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Fast and efficient sensitivity calculation using adjoint method for three-phase field-scale history matching

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ARTICLE INFO

ABSTRACT

Adjoint method-based sensitivity for field-scale history matching with large numbers of parameters suffers from several limitations. First, the CPU time depends on the data points which are large for any brown fields of long history; second, it requires large memory to save the gridblock pressure and saturation per each time step used in the forward model. Third, it is computationally expensive as it requires solving the Adjoint system of equations backward in time per each forward time step which is usually of high magnitude in case of field scale applications of long history. Lastly, the solver used for solving the Adjoint system of equations needs to be efficient for large-scale applications.

We propose an efficient and fast approach for sensitivity calculation based on the Adjoint method to overcome much of the current limitations. First, we use a commercial finite difference simulator, ECLIPSE, as a forward model, which is general and can account for complex physical behavior that dominates most field applications. Second, the production data misfit is represented by a single generalized travel time misfit per well, thus effectively reducing the number of data points into one per well. Third, we solve the Adjoint system of equations backward in time in a larger time step that is equivalent to the time of severe changes in pressure and saturation due to changing well conditions or introducing new infill wells rather than using the forward model time steps. This approach reduces the computational effort and memory allocated for the sensitivity calculation. Fourth, we use an iterative sparse matrix solver, LSQR, for solving the Adjoint system of equations which shows high stability for field-scale applications.

We demonstrate the power and utility of our approach using synthetic and pseudo field examples. The synthetic examples show the robustness and efficiency of our sensitivity calculation approach compared to the perturbation. The pseudo-field example had 10 years of production history with an original gas cap and a strong aquifer support. Using well log data, core data, water cut and gas–oil ratio history from producing wells; we characterize the permeability at each cell, thus demonstrating the feasibility of our approach for field applications.

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1. Introduction

Conditioning geological models to production data is an important step in reservoir modeling to build a reliable model to be used in predicting the reservoir performance and in proposing the optimum field development plan. Conditioning the geological or the static model to production data is typically known as "History Matching". Any reduction in the time taken for the "History Matching" phase is very important to speed up the modeling process as the majority of the development plans should be based on examining it on a model before accepting it for practical application. Accordingly, building reservoir model for each field has become a commonly used practice in the industry, and any improvement in speeding up the reservoir modeling process is highly demanded.

Different techniques are proposed to speed up the history matching process, where all are grouped under what is called computer assisted or automatic history matching (Anterion et al., 1989; Chavent et al., 1975; Chen et al., 1974; Cheng et al., 2004; Daoud, 2004; Daoud and Vega, 2006; Gavalas et al., 1976; He et al., 2002; Li et al., 2001; Vasco et al., 1999; Vasco and Datta-Gupta, 1997; Vega, 2003; Wasserman et al., 1975; Watson et al., 1980; Wu et al., 1999; Wu and Datta-Gupta, 2002; Zhang et al., 2003). The automatic history matching procedure involves the following steps; First, the forward model formulation, second, the data misfit calculation, third, the sensitivity coefficient calculation, and finally an optimization algorithm.

First, the forward model used is the commonly used finite difference simulation, ECLIPSE (Schlumberger, 2003) which is general, robust, and can tackle different physical problems.

Second, the data misfit is the way of representing the error between the actual field response and the calculated response from the model which is the objective function required to be minimized using an optimization algorithm. Different ways are proposed to represent the

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data misfit; the amplitude misfit (Li et al., 2001; Wu et al., 1999; Zhang et al., 2003), travel time misfit (Vasco et al., 1999), and generalized travel time misfit (Cheng et al., 2004; Daoud, 2004; Daoud and Vega, 2006; He et al., 2002; Vega, 2003; Wu and Datta-Gupta, 2002). The generalized travel time is well suited for field-scale history matching as it considers all the observed data points, and treats them as one objective function per well which is computationally efficient (Daoud, 2004; Daoud and Vega, 2006; Wu and Datta-Gupta, 2002). In addition, it has a linear convergence criteria compared to the amplitude misfit (Cheng et al., 2005).

