KING SAUD UNIVERSITY
College of Science
Department of Mathematics

First Semester (1433/1434)

Final Exam, M-106

Programmable Calculators are Not Authorized

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 40 Time: Three hours

The Exam paper contains 8 pages
(**15 Multiple choice questions and 6 Full questions**)

Multiple Choice (1-15)	
Question # 16	
Question # 17	
Question # 18	
Question # 19	
Question # 20	
Question # 21	
Total	

Multiple Choice

Q. No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\{a,b,c,d\}$	b	c	c	b	a	d	d	b	c	b	a	d	b	b	b

Q. No: 1 $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2k}{n^2}$ is equal to:

(a) 0 $\quad(b)$ 1 $\quad(c)$ 2 $\quad(d)$ ∞

Q. No: 2 The average value of the function $f(x)=\sin(3x)$ on $[0,\frac{\pi}{3}]$ is equal to:

$(a)\ -\frac{2}{\pi}$ $\quad(b)\ \frac{2}{3}$ $\quad(c)\ \frac{2}{\pi}$ $\quad(d)\ -\frac{2}{3}$

Q. No: 3 The integral $\int_0^1\left|x-\frac{1}{2}\right|dx$ is equal to:

(a) 0 $\quad(b)$ 1 $\quad(c)\ \frac{1}{4}$ $\quad(d)\ \frac{1}{2}$

Q. No: 4 The intergral $\int\frac{\sin x}{1+\cos^2 x}dx$ is equal to:

$(a)\ -\tan^{-1}(\sin x)+c$ $\quad(b)\ -\tan^{-1}(\cos x)+c$
$(c)\ \tan^{-1}(\sin x)+c$ $\quad(d)\ \tan^{-1}(\cos x)+c$

Q. No: 5 The integral $\int 2^{-x}\tanh(2^{1-x})dx$ is equal to:

$(a)\ \frac{1}{-2\ln 2}\ln\cosh(2^{1-x})+c$ $\quad(b)\ \frac{1}{-\ln 2}\ln\cosh(2^{1-x})+c$
$(c)\ \frac{1}{-2\ln 2}\tanh^2(2^{1-x})+c$ $\quad(d)\ \frac{1}{-2\ln 2}\ln\sinh(2^{1-x})+c$

Q. No: 6 If $F(x)=\pi^x\int_0^{x^2}\tan^{-1}(t)dt$, then $F'(x)$ is equal to:

$(a)\ \pi^x F(x)\ln\pi+\ 2x\pi^x\tan^{-1}x$ $\quad(b)\ \pi^x F(x)\ln\pi+\ 2x\pi^x\tan^{-1}x^2$
$(c)\ F(x)\ln\pi+\ 2x\pi^x\tan^{-1}x$ $\quad(d)\ F(x)\ln\pi+\ 2x\pi^x\tan^{-1}x^2$

Q. No: 7 $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\cos(2x)}$ is equal to:

(a) 2 $\quad(b)\ -2$ $\quad(c)\ -1$ $\quad(d)$ 1

Q. No: 8 If a point has $xy-$ coordinates $(x, y) = (\sqrt{2}, \sqrt{2})$ then one of its polar coordinates (r, θ) is:

(a) $(1, \frac{\pi}{4})$ (b) $(2, \frac{\pi}{4})$ (c) $(\sqrt{2}, \frac{\pi}{4})$ (d) $(\sqrt{2}, \frac{3\pi}{4})$

Q. No: 9 The integral $\int_1^2 x \ln(x) dx$ is equal to:

(a) $2\ln 2 - \frac{5}{4}$ (b) $2\ln 2 + \frac{5}{4}$ (c) $2\ln 2 - \frac{3}{4}$ (d) $2\ln 2 + \frac{3}{4}$

Q. No: 10 The slope of the tangent line at the point corresponding to $t = 1$ on the curve given parametrically by the equations $x = t^3 + t$, $y = -3t$, is:

(a) $-\frac{1}{2}$ (b) $-\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Q. No: 11 If a graph has polar equation $r = -4\cos\theta$, then its equation in $xy-$system is:

(a) $x^2 + 4x + y^2 = 0$ (b) $x^2 - 4x + y^2 = 0$
(c) $x^2 - x + y^2 = 0$ (d) $x^2 + y^2 = 0$

Q. No: 12 The arc length of the curve $C : x = 4\cos(t)\,,\ y = 4\sin(t)\,,\ -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ equals:

