KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS
TIME: 3H, FULL MARKS: 40, 27/02/1434
MATH 204

Question 1. a) [3] Determine the largest region for which the following initial value problem admits a unique solution

$$\ln(x-2).\frac{dy}{dx} = \sqrt{y-2}\ , \qquad y(\frac{5}{2}) = 4.$$

b) [3]. Solve the linear first order differential equation:

$$(y - x + xy\cot x)dx + xdy = 0, \quad x \in (0, \pi).$$

Question 2. a) [4]. Verify that $\mu(x,y) = xy^2$ is an integrating factor for the equation

$$(4x^2y + 2y^2)dx + (3x^3 + 4xy)dy = 0,$$

and hence solve it.

b) [4]. Find the family of orthogonal trajectories for the family of curves: $x^2 - y^2 = C$. Which curve of the orthogonal family passes through $(0,0)$.

Question 3. a) [4]. Find the general solution of the differential equation

$$y^{''} - 2y' + y = \frac{e^x}{x}, \qquad x > 0.$$

b) [4]. Write down the general form of the particular solution y_p for the differential equation

$$y^{(4)} - y^{''} = x + xe^x + xe^{-x}.$$

Question 4. a) [4] Solve the initial value problem:

$$x^2y'' + 3xy' + 2y = 0, \quad y(1) = 0, \ y'(1) = 1, \qquad x > 0.$$

b) [4]. Solve the following differential equation by using the method of power series about $x = 0$.

$$y'' - 2x^2y' + 8y = 0$$

Question 5. a) [5]. Expand in Fourier series the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$

and deuduce that $\sum\limits_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$.

b) [5]. Find the Fourier integral representation of the function

$$g(x) = \begin{cases} 0, & -\infty < x < -1 \\ 2, & -1 < x < 1 \\ 0, & 1 < x < \infty \end{cases}$$

and deduce that $\int\limits_0^{\infty} \frac{\sin\lambda}{\lambda} d\lambda = \frac{\pi}{2}$.