TIME: 3hours **KING SAUD UNIVERSITY** **FULL MARKS :100**

M - 107 **DEPARTMENT OF MATHEMATICS**

(SEMESTER I, 1429-1430) FINAL

NOTE: Attempt all Questions.

Question: 1 . (a) **Use Cramer's rule to solve the system of equations**

$$x + y - 2z = 1$$
$$2x - y + z = 2 \qquad \textbf{[6]}$$
$$x - 2y - 4z = -4$$

(b) For what values of λ, the following matrix is invertible

$$A = \begin{bmatrix} \lambda & 1 & 2\lambda \\ 0 & \lambda - 1 & 0 \\ 1 & 3 & \lambda^2 + 3 \end{bmatrix} \qquad \textbf{[6]}$$

Question: 2 .(a) Find value of a, b **and** c **such that** $U = < a, b, c >$ **is orthogonal to** $V = < 1, 2, 1 >$ **and** $W = < 1, -1, 1 >$. **[6]**

(b) Let $a = < 2, 3, 4 >$ **and** $b = < -1, 3, 5 >$ **and** $c = 2a + 3b$ **,**

show that $(a \text{ x } b).\ c = 0.$ **[6]**

Question: 3 . (a) Find equation of the plane containing points P(1,1,1), Q(1,0,1) and R(0,1,0). Also find distance of the plane from the point S(2,3,5). **[8]**

(b) Use differentials to approximate the change in temperature T = xy + yz + xz, if the point (x , y, z) moves from the point P (2, −1, 3) to the point Q (1.98, −0.98, 3.02) . **[8]**

Question: 4 . (a) A particle starts at an initial position $r(0) = < 1, 0, 0 >$ **with initial velocity** $v(0) = < 1, -1, 1 >$**. Its acceleration is** $a(t) = 4ti + 6tj + k$ **. Find its velocity and position at time t.** **[10]**

(b) Let $r(t) = < t^2, 2t, t >$ **be the position vector of a moving point P. Find tangential and normal components of acceleration (** a_T and a_N **) at point Q(1,2,1). Also find curvature** κ **.** **[10]**

Question: 5 .(a) Show that the function $z = \ln(x^2 + y^2)$ $, x \neq 0, y \neq 0$

satisfies the Laplace equation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$. **[10]**

(b) Find the equation of the tangent plane and normal line to the graph of the surface $x^2 + y^2 - z^2 = 18$ **at the point P (3, 5, −4) .** **[10]**

Question: 6.(a) Find a point on the surface $z = 3x^2 - y^2$ **,where the tangent plane is parallel to the plane** $6x + 4y - z = 5.$ **[10]**

(b) Use Lagrange multipliers to find the maximum value of the function $f(x, y) = x^2 + y$ **subject to constraint** $x^2 + y^2 = 1$. **[10]**