TIME: 3hours **KING SAUD UNIVERSITY** FULL MARKS :100

**M - 107** **DEPARTMENT OF MATHEMATICS**

**(SEMESTER I, 1431-1432) FINAL**

---

**Question: 1 . (a) For what values of λ does the system of equations have**
**[10+4+10] (i) unique solution, (ii) infinitly many solutions and (iii) no solution.**

$$\begin{aligned} 3x + \lambda z &= 2 \\ 3x + 3y + 4z &= 4 \\ y + 2z &= 3 \end{aligned}$$

**(b) By using properties of determinant, show that**

$$\begin{vmatrix} y+z & z+x & y+x \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

**(c) For the given system of linear equations:**

$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + y + 3z &= 3 \\ x + 2y + 4z &= 1 \end{aligned}$$

i. **Write the system of equation in the form AX=B,**
ii. **Find adj (A),**
iii. **Use adj (A) to find $A^{-1}$ , if exists, and**
iv. **Use $A^{-1}$ to solve the given system.**

**Question:2.(a) Find the** $\text{comp}_b^a$ and $\text{Proj}_b^a$

**[6+6+6]** if $a = -2i + j + k$ and $b = 4i - 3j + k$ .

**(b) Find the point at which the line**

$x = 2 + 3t, \quad y = -4t, \quad z = 5 + t, \quad t \in R$

**intersects the plane** $4x + 5y - 2z = 18$.

**(c) Find the direction cosines and direction angles of the vector**

$\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k}$

**Question: 3 . (a) Show that** $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ **does not exist.**

**[6+8+6] (b) A particle starts from position** $r(1) = i + j$ **. Its velocity is** $v(t) = 2ti + 3t^2 j + \sqrt{t}k$ **. Find its position at time t.**

**(c) Find the parametric equations of the tangent line to the curve with parametric equations** $x = t^5, y = t^4, z = t^3,$ **at the point ( 1, 1, 1).**

**Question: 4 .(a) Show that the function** $f(x,y) = \ln(x^2 + y^2) \quad x \neq 0, y \neq 0$

**[6+6+8] satisfies the Laplace equation** $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$

**(b) If** $3xy^3 + 3x^2y - 6y + 9xy = 8$ **find** $\dfrac{dy}{dx}$

**(c) Find the tangential and normal components of the acceleration of a particle moving along th curve** $x = t^3$ **,** $y = t+2$**,** $z = 4$ **,** $t \in R$ **at** $t = 1$**. Also find the curvature at** $t = 1$**.**

**Question: 5 . (a) The base radius and the height of the right circular cone are measured to be 10cm and 25cm respectively, with a possible error in measurement of 0.1cm in each. Use differentials to estimate the maximum error in the calculated volume of the cone.** $\left[V = \frac{1}{3}\pi r^2 h\right]$

**[6+8+6] (b) Find the directional derivative of** $f(x,y) = xy$ **at the point P( 3, 2 ) in the direction of < 1, 2>. In which direction is the direction of the derivative maximum? What is the maximum value of the derivative?**

**(c) Use Lagrange multipliers to find the maximum and minimum values of the function** $f(x,y) = x + 2y$ **subject to constraint** $x^2 + y^2 = 5$**.**

**NOTE: For solution of the paper visit http://faculty.ksu.edu.sa/khawaja/default.aspx**