Third, the sensitivity calculations are the main core of the history match procedure and the highest computational time in the process. Many methods were developed to calculate the sensitivity coefficients. The adjoint method or the optimal control theory becomes more popular (Li et al., 2001; Wu et al., 1999; Zhang et al., 2003) due to its computational advantages, particularly when the number of model parameters is higher than the number of the data points.

Finally, an optimization algorithm is required to solve such an inverse problem. The gradient-based algorithms are commonly used in the automatic or computer assisted history matching process because the gradient-free algorithms can be computationally prohibitive for field applications, particularly when a large number of parameters are involved. The gradient-based algorithms are classified according to their search direction into steepest descent, Newton, quasi-Newton, and conjugate gradient (Nocedel and Wright, 1999). The fastest among those is the Newton-type search as it has a quadratic rate of convergence in the vicinity of the solution compared to the quasi-Newton method which has a super linear rate of convergence, and conjugate gradient or

the steepest descent which have a linear rate of convergence. The Gauss-Newton and its variations (Nocedel and Wright, 1999) are commonly used to solve the reservoir history matching problems.

Any reduction in the CPU time taken for the above mentioned steps of the automatic history matching is considered as a major step in reducing the time spent in conducting field study and building a reliable model that honors both the geological and the production data. In our work, we propose an approach to speed up the sensitivity calculation using adjoint method. All the previous work of the history match using the adjoint method obtains the sensitivity by solving the adjoint system of equations at time steps identical to the forward model time steps which is computationally expensive. In our work, we propose a fast and robust sensitivity calculation approach that overcomes this issue and is well suited for large-scale field applications. Our approach is based on solving the adjoint system of equations backward in time at larger timestep with reasonable accuracy compared to the conventional adjoint method. Our proposed approach exhibits a highly reduction of CPU time, which reaches up to 90% of the time required for the sensitivity calculations using the conventional adjoint method approach. Thus, it is considered an efficient way in reducing the time required for the history match process.

The outline of our paper is as follows. First, we give the mathematical formulation for the sensitivity of generalized travel time with respect to the model parameters using the adjoint method for three phase problems. Second, we propose a new approach of solving the adjoint system of equations at larger time steps than the forward model time steps. Third, we demonstrate the power of our proposed approach in terms of quality of sensitivity, and savings in CPU time and memory by comparing it with the conventional approach. Finally, we conduct a full history match process on a pseudo field example to illustrate the efficiency and utility of our approach.

2. Adjoint-method based sensitivity for three phase three dimensions reservoir models

The sensitivity computation using the adjoint method requires three major steps (Daoud and Vega, 2006; Li, 2001; Li et al., 2001; Wu et al., 1999; Wu and Datta-Gupta, 2002; Zhang et al., 2003): the forward model formulation, adjoint system of equations, and the sensitivity coefficient calculation. First, the forward model used is the commercial finite difference simulator ECLIPSE as it is robust and efficient for field-scale applications. The required output from the ECLIPSE forward model for the generalized travel time sensitivity calculation are the generalized travel time misfit at each producer, the pressure at each grid block for each timestep, water and gas saturations at each grid block per each time step, and the bottom hole flowing pressures at the wells at each timestep.

