King Saud University
College of Engineering
Department of Civil Engineering

FINAL EXAM

CE302 Mechanics of Materials – 2^{nd} Semester 1431- 32H

Sunday, 10^{th} Rajab 1432 H – 12^{th} June 2011
Time allowed: 3 hours

Student Name	**SOLUTIONS**
Student Number	
Section (put X please)	☐ **30629 (from 9:00 to 10:00 A.M.)** ☐ **30170 (from 10:00 to 11:00 A.M.)**

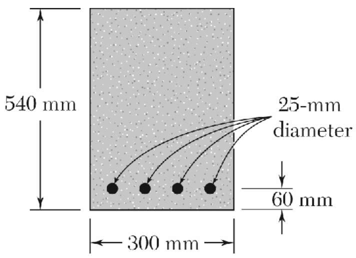
Questions	Maximum Marks	Marks obtained
Q ≠ 1	7	
Q ≠ 2	7	
Q ≠ 3	8	
Q ≠ 4	8	
Q ≠ 5	10	
Q ≠ 6	10	

Total marks ______ / 50

Total marks obtained (in words):____

Instructor's Signature

Question ≠ 1 (7 points):



The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine;

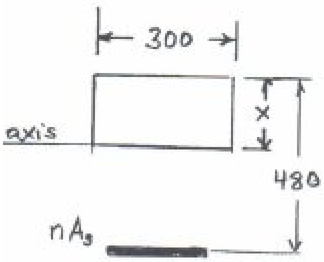
(a) the stress in the steel,

(b) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200\ GPa}{25\ GPa} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)(\frac{\pi}{4})(25)^2 = 1.9635 \times 10^3\ mm^2$$

$$nA_s = 15.708 \times 10^3\ mm^2$$



Locate the neutral axis.

$$300\ x \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150\ x^2 + 15.708 \times 10^3\ x - 7.5398 \times 10^6 = 0$$

Solve for x.

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87\ mm, \qquad 480 - x = 302.13\ mm$$

$$I = \frac{1}{3}\ 300\ x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9\ mm^4 = 1.9966 \times 10^{-3}\ m^4$$

$$\sigma = -\frac{nMy}{I}$$

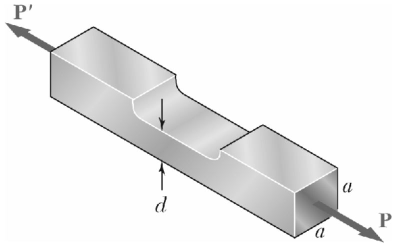
(a) Steel: $y = -302.45\ mm = -0.30245\ m$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6\ Pa = 212\ MPa$$

(b) Concrete: $y = 177.87\ mm = 0.17787\ m$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6\ Pa = -15.59\ MPa$$

Question ≠ 2 (7 points):



A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that a = 30 mm, d = 20 mm and σ_{all} = 60 MPa, determine the magnitude P of the largest load that can be safely applied at the centers of the ends of the bar.

$$A = ad, \qquad I = \frac{1}{12} a d^3, \qquad c = \frac{1}{2} d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

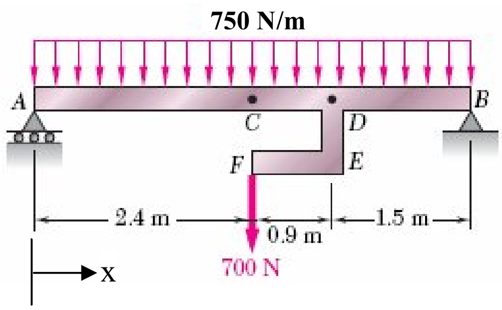
$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \qquad \text{where } K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

Data: $a = 30\text{ mm} = 0.030\text{ m} \qquad d = 20\text{ mm} = 0.020\text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3\ \text{N} \qquad P = 14.40\ \text{kN}$$

Question ≠ 3 (8 points):