(a) 2π (b) 8π (c) π (d) 4π

Q. No: 13 The surface area resulting by revolving the graph of the equation $x = y$, $0 \leq y \leq 1$ around the $y-$axis is equal to:

(a) $8\sqrt{2}\pi$ (b) $\sqrt{2}\pi$ (c) $24\sqrt{2}\pi$ (d) $\frac{9}{2}\sqrt{2}\pi$

Q. No: 14 The improper integral $\int_0^\infty \frac{1}{x+1} dx$

(a) converges to 0 (b) diverges (c) converges to 1 (d) converges to -1

Q. No: 15 The graph of the curve $\mathcal{C}$ defined by the parametric equations $x = t + 1$; $y = t^2 + 1$, $-3 \leq t \leq 1$ is a:

(a) a straight line (b) a parabola (c) an ellipse (d) a circle

Full Questions

Question No: 16 **Evaluate** $\int \frac{1}{x^2\left(x^2+1\right)} dx$

Method 1: We can get

$$\frac{1}{x^2\left(x^2+1\right)}=\frac{1}{x^2}-\frac{1}{x^2+1} \text{ [2]}$$

and then we will have

$$\int \frac{1}{x^2\left(x^2+1\right)} dx=-\frac{1}{x}-\tan^{-1}(x)+c \text{ [2]}$$

Method 2: Let

$$x=\tan\theta,-\frac{\pi}{2}<\theta<\frac{\pi}{2} \text{ then } dx=\sec^2\theta d\theta \text{ [1]}$$

we get

$$\begin{aligned}
\int \frac{1}{x^2\left(x^2+1\right)} dx &= \int \frac{\sec^2(\theta)}{\tan^2(\theta)\sec^2(\theta)} d\theta=\int \cot^2(\theta) d\theta=\int(\csc^2(\theta)-1) d\theta \text{ [1]}\\
&= -\cot(\theta)-\theta+c \text{ [1]}\\
&= -\frac{1}{x}-\tan^{-1} x+c \text{ [1]}
\end{aligned}$$

1. **Evaluate** $\int \frac{x+2}{\sqrt{x^2+2x+2}} dx$

$$\begin{aligned}
\int \frac{x+2}{\sqrt{x^2+2x+2}} dx &= \int \frac{x+2}{\sqrt{(x+1)^2+1}} dx=\int \frac{x+1+1}{\sqrt{(x+1)^2+1}} dx \quad \text{[0.5]}\\
&= \int \frac{u+1}{\sqrt{u^2+1}} du \text{ where } u=x+1 \quad \text{[1]}\\
&= \int \frac{2u}{2\sqrt{u^2+1}} du+\int \frac{1}{\sqrt{u^2+1}} du \quad \text{[1]}\\
&= \sqrt{u^2+1}+\sinh^{-1}(u)+c \quad \text{[1]}\\
&= \sqrt{(x+1)^2+1}+\sinh^{-1}(x+1)+c \quad \text{[0.5]}
\end{aligned}$$

Question No: 18 **a) Evaluate** $F(x) = \dfrac{d}{dx}(\sqrt{x}\sinh(\sqrt{x}))$.

$$\begin{aligned} F(x) &= \frac{d}{dx}(\sqrt{x}\sinh(\sqrt{x})) \\ &= \frac{1}{2\sqrt{x}}\sinh(\sqrt{x}) + \sqrt{x}\frac{1}{2\sqrt{x}}\cosh(\sqrt{x}) \ [2] \\ &= \frac{1}{2\sqrt{x}}\sinh(\sqrt{x}) + \frac{1}{2}\cosh(\sqrt{x}) \qquad [1] \end{aligned}$$

b) Find $\int \cosh(\sqrt{x})dx$ by using $F(x)$.

we have from a)

$$\cosh(\sqrt{x}) = 2F(x) - \frac{1}{\sqrt{x}}\sinh(\sqrt{x})$$

Then

$$\begin{aligned} \int \cosh(\sqrt{x})dx &= 2\int F(x)dx - \int \frac{1}{\sqrt{x}}\sinh(\sqrt{x})dx \quad [1] \\ &= 2\sqrt{x}\sinh(\sqrt{x}) - 2\cosh(\sqrt{x}) + c \quad [1] \end{aligned}$$

Question No: 19 **Evaluate** $\displaystyle\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}dx$

