1) [3 Marks] If the survival function is given by $s(x) = \left(\frac{1}{x+1}\right)^5$ for $x \geq 0$, how much longer can a 43 year old expected to live, as measued by complete life expectancy?

$$\cancel{\mathring{e}_{43} = \int_0^\infty \frac{1}{x+1}}$$

$$_tP_x = \frac{S_0(x+t)}{S_0(x)} = \frac{\left(\frac{1}{x+t+1}\right)^5}{\left(\frac{1}{x+1}\right)^5} = \left(\frac{x+1}{x+t+1}\right)^5$$

3

$$\mathring{e}_{43} = \int_0^\infty \left(\frac{43+1}{43+1+t}\right)^5 dt = \int_0^\infty \left(\frac{44}{44+t}\right)^5 dt$$

$$= 44^5 \int_0^\infty (44+t)^{-5}\, dt = \frac{44^5 (44+t)^{-4}}{-4}\Big|_0^\infty$$

$$= \frac{44^5}{-4(44+t)^4}\Big|_0^\infty = \frac{44^5}{4(44)^4} = \underline{11}$$

$l_{93} = 825 \quad l_{92} = 1000$

$p_{92} = \frac{l_{93}}{l_{92}} = 0.825$

$p_{95} = \frac{l_{96}}{l_{95}}$

$p_{96} = \frac{l_{97}}{l_{96}} =$

$p_{95} = \frac{l_{96}}{l_{95}}$

2) [4 Marks] You are given the following information from a life table:

i)

x	l_x	d_x	p_x	q_x
95	600	240	0.60	0.40
96	360	288	0.20	0.80
97	72	72	0	1.00

ii) $l_{92} = 1000$ and $l_{93} = 825.$

iii) Deaths are uniformly distributed over year of age.

Calculate the probability that (92) dies between ages 93 and 95.5.

$1|2.5q_{92} = p_{92} - 3.5p_{92}$

$= p_{92} \cdot p_{92}\ 2.5p_{93}$

$= \cancel{p_{92} - p_{92}\ 2p_{93}\ p_{95}}$

$= p_{92} - p_{92}\ 2p_{93}\ 0.5p_{95}$

4

$\cancel{1|2.5q_{92} = p_{92} - 3.5p_{92} = p_{92} \cdot 2p_{92}\ 1.5}$

$\cancel{1|2.5q_{92} = p_{92}\ 2.5q_{93} = p_{92}\ 0.5q_{93}\ 2q_{93.5}}$

$\cancel{= \frac{l_{93}}{l_{92}}}$

$1|2.5q_{92} = p_{92} - 3.5p_{92} = p_{92} \cdot p_{92}\ 2.5p_{93} = \cancel{p_{92}\ p_{92}\ 0.5p_{93}\ 2p_{93.5}}$

$= p_{92} - p_{92}\ 2p_{93}\ 0.5p_{95} = \frac{l_{93}}{l_{92}} - \frac{l_{93}}{l_{92}}\left(\frac{l_{95}}{l_{93}}\right)(1 - 0.5q_{95})$

$= 0.825 - 0.825(0.727)(1 - 0.5(1 - \frac{l_{96}}{l_{95}}))$

$= 0.825 - 0.825(0.727)(0.8)$

$= 0.34518$

3) [3 Marks] For a 2-year select and ultimate mortality model, you are given:

i) $q_{[x]+1} = 0.95 q_{x+1}$
ii) $l_{76} = 98.153$
iii) $l_{77} = 96.124$

Calculate $l_{[75]+1}$.

$q_{[75]+1} = 0.95(1-0.9793)$
$= 0.0197$

$p_{[75]+1} = 0.9803$

$\cancel{l_{[75]+1}}$

$\cancel{p_{[75]} = \frac{p_{77}\, l_{[75]+1}}{l_{[75]}}}$

$p_{76} = \frac{l_{77}}{l_{76}} = 0.9793$

3

$p_{[75]+1} = \frac{l_{[75]+2}}{l_{[75]+1}} = \frac{l_{77}}{l_{[75]+1}}$

$l_{[75]+1} = \frac{l_{77}}{p_{[75]+1}} = \frac{96.124}{0.9803} = 98.056$ ✓

4) [4 Marks] You are given:

i) Deaths are uniformly distributed over each year of age.
ii) $i = 0.12$
iii) $q_x = 0.1$ and $q_{x+1} = 0.2$
iv) $Z = \begin{cases} v^{K_x^{(3)}+\frac{1}{3}} & T_x < 2 \\ v^2 & T_x \geq 2 \end{cases}$