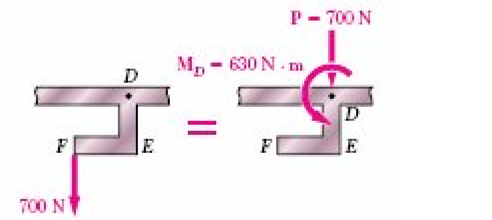


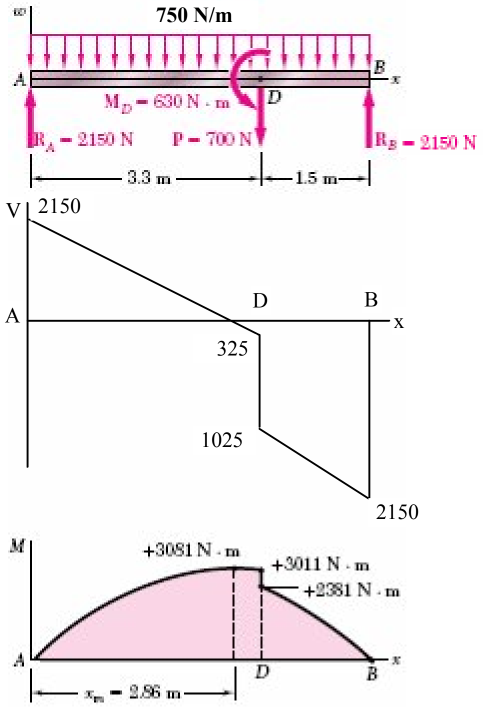
The rigid bar DEF is welded at point D to the steel beam AB. For the loading shown, determine;

(a) the equations defining the shear and bending at portion AD of the steel beam AB,

(b) the location and magnitude of the largest bending moment.

(Hint: Replace the 700 N load applied at F by an equivalent force-couple system at D)



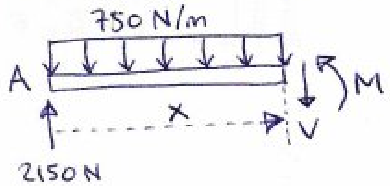


SOLUTION

Reactions. We consider the beam and bar as a free body and observe that the total load is 4300 N. Because of symmetry, each reaction is equal to 2150 N.

Modified Loading Diagram. We replace the 700-N load applied at *F* by an equivalent force-couple system at *D*. We thus obtain a loading diagram consisting of a concentrated couple, three concentrated loads (including the two reactions), and a uniformly distributed load

a) Cut the beam somewhere in between portion AD



$\Sigma F_y = 0:\quad 2150 - 750x - V = 0$

$$\boxed{V = 2150 - 750x} \qquad 0 < x < 3.3\,m$$

$\Sigma M_{cut} = 0:\quad M + 750x\left(\frac{x}{2}\right) - 2150x = 0$

$$\boxed{M = -375x^2 + 2150x} \qquad 0 < x < 3.3\,m$$

b) Moment will be maximum when $V = 0$

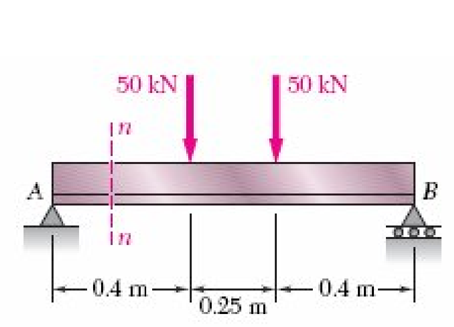
$V = 2150 - 750x = 0$

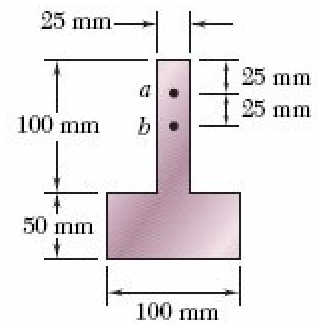
$x = \frac{2150}{750} = 2.86\,m$

$\therefore\ M_{max} = -375(2.86)^2 + 2150(2.86)$

$$\boxed{M_{max} = +3081\ N.m}$$

Question ≠ 4 (8 points):

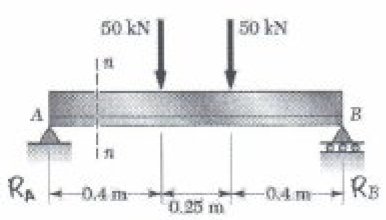


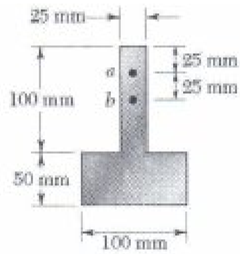


For the beam & loading shown and considering the given cross-section through section n-n, determine;

(a) the shearing stresses at points a and b,

(b) the largest shearing stress.

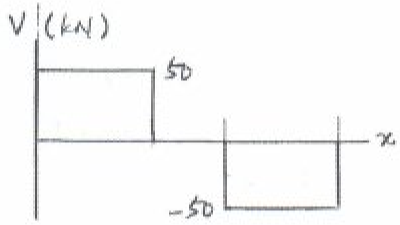




$R_A = R_B = 50\ kN$

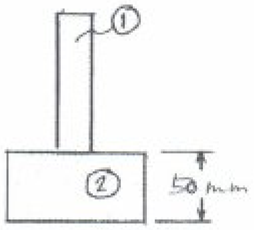
Draw shear diagram.

