Department of Mathematics
King Saud University

(B)

M-106

Second Semester(1428/1429)

Final Exam

Name:	**Roll Number**:
Name of Teacher:	**Group No:**

Max Marks: 50 Time: Three hours

Marks:

Multiple Choice(1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Mark {a,b,c,d} for the correct answers in the space given below for Q.1-to-Q.20 $[20\times 1 = 20]$

Q.N0:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
{a,b,c,d}	a	b	a	a	c	d	a	b	d	a	b	c	b	a	a	a	b	b	a	a

Q.No:1 The number z that satisfies the conclusion of the Mean-value theorem for integral $\int_{-2}^{1}(x^2+1)dx$ is

(a) -1 (b) 0 (c) 1 (d) None of these.

Q.No: 2 $\frac{d}{dx}\left(\int_{-x}^{x}\tan(t^2)dt\right)$ is equals to

(a) 0 (b) $2\tan(x^2)$ (c) $Sec^2(x)$ (d) None of these

Q.No: 3 The domain of the function $f(x)=\ln\left(\frac{2}{x-2}\right)$ is

(a) $(2,\infty)$ (b) $[2,\infty)$ (c) $(-\infty,2)$ (d) None of these

Q.No: 4 If $\log_2\left(\frac{x}{x-1}\right)=1$ then x is equal to

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

Q.No:5 The slope of the tangent line to the graph of $y=\ln(x^4)$ at the $(1,0)$ is

(a) 0 (b) 2 (c) 4 (d) None of these

Q.No:6 If $f(x)=x^{\pi}+\pi^{x}$ then $f'(x)$ is

(a) $\pi x^{\pi-1}+x\pi^{x-1}$ (b) $(\pi-1)x^{\pi-1}+x\pi^{x-1}$ (c) $\pi x^{\pi-1}+\ln(\pi)\pi^{x-1}$ (d) None of these.

Q.No:7 To evaluate the integral $\int\frac{dx}{x\sqrt{9-x^2}}$ we put

(a) $x=3\sin\theta$ (b) $x=3\sinh\theta$ (c) $x=3\tan\theta$ (d) None of these.

Q.No:8 To evaluate the integral $\int\frac{x^{1/2}}{1+x^{1/4}}dx$ we put

(a) $u=x^{1/2}$ (b) $u=x^{1/4}$ (c) $u=x^{1/8}$ (d) None of these.

Q.No:9 The partial fraction decomposition of $\frac{x^3}{(x^2+x-12)^2}$ takes the form

(a) $\frac{A}{x+4}+\frac{B}{x-3}$ (b) $\frac{A}{x+4}+\frac{B}{(x-3)^2}$ (c) $\frac{A}{(x+4)^2}+\frac{B}{(x-3)^2}$ (d) None of these.

Q.No:10 The derivative of the function $f(x)=\sinh^{-1}(\tan x)$ is

(a) $\frac{\sec^2 x}{\sqrt{\tan^2 x+1}}$ (b) $\frac{\sec^2 x}{\sqrt{\tan^2 x-1}}$ (c) $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$ (d) None of these.

Q.No:11 The integral $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to

(a) $\frac{\sin^{-2}(x)}{2} + c$ (b) $\frac{(\sin^{-1} x)^2}{2} + c$ (c) $\frac{(1-x^2)^{3/2}}{3/2}$ (d)None of these.

Q.No:12 $\lim_{x \to \left(\frac{\pi}{2}\right)^-} \frac{Sin(2x)}{4x^2 - \pi^2}$ is equal to

(a) $\frac{1}{2\pi}$ (b) 2π (c) $-\frac{1}{2\pi}$ (d)None of these.

Q.No:13 The integral $\int_0^2 \frac{dx}{(x-2)^2}$ is equal to

(a) 0 (b) ∞ (c) $-\infty$ (d)None of these.

Q.No:14 The area of the region bounded by the graph of the curves $y = x^2, y = 8 - x^2$ is

(a) $\frac{64}{3}$ (b) 64 (c) $\frac{128}{3}$ (d)None of these

Q.No:15 The volume of the solid obtained by revolving the region bounded by the curves $y = x^3, y = 0, x = 1$ about the y-axis is equal to

(a) $\frac{2\pi}{5}$ (b) $\frac{5\pi}{2}$ (c) $\frac{3\pi}{2}$ (d)None of these

Q.No:16 The arc length of the curve $x = t, y = \cosh t, 0 \le t \le \ln 2$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{e^2}{2}$ (d) $\frac{1}{2}$

Q.No:17 The area of the surface generated by revolving the graph of $x = \sqrt{9-y^2}$, $-2 \le y \le 2$ about the y-axis is equal to

(a) 24π (b) 12π (c) 32π (d) $\frac{15\pi}{2}$

Q.No:18 If $(r,\theta) - coordinate$ of points are $\left(\sqrt{3}, \frac{\pi}{6}\right)$ then $(x,y) - coordinates$ are

(a) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ (c) $\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$ (d)None of these

Q.No:19 If $(x,y) - coordinate$ of a point are $(0,2)$ then the $(r,\theta) - coordinates$ are

(a) $\left(2, \frac{\pi}{2}\right)$ (b) $\left(2, -\frac{\pi}{2}\right)$ (c) $\left(2, \frac{3\pi}{2}\right)$ (d)None of these

Q.No:20 If $C: x = t+1, y = t^2 + 3t$, then $\frac{d^2y}{dx^2}$ at t=2 is equal to

(a) 2 (b) t=-2, (c) t=1, (d) None of these.

