathbf{1.5})$$

$$= \frac{1}{12}\left(1 + \frac{16}{5} + \frac{4}{3} + \frac{16}{7} + \frac{1}{2}\right) \approx 0.69325 \quad (\mathbf{0.5})$$

$|Error| \leq \frac{M}{180.4^4}$, $\quad M = \sup\{\left(\frac{24}{x^5}\right) \ 1 \leq x \leq 2\} = 24 \quad (\mathbf{1}) + (\mathbf{0.5})$

So $|Error| \leq \frac{24}{46080} \approx 0.00052 \quad (\mathbf{0.5})$

Exercise2

a) $\int (x+1)3^{-x^2-2x}dx = -\frac{1}{2}\int 3^u \, du = -\frac{1}{2}\frac{3^u}{ln3} + C = -\frac{3^{-x^2-2x}}{2ln3} + C$

$(\mathbf{1}) + (\mathbf{1})$

b) $(\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$

$$\int \frac{x^2dx}{\cosh(x^3)} = \frac{1}{3}\int \frac{du}{cosh^2u} = \frac{1}{3}\int sech^2u du = \frac{1}{3}tanhu + C$$

$$= \frac{1}{3}\, tanh(x^3) + C$$

c) $\int \frac{dx}{x\sqrt{x-1}} = 2\int \frac{dx}{2\sqrt{x}\sqrt{x}\sqrt{x-1}} = 2\int \frac{du}{u\sqrt{u^2-1}} = 2sec\sqrt{x} + C$

$(\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$

<u>Or also</u> $u = \sqrt{x-1}\,, \int \frac{dx}{x\sqrt{x-1}} = 2\int \frac{du}{u^2+1} = 2\tan^{-1}\sqrt{x-1} + C$

$$(\mathbf{2}) + (\mathbf{1})$$

<u>Exercise3</u>

a) $\int e^{\sqrt{x}}\ dx = 2\int ue^u du = 2(ue^u - \int e^u du) = 2(\sqrt{x}\,e^{\sqrt{x}} - e^{\sqrt{x}}) + C$

$(\mathbf{1}) \quad + (\mathbf{1}) + \quad (\mathbf{1})$

b) $\int (sinx)^{\frac{1}{2}} cos^5 x dx = \int u^{\frac{1}{2}}(1-u^2)^2 du$

$= \int u^{\frac{1}{2}}(u^4 - 2u^2 + 1)du = \frac{2}{11}(sinx)^{11/2} - \frac{4}{7}(sinx)^{7/2} + \frac{2}{3}(sinx)^{3/2} + C$

$(\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$

c) $\int_0^{\pi/2} \sin(3x)\cos(2x)\,dx = \frac{1}{2}\int_0^{\pi/2}(\sin(5x) + sinx)dx$

$= -\frac{1}{2}[\frac{\cos(5x)}{5} + cosx]_0^{\pi/2} = \frac{3}{5} \quad (\mathbf{1}) + (\mathbf{1})$

<u>Exercise4</u>

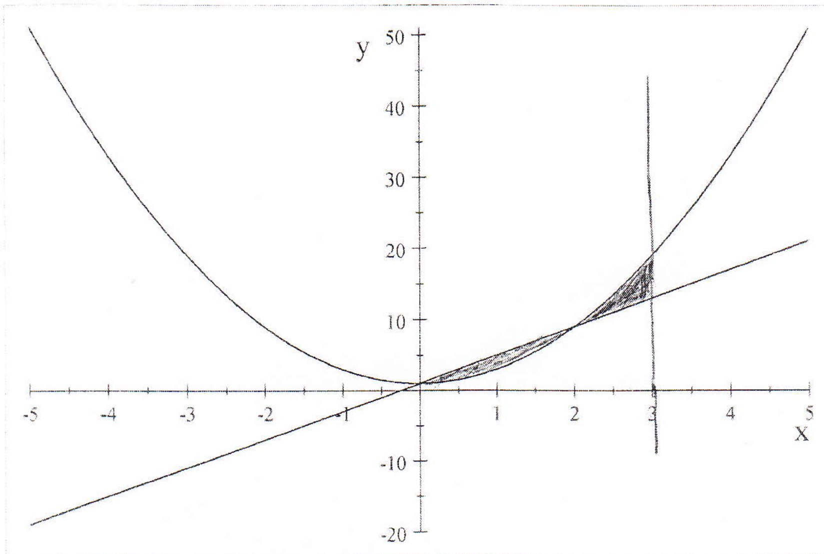
a) $2x^2 + 1 = 4x + 1 \Leftrightarrow x = 0 \ or\ x = 2 \quad (\mathbf{0.5})$