$V = 50\ kN$



Determine section properties.

Part	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$	$d\ (mm)$	$Ad^2\ (mm^4)$	$\bar{I}\ (mm^4)$
①	2500	100	250000	50	6250000	2083333
②	5000	25	125000	-25	3125000	1041667
Σ	7500		375000		9375000	3125000



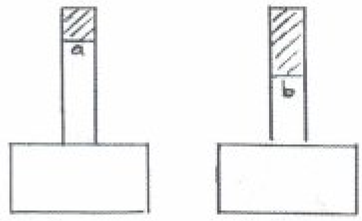
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{375000}{7500} = 50\ mm$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 12.5 \times 10^6\ mm^4$$

(a) $A = 625\ mm^2 \quad \bar{y} = 87.5\ mm \quad Q_a = A\bar{y} = 54687.5\ mm^3$

$t = 25\ mm$

$$\tau_a = \frac{VQ_a}{It} = \frac{(50000)(54687.5)}{(12.5\times10^6)(25)} = 8.75\ MPa$$



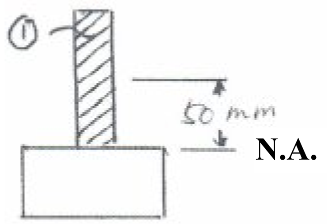
(a) (b)

$A = 1250\ mm^2 \quad \bar{y} = 75\ mm \quad Q_b = A\bar{y} = 93750\ mm^3$

$t = 25\ mm$

$$\tau_b = \frac{VQ_b}{It} = \frac{(50000)(93750)}{(12.5\times10^6)(25)} = 15\ MPa$$

(b)

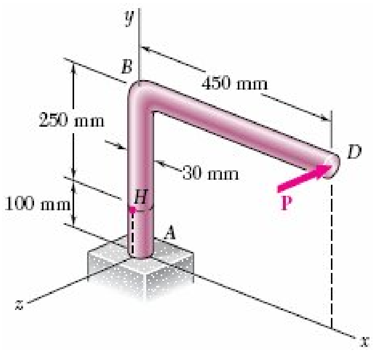


$Q = A_1\bar{y}_1 = (2500)(50) = 125000$

$t = 25\ mm$

$$\tau_{max} = \frac{VQ}{It} = \frac{(50000)(125000)}{(12.5\times10^6)(25)} = 20\ MPa$$

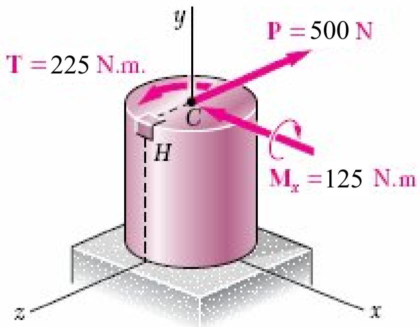
Question ≠ 5 (10 points):



A single horizontal force P of magnitude 500 N is applied to end D of lever ABD. Knowing that portion AB of the lever has a diameter of 30 mm, determine;

(a) the state of plane stress (i.e. the normal and shearing stresses) on an element located at point H, 100 mm above point A and having sides parallel to the x and y axes,

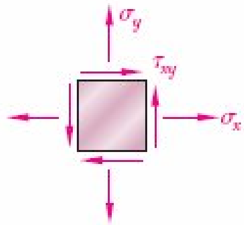
(b) the principal planes (i.e. Θ_{p1} & Θ_{p2}) and the principal stresses (i.e. σ_{max} & σ_{min}) at the same point H.



SOLUTION

Force-Couple System. We replace the force **P** by an equivalent force-couple system at the center *C* of the transverse section containing point *H*:

$$P = 500\ \text{N} \qquad T = (500\ \text{N})(0.45\ \text{m}) = 225\ \text{N}\cdot\text{m}$$
$$M_x = (500\ \text{N})(0.25\ \text{m}) = 125\ \text{N}\cdot\text{m}$$

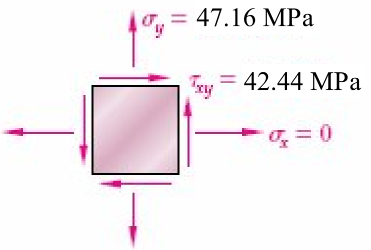


***a*. Stresses σ_x, σ_y, τ_{xy} at Point H.** Using the sign convention shown in Fig. 7.2, we determine the sense and the sign of each stress component by carefully examining the sketch of the force-couple system at point *C*:

$$\sigma_x = 0 \qquad \sigma_y = +\frac{Mc}{I} = +\frac{(125\ \text{N}\cdot\text{m})(0.015\ \text{m})}{\frac{1}{4}\pi\,(0.015)^4} \qquad \sigma_y = 47.16\ \text{MPa} \blacktriangleleft$$

$$\tau_{xy} = +\frac{Tc}{J} = +\frac{(225\ \text{N}\cdot\text{m})(0.015)}{\frac{1}{2}\pi\,(0.015)^4} \qquad \tau_{xy} = 42.44\ \text{MPa} \blacktriangleleft$$

We note that the shearing force **P** does not cause any shearing stress at point *H*.