Calculate:

a) $\bar{A}_{x:\overline{2}|}$.
b) $A^{(3)}_{x:\overline{2}|}$.
c) The standard deviation of Z.

a) $\bar{A}_{x:\overline{2}|} = \frac{i}{\delta} A^{1}_{x:\overline{2}|} + {}_2E_x$

$A^{1}_{x:\overline{2}|} = \sum_{K=0}^{1} v^{K+1} {}_K p_x \, q_{x+K} = v q_x + v^2 p_x \, q_{x+1}$

$= \frac{0.1}{1.12} + \frac{(0.9)(0.2)}{(1.12)^2} = 0.2327806$

3

${}_2E_x = v^2 \, {}_2p_x = v^2 \, p_x \, p_{x+1} = 0.5739795918$

$\delta = \ln(1.12) = 0.1133287$

$\bar{A}_{x:\overline{2}|} = \frac{0.12}{0.1133287}(0.2327806) + 0.5739795918$

$\boxed{= 0.820463}$

b)

$A^{(3)}_{x:\overline{2}|} = \frac{i}{i^{(3)}} A_{x:\overline{2}|} = \frac{0.12}{0.115496}(0.820463) = \boxed{0.8524586133}$

$i^{(3)} = 0.115496$

$(1+i) = (1+\frac{i^{(3)}}{3})^3$

4) [4 Marks] You are given:

i) Deaths are uniformly distributed over each year of age.
ii) $i = 0.12$
iii) $q_x = 0.1$ and $q_{x+1} = 0.2$
iv) $Z = \begin{cases} v^{K_x^{(3)}+\frac{1}{3}} & T_x < 2 \\ v^2 & T_x \geq 2 \end{cases}$

Calculate:

a) $\bar{A}_{x:\overline{2}|}$.
b) $A^{(3)}_{x:\overline{2}|}$.
c) The standard deviation of Z.

a) $A_{x:\overline{2}|} = A^1_{x:\overline{2}|} + A_{x:\overset{1}{\overline{2}|}}$

$A^1_{x:\overline{2}|} = \sum_{k=0}^{1} v^{k+1} {}_kp_x q_{x+k} = v q_x + v^2 p_x q_{x+1}$

$= (1.12)^{-1}(0.1) + (1.12)^{-2}(0.9)(0.2)$

$= 0.2328$

0.5

$A_{x:\overset{1}{\overline{2}|}} = v^2 {}_2p_x = v^2 p_x p_{x+1} = (1.12)^{-2}(0.9)(0.8)$

$= 0.57398$

$A_{x:\overline{2}|} = 0.57398 + 0.2328 = 0.8068$

$\bar{A}_{x:\overline{2}|} = \frac{i}{\delta} A_{x:\overline{2}|} = \frac{0.12}{\ln(1.12)}(0.8068) = 0.8543$

b) $A^{(3)}_{x:\overline{2}|} = \frac{i}{i^{(3)}} A_{x:\overline{2}|}$

$1+i = \left(1 + \frac{i^{(3)}}{3}\right)^3 \Rightarrow i^{(3)} = 0.115$

$\Rightarrow A^{(3)}_{x:\overline{2}|} = \frac{0.12}{0.115}(0.8068) = 0.8419$

00

c)

5) [3 Marks] The following table gives the survival probabilities of a certain population:

t	0	1	2	3	4	5	6
p_{x+t}	0.8	0.9	0.95	0.96	0.95	0.95	0.9

Assume $i = 4\%$.

a) Calculate the APV of a term life insurance of 1SR issued to (x), payable at the end of the first policy year if death occurs in the first policy year.

b) Calculate the APV of a term life insurance of 3SR issued to $(x+1)$, payable at the end of the third policy year if death occurs in the third policy year.

c) An endowment insurance issued to $(x+2)$, which pays 1SR at the end of the year of death if death occur in the first two years, and 2SR at the end of the second policy year if the life survives.