$A = \int_0^2 (4x+1) - (2x^2+1)dx + \int_2^3 (2x^2+1) - (4x+1)dx \quad (\mathbf{1})$

$A = [2x^2 - \frac{2x^3}{3}]_0^2 + [\frac{2x^3}{3} - 2x^2]_2^3 = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} \quad (\mathbf{0.5})$

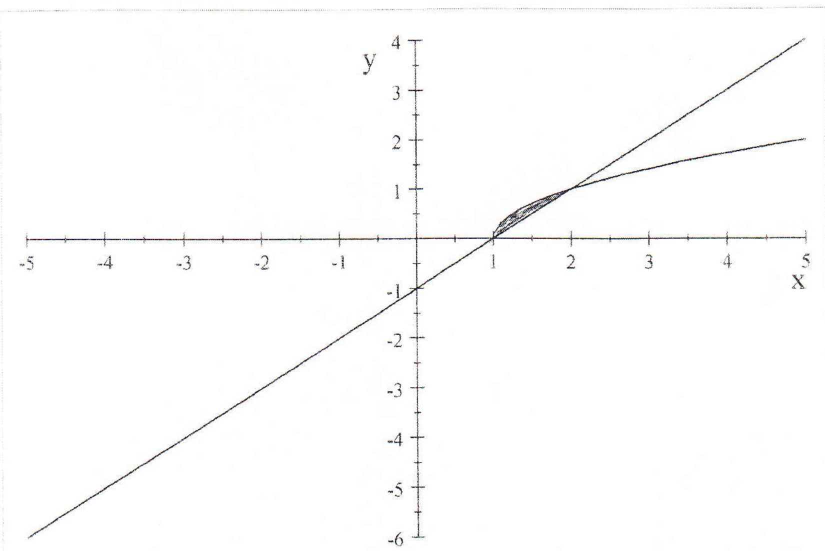
Q4) a)

(1)



b)

(1)



b) i) $\sqrt{x-1} = x-1 \Leftrightarrow x=1 \; or \; x=2$ $(\mathbf{0.5})$

$$V = \pi \int_1^2 (x-1)-(x-1)^2 dx$$

$$= \pi\left[\frac{(x-1)^2}{2} - \frac{(x-1)^3}{3}\right]_1^2 = \frac{\pi}{6} \quad (\mathbf{1.5})$$

Graph(previous page) $(\mathbf{1})$

ii) $V = \int_1^2 2\pi(3-x)(\sqrt{x-1}-x-1)dx \quad (\mathbf{1})$

c) $\frac{2x+1}{x(x+1)^2} = \frac{a}{x} + \frac{b}{x+1} + \frac{c}{(x+1)^2}$

$a=1, b=-1, c=1 \quad (\mathbf{1})$

$$\int \frac{2x+1}{x(x+1)^2} dx = \ln|x| - \ln|x+1| - \frac{1}{x+1} + C \;\; (\mathbf{1})$$

__Exercise5__

a) $L = \int_0^1 \sqrt{t^2+t^4}\, dt = \int_0^1 t\sqrt{1+t^2}\; dt = \frac{1}{2}\int_1^2 \sqrt{u}\, du$

$= \frac{1}{3}\left(2\sqrt{2}-1\right)$

$(\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$

b) $S = \int_1^2 2\pi 2\sqrt{x}\sqrt{1+\frac{1}{x}}\; dx = 4\pi \int_1^2 \sqrt{x+1}\, dx$

$= \frac{8\pi}{3}\left(3^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) \;\; (\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$

c) $cos\theta = sin\theta$ and R in first quadrant gives $\theta = \frac{\pi}{4}$ $\mathbf{(0.5)}$

$$A = 1/2\int_0^{\pi/4} 4cos^2\theta - 4sin^2\theta d\theta = 2\int_0^{\pi/4} cos2\theta d\theta$$

$$= 1 \quad \mathbf{(1,5)}$$

graph $\mathbf{(1)}$