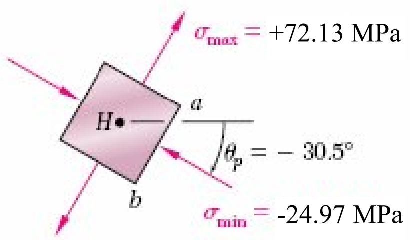
***b*. Principal Planes and Principal Stresses.** Substituting the values of the stress components into Eq. (7.12), we determine the orientation of the principal planes:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(42.44)}{0 - 47.16} = -1.80$$
$$2\theta_p = -61.0^\circ \quad \text{and} \quad 180^\circ - 61.0^\circ = +119^\circ$$
$$\theta_p = -30.5^\circ \quad \text{and} \quad +59.5^\circ \blacktriangleleft$$

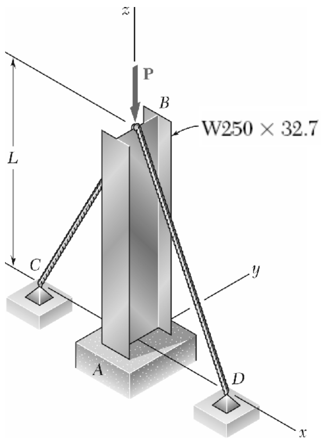
Substituting into Eq. (7.14), we determine the magnitudes of the principal stresses:

$$\sigma_{max,\,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{0 + 47.16}{2} \pm \sqrt{\left(\frac{0 - 47.16}{2}\right)^2 + (42.44)^2} = 23.58 \pm 48.55$$

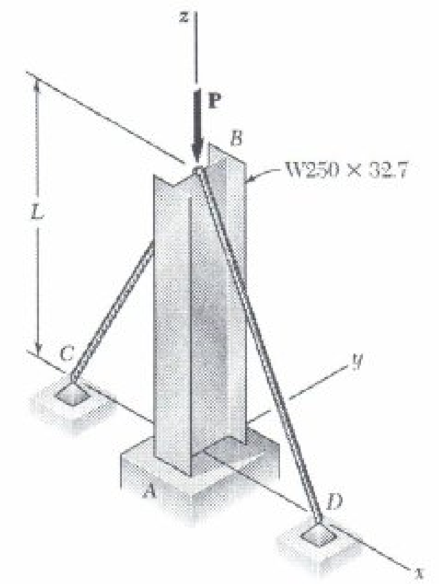
$$\sigma_{max} = +72.13\ \text{MPa} \blacktriangleleft$$
$$\sigma_{min} = -24.97\ \text{MPa} \blacktriangleleft$$



Question ≠ 6 (10 points):



A wide-flange shape column *AB* carries a centric load P of magnitude 60 kN. Cables *BC* and *BD* are taut and prevent motion of Point *B* in the *xz* plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length *L*. Take E = 200 GPa and I_x= 48.9x10^6 mm^4 & I_y= 4.73x10^6 mm^4 for W250 x 32.7. (Hint: Consider buckling in xz-plane and yz-plane separately)



W250x32.7: $I_x = 48.9 \times 10^6\ mm^4$, $I_y = 4.73 \times 10^6\ mm^4$

$P = 60$ kN.

$P_{cr} = (F.S.)P = (2.2)(60) = 132$ kN

Buckling in xz-plane. $L_e = 0.7L$

$P_{cr} = \dfrac{\pi^2 E I_y}{(0.7L)^2}$ $\qquad L = \dfrac{\pi}{0.7}\sqrt{\dfrac{EI_y}{P_{cr}}}$

$L = \dfrac{\pi}{0.7}\sqrt{\dfrac{(200 \times 10^9)(4.73 \times 10^{-6})}{132000}} = 12.01$ m.

Buckling in yz-plane. $L_e = 2L$

$P_{cr} = \dfrac{\pi^2 E I_x}{(2L)^2}$ $\qquad L = \dfrac{\pi}{2}\sqrt{\dfrac{EI_x}{P_{cr}}} = \dfrac{\pi}{2}\sqrt{\dfrac{(200 \times 10^9)(48.9 \times 10^{-6})}{132000}} = 13.52$ m.

Smaller value for L governs. $\qquad L = 12.01$ m