a) $A^{1}_{x:\overline{1}|} = \sum_{k=0}^{0} v^{k+1}\, {}_kp_x\, q_{x+k} = v q_x = (1.04)^{-1}(1-0.8)$

$= 0.1923$

b) $\cancel{A^{1}_{x+1:\overline{3}|} = \sum_{k=2}^{2} \; {}_{2|}A_{x+1}}$

3

$= 3\sum_{k=2}^{2} v^{k+1}\, {}_kp_{x+1}\, q_{x+1+k} = 3 v^3\, {}_2p_{x+1}\, q_{x+3}$

$= 3 v^3 p_{x+1} p_{x+2} q_{x+3} = 3(1.04)^{-3}(0.9)(0.95)(1-0.96)$

$= 0.0912$

c) $\cancel{A}$: $A_{x+2:\overline{2}|} = A^{1}_{x+2:\overline{2}|} + 2A_{x+2:\overline{2}|}^{\;\;\;\,1}$

$A^{1}_{x+2:\overline{2}|} = \sum_{k=0}^{1} v^{k+1}\, {}_kp_{x+2}\, q_{x+2+k} = v q_{x+2} + v^2 p_{x+2} q_{x+3}$

$= (1.04)^{-1}(1-0.95) + (1.04)^{-2}(0.95)(1-0.96)$

$= 0.0832$

$A_{x+2:\overline{2}|}^{\;\;\;\,1} = v^2\, {}_2p_{x+2} = v^2 p_{x+2} p_{x+3} = (1.04)^{-2}(0.95)(0.96)$

$= 0.8432$

$A_{x+2:\overline{2}|} = 0.0832 + 2(0.8432) = 1.7696$

$$\cancel{v\sum v^{k+1}{}_{k}p_x q} \;=\; v\sum v^k({}_{k+1}q_x - {}_kq_x)$$
$$= v\sum v^k(q_x\ {}_kq_{x+1} - {}_kq_x)$$

6) [3 Marks] Prove the following recursion formaulas:

a) $A^1_{x:\overline{n}|} = vq_x + vp_xA^1_{x+1:\overline{n-1}|}$.
b) $A_{x:\overline{n}|} = vq_x + vp_xA_{x+1:\overline{n-1}|}$.
c) $(DA)^1_{x:\overline{n}|} = vnq_x + vp_x(DA)^1_{x+1:\overline{n-1}|}$

a) $A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}\,{}_kp_x\,q_{x+k}$ $\cancel{= v\sum_{k=0}^{n-1} v^k\,{}_{k|}q_x}$

$$\cancel{= v\sum_{k=0}^{n-1} v^k({}_{k+1}q_x - {}_kq_x) = v\sum_{k=0}^{n-1} v^k(q_x\ {}_kq_{x+1} - {}_kq_x)}$$

$$\cancel{= v\sum_{k=0}^{n-1} v^k\,{}_{k|}q_x = v\sum_{k=1}^{n} v^{k+1}\,{}_{k|}q_{x+1}}$$

$$\cancel{= v\sum_{k=0}^{n} v^{k+p}}$$

$$= v\sum_{k=0}^{n-1} v^k\,{}_kp_x\,q_{x+k}$$

2

$$= vq_x + v^2p_xq_{x+1} + v^3\,{}_2p_x\,q_{x+2} + \dots$$
$$+ v^n\,{}_{n-1}p_x\,q_{x+n-1}$$

$$= vq_x + vp_x(vq_{x+1} + v^2p_{x+1}q_{x+2} + \dots + v^{n-1}\,{}_{n-2}p_x\,q_{x+n-1})$$

$$= vq_x + vp_x\sum_{k=0}^{n-2} v^{k+1}\,{}_kp_{x+1}\,q_{x+k+1} = vq_x + vp_xA^1_{x+1:\overline{n-1}|}$$

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b) $A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + v^n\,{}_np_x$