Method 1: By using integration by part

$$u = \sin^{-1}(x) \text{ and } v' = \frac{x}{\sqrt{1-x^2}} \ [1]$$

then

$$u' = \frac{1}{\sqrt{1-x^2}} \text{ and } v = -\sqrt{1-x^2} \ [1]$$

and we can get

$$\begin{aligned} \int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}dx &= -\sqrt{1-x^2}\sin^{-1}(x) + \int dx \ [1] \\ &= -\sqrt{1-x^2}\sin^{-1}(x) + x + c \ [1] \end{aligned}$$

Method 2: Let $u = \sin^{-1}(x)$ where $-\dfrac{\pi}{2} < u < \dfrac{\pi}{2}$ then

$$u = \sin^{-1}(x) \text{ where } -\frac{\pi}{2} < u < \frac{\pi}{2} \text{ then, } \sin u = x \text{ and } dx = \cos u\, du \quad [1]$$

and then

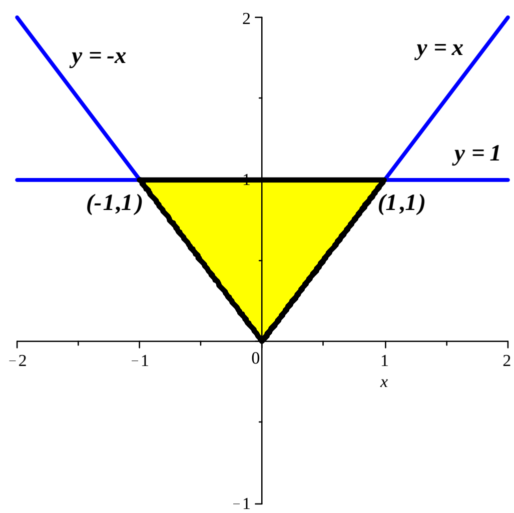
$$\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}dx = \int \frac{u\sin u}{\cos u}\cos u du = \int u\sin u du \text{ [1]}$$

and by using integration by part and $\cos u = \sqrt{1-x^2}$ we will have

$$\begin{aligned} \int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}dx &= \sin u - u\cos u + c \quad \text{[1]} \\ &= x - \sin^{-1}(x)\sqrt{1-x^2} + c \text{ [1]} \end{aligned}$$

Question No: 20 Let R be the region bounded by the graph $y = x$, $y = -x$, and $y = 1$. **Sketch** the region R and **Find** the **volume** of the solid generated by revolving the region R about the $x-$axis.

Solution:



[1]

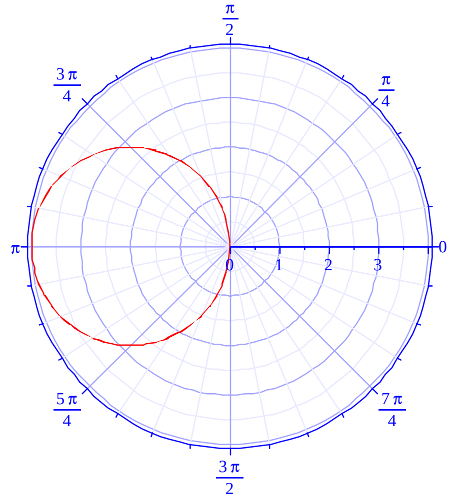
Method 1: We have

$$\begin{aligned} V &= 2\pi \int_0^1 (2y)y dy \quad = 4\pi \int_0^1 y^2 dy \qquad \text{[2]} \\ &= \frac{4\pi}{3} \qquad \text{[1]} \end{aligned}$$

Method 2: We can get also

$$\begin{aligned} V &= V_1 + V_2 \\ &= \pi \int_{-1}^{0} (1^2 - (-x)^2) dx + \pi \int_{0}^{1} (1^2 - x^2) dx = \pi \int_{-1}^{1} (1^2 - x^2) dx \text{ [2]} \\ &= \frac{4\pi}{3} \text{ [1]} \end{aligned}$$

Question No: 21 : **Sketch** and **Find** the surface area genarated by revolving the graph of the polar equation $r = -4\cos(\theta)$ about the vertical line $\theta = \frac{\pi}{2}$.



[1]

$$A = 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |x| \, ds \text{ [1]}$$

where

$$|x| = r\,|\cos\theta| = -r\cos\theta = 4\cos^2(\theta) \text{ [0.5]}$$

and

$$ds = \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta = \sqrt{16\cos^2\theta + 16\sin^2\theta} d\theta = 4d\theta \text{ [0.5]}$$

Then

$$\begin{aligned} A &= 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 4\cos^2(\theta) 4 d\theta \text{ [0.5]} \\ &= 16\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1 + \cos(2\theta)) d\theta = 16\pi^2 \text{ [0.5]} \end{aligned}$$