Full Questions

Question No: 21 Find the limit $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln(x)}\right)$ if it exists. [4]

Solution: $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln(x)}\right)$ is a $(\infty-\infty)-form$

$$\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln(x)}\right)=\lim_{x\to 1}\left(\frac{\ln(x)-(x-1)}{(x-1)\ln(x)}\right)$$ is now a $\frac{0}{0}-form$

We can apply L'Hopital rule

$$\lim_{x\to 1}\left(\frac{\frac{1}{x}-1}{\ln(x)+(x-1)\frac{1}{x}}\right)=\lim_{x\to 1}\left(\frac{1-x}{x\ln(x)+(x-1)}\right)$$ it is still $\frac{0}{0}-form$

$$=\lim_{x\to 1}\frac{-1}{x\frac{1}{x}+\ln(x)+1}=\lim_{x\to 1}\frac{-1}{1+\ln(x)+1}=-\frac{1}{2}$$

Question No: 22 Evaluate the integral $\int\frac{x^3}{\sqrt{x^2+49}}dx$. [6]

Solution: Put $x=7\tan\theta\Rightarrow dx=7Sec^2\theta d\theta$

$$\sqrt{x^2+49}=7Sec\theta$$

$$\int\frac{x^3}{\sqrt{x^2+49}}dx=\int\frac{(7\tan\theta)^3}{7Sec\theta}7Sec^2\theta d\theta=(7)^3\int\tan^3\theta Sec\theta d\theta$$

$$=343\int\left(Sec^2\theta-1\right)\tan\theta Sec\theta d\theta=\frac{343}{2}\frac{\left(Sec^2\theta-1\right)^2}{2}+c=\frac{343}{4}\left(\frac{49+x^2}{49}-1\right)^2+c$$

Question No: 23 **Set up the integral that can be used to find the volume of the Solid generated by revolving the region $y = x^2$, $y = 6 - x$, and $y = 0$ about the x-axis.**

Solution: **Using Disc method**

$$V= \pi\int_0^2 (x^2)^2\,dx + \int_2^6 (6-x)\,dx \qquad [4]$$

Question No: 24 **Find the length of the curve given by parametric equations $C: x = t^2, y = 2t^3 + 1\ dx$, $0 \le t \le 1$.**

Solution:

$$L_0^1 = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{(2t)^2 + (6t^2)^2}\,dt = \int_0^1 \sqrt{4t^2 + 36t^4}\,dt$$

$$= \int_0^1 2t\sqrt{1+9t^2}\,dt = \frac{1}{9}\int_0^1 (1+9t^2)^{1/2}(18t)\,dt = \frac{1}{9}\left[\frac{(1+9t^2)^{3/2}}{3/2}\right]_0^1$$

$$= \frac{2}{27}\left[(10)^{3/2} - 1\right]$$

Question N0: 25 **Find the surface area of the solid generated by revolving the**

Curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \le x \le 2$ **about the x-axis.**

Solution:

Surface Area$= = 2\pi\int_1^2 y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\pi\int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{1+\left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}\,dx$

$$= 2\pi\int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{1+\left(\frac{x^4}{4} - 2\left(\frac{x^2}{2}\right)\left(\frac{1}{2x^2}\right) + \frac{1}{4x^4}\right)}dx$$

$$= 2\pi\int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{1+\left(\frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}\right)}dx$$

$$= 2\pi\int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\ dx$$

$$= 2\pi\int_1^{21}\left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)\ dx = 2\pi\int_1^2\left(\frac{x^5}{12}+\frac{x}{12}+\frac{x}{4}+\frac{1}{4x^3}\right)dx$$

$$= 2\pi\int_1^2\left(\frac{x^5}{12}+\frac{x}{3}+\frac{1}{4}x^{-3}\right)\ dx = 2\pi\left[\frac{x^6}{72}+\frac{x^2}{6}-\frac{x^{-2}}{2}\right]_1^2$$

$$= 2\pi\left[\frac{(2)^6}{72}+\frac{4}{6}-\frac{1}{8}-\left(\frac{1}{72}+\frac{1}{6}-\frac{1}{2}\right)\right]$$

$$= 2\pi\left[\frac{64}{72}+\frac{4}{6}-\frac{1}{8}-\frac{1}{72}-\frac{1}{6}+\frac{1}{2}\right] = 2\pi\left[\frac{63}{72}+\frac{3}{6}+\frac{3}{8}\right] = 2\pi$$

Question No: 26 Find the area of the region inside $r = 2\cos\theta$ and out side $r = 1$.

Solution: **Area=** $2\pi\int_0^{\pi/3}\left[(2Cos\theta)^2-(1)^2\right]d\theta = 2\pi\int_0^{\pi/3}\left[4Cos^2\theta-1\right]d\theta$

$$= 2\pi\int_0^{\pi/3}\left[\frac{4(1+Cos2\theta)}{2}-1\right]d\theta = 2\pi\int_0^{\pi/3}\left[2+2Cos2\theta-1\right]d\theta$$

$$= 2\pi\int_0^{\pi/3}\left[1+2Cos2\theta\right]d\theta = 2\pi\left[\theta+2\frac{Sin2\theta}{2}\right]_0^{\pi/3}$$

$$= 2\pi\left[\frac{\pi}{3}+Sin\left(\frac{2\pi}{3}\right)\right] = 2\pi\left[\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right]$$