2

~~$A^1_{x:\overline{n}|}$ from before~~

$$A_{x:\overline{n}|} = vq_x + vp_x(vq_{x+1} + v^2p_{x+1}q_{x+2} + \dots + v^{n-1}\,{}_{n-2}p_x\,q_{x+n-1})$$
$$+ v^n\,{}_np_x$$

$$= vq_x + vp_xA^1_{x+1:\overline{n-1}|} + v^n\,{}_np_x$$

$$= vq_x + vp_xA_{x+1:\overline{n-1}|}$$

$$4000\,\ddot{a}_x = 72000$$

$$3000\,\ddot{a}_{x:\overline{20}|} + 6000\,{}_{20|}\ddot{a}_x = 80000$$

7) [3 Marks] You are given:

i) For 72 000, (x) can buy a life annuity due that pays 4000 a year.
ii) For 80 000, (x) can buy a life annuity due that pays 3000 a year for the first 20 years and 6000 a year thereafter.

Calculate the PV of a 20-year temporary life annuity due that pays 5000 a year.

$$5000\,\ddot{a}_{x:\overline{20}|} \; ?$$

$$4000\,\ddot{a}_x = 72000 \Rightarrow \ddot{a}_x = 18$$

$$3000\,\ddot{a}_{x:\overline{20}|} + 6000\,{}_{20|}\ddot{a}_x = 80000$$

$$\Rightarrow 3000\,\ddot{a}_{x:\overline{20}|} + 6000(\ddot{a}_x - \ddot{a}_{x:\overline{20}|}) = 80000$$

$$3000\,\ddot{a}_{x:\overline{20}|} + 6000(18) - 6000\,\ddot{a}_{x:\overline{20}|} = 80000$$

$$-3000\,\ddot{a}_{x:\overline{20}|} = -28000$$

$$\ddot{a}_{x:\overline{20}|} = 9.33$$

3

$$\Rightarrow 5000\,\ddot{a}_{x:\overline{20}|} = 5000(9.33) = 46650$$

1 3 5 7 9

0 1 2 3 4 5

$x = 20$ $n = 5$

8) [3 Marks] A person aged 20 buys a special five-year temporary life annuity-due, with payment of 1,3,5,7 and 9. You are given:

$3.04 = 3.41 - 1 + {}_4E_{20}$

i) $\ddot{a}_{20:\overline{4}|} = 3.41$

ii) $a_{20:\overline{4}|} = 3.04$

iii) $(I\ddot{a})_{20:\overline{4}|} = 8.05$

iii) $(Ia)_{20:\overline{4}|} = 7.17$

${}_4E_{20} = 0.63$

Find the APV of this annuity

$(I\ddot{a})_{20:\overline{4}|} = (I\ddot{a})^{1}_{20:\overline{4}|} + 4\,{}_4E_{20}$

$8.05 = (I\ddot{a})^{1}_{20:\overline{4}|} + 4(0.63)$

$(I\ddot{a})^{1}_{20:\overline{4}|} = 5.53$

9) [4 Marks] For a special insurance is designed to pay a benefit in the event a product fails. You are given:

i) Benefits are payable at the moment of faillure.

ii) $b_t = \begin{cases} 300, & 0 \le t < 25. \\ 100, & t \ge 25. \end{cases}$

iii) $\mu_t = 0.04, \; t \ge 0.$

iv) $\delta_t = \begin{cases} 0.02, & 0 \le t < 25. \\ 0.03, & t \ge 25. \end{cases}$

Calculate the APV of this special insurance.

$$\bar{A}_x = \int_0^\infty v^t \, {}_tp_x \, \mu_{x+t} \, dt$$

if $0 \le t \le 25$

$$v^t = e^{-0.02t}$$

$${}_tp_x = e^{-0.04t}$$

4

if $t \ge 25$

$$v^t = e^{-\left(\int_0^{25} 0.02\,dr + \int_{25}^{t} 0.03\,dr\right)} = e^{-(0.5 + 0.03t - 0.75)} = \cancel{e} \; e^{0.25 - 0.03t}$$

$$\cancel{{}_tp_x = e^{-\int_0}} \qquad {}_tp_x = e^{-0.04t}$$

$$\bar{A}_x = 300 \int_0^{25} e^{-0.02t} e^{-0.04t} (0.04)\,dt + 100 \int_{25}^{\infty} e^{0.25 - 0.03t} e^{-0.04t} (0.04)\,dt$$

$$= 155.37 + 4 e^{0.25} \int_{25}^{\infty} e^{-t(0.03 + 0.04)}\,dt$$

$$= 155.37 + \left(\frac{-4 e^{0.25}}{0.07} e^{-t(0.07)} \Big|_{25}^{\infty} \right)$$

$$= 155.37 + \frac{4 e^{0.25}}{0.07} e^{-1.75} = 168.12$$

10) [3 Marks] You are given:

i) $i = 6\%$

ii) ${}_{10}E_{40} = 0.540$

iii) $1000A_{40} = 168$

iv) $1000A_{50} = 264$

Calculate $1000\,{}_{10}P(A_{40})$, the net premium for a 10-payment fully discrete life insurance of 1000 on (40).

$${}_{10}P(A_{40}) = \cancel{\frac{A_{40}}{\ddot{a}_{40:\overline{10|}}}} \quad \frac{A_{40}}{\ddot{a}_{40:\overline{10|}}}$$

$A_{40} = 0.168 \qquad A_{50} = 0.264$

$$\ddot{a}_{40:\overline{10|}} = \frac{1 - A_{40:\overline{10|}}}{d}$$

$$A_{40:\overline{10|}} = A^1_{40:\overline{10|}} + {}_{10}E_{40}$$

$$A^1_{40:\overline{10|}} = A_{40} - {}_{10}E_{40}\, A_{50}$$

$$A^1_{40:\overline{10|}} = 0.168 - (0.540)(0.264) = 0.02544$$

$$A_{40:\overline{10|}} = 0.02544 + 0.540 = 0.5654$$

$$\ddot{a}_{40:\overline{10|}} = \frac{1 - 0.5654}{\frac{0.06}{1.06}} = 7.678$$

$$1000\,{}_{10}P(A_{40}) = 1000\left(\frac{0.168}{7.678}\right) = 21.8807$$

11) [3 Marks] You are given:

i) ${}_{k|k+1}q_x = \frac{0.9^{2k}}{9}$

ii) $i = 8\%$

iii) The force of mortality is constant

Calculate $1000(\bar{P}(\bar{A}_x) - P_x)$.

$$\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} = \frac{\frac{\mu}{\mu+\delta}}{\frac{1}{\mu+\delta}} = \mu = -\ln(p_x)$$

2

$$~~P_x~~$$

$$_{0|1}q_x = q_x = \frac{0.9^{(0)}}{9} = \frac{1}{9} = 0.10$$

$$~~= 1 - p_x = 0.89~~$$

$$~~\bar{P}(\bar{A}_x) = -\ln(0.89) - \ln\left(\frac{1}{0.89}\right) = 0.1165~~$$

$$P_x = v\,q_x = (1.08)^{-1}(0.11) = 0.1019$$

$$1000(\bar{P}(\bar{A}_x) - P_x) = 1000(0.1165 - 0.1019)$$

$$= 14.6$$

12) [4 Marks] The distribution of Ahmed future lifetime is a mixture of two-point:

i) With probability 0.60, Ahmed future lifetime follows the ILT, with deaths UD ever each year of age.
ii) With probability 0.40, Ahmed future lifetime follows a constant force of mortality $\mu = 0.02$.

A fully continuous whole life imsurance of 1000 is issued on Ahmed at age 62. Calculate the net premium for this insurance at $i = 6\%$.

$\bar{P}(\bar{A}_{62}) = 0.60\ {}_{I}\bar{P}(\bar{A}_{62}) + 0.40\ {}_{c}\bar{P}(\bar{A}_{62})$

i) ii)

4

for i) $\Rightarrow {}_{I}\bar{P}(\bar{A}_{62}) = \dfrac{\bar{A}_{62}}{\bar{a}_{62}} = \dfrac{\frac{i}{\delta}(0.3693)}{\dfrac{1 - \frac{i}{\delta}(0.3693)}{\delta}} = 0.0358$

for ii) ${}_{c}\bar{P}(\bar{A}_{62}) = \dfrac{\bar{A}_{62}}{\bar{a}_{62}} = \mu = 0.02$

$1000\ \bar{P}(\bar{A}_{62}) = 1000(0.6(0.0358) + 0.40(0.02))$

$= 29.48$