Second, the adjoint system of equations in a complete discretized form (Daoud, 2004; Daoud and Vega, 2006; Li, 2001; Wu et al., 1999)
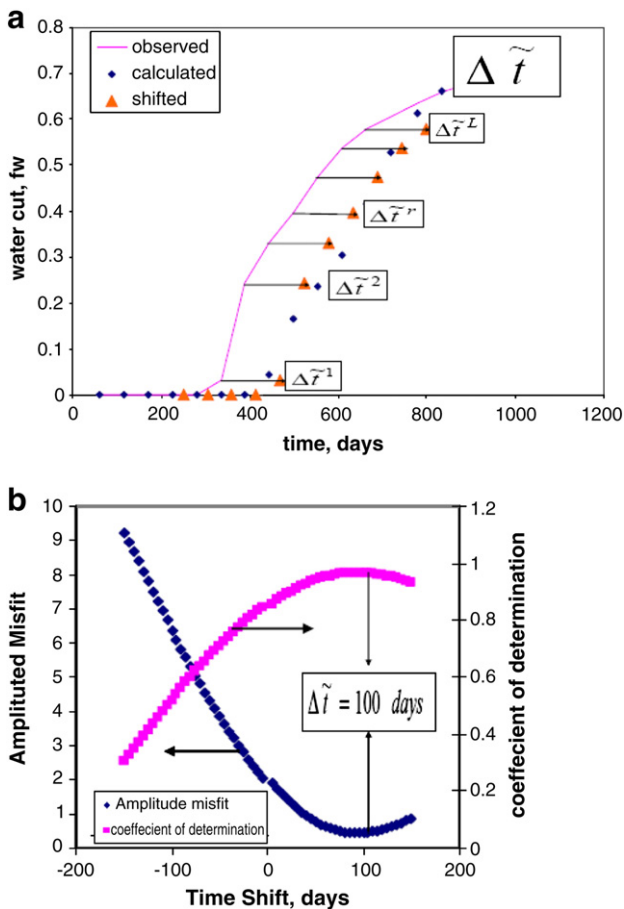

Fig. 1. Illustration of generalized travel-time inversion: (a) history-matching by systematically shifting the observed water cut to the calculated history, (b) best shift-time which maximizes the coefficient of determination or minimizes the amplitude misfit (After Daoud and Vega, 2006).

Table 1
Data for case 1.

|	
Grid blocks	$15 \times 15 \times 2$
Grid size	$\Delta x = 40$ ft, $\Delta y = 40$ ft, $\Delta z = 30$ ft
Porosity	0.22
Permeability	100 md
Oil and water viscosity	$\mu_o = 0.82$ cp, $\mu_w = 1.0$ cp
Oil formation volume factor, B_o at P_i	1.24 bbl/stb
Water formation volume factor, B_w at P_i	1.0 bbl/stb
Initial water saturation	0.2
Initial pressure	5500 psi
Wells	Producer at grid 3, 3, 1:2 (Liquid rate control of 500 stb/d) Injector at grid 13, 13, 1:2 (BHP control of 6100 psi)
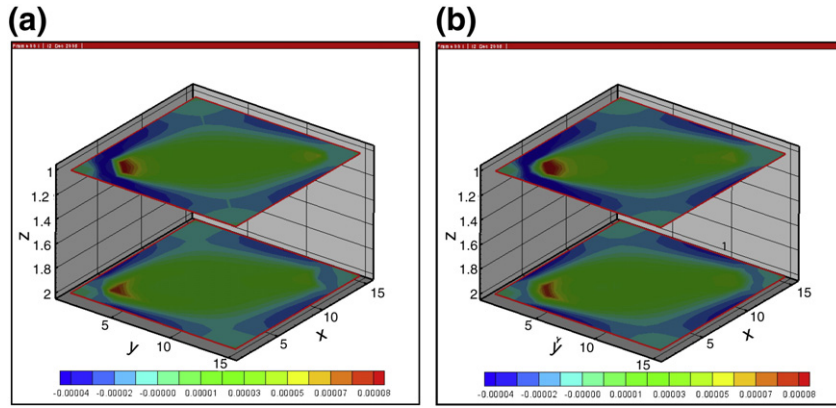
Fig. 2. Comparison of sensitivity for the water cut with respect to horizontal permeability for case 1, (a) Perturbation, (b) the conventional Adjoint method.

as given in Eqs. (1)–(3) is solved backward in time to obtain the adjoint variables at each grid block per each time step.

$$\sum_{m=o,w,g}\left[\left(\nabla_{P^l}\left[f_m^l-A_m^l-q_m^l\right]^T\right)\lambda_m^l + \nabla_{P^l}\left[A_m^l\right]^T\lambda_m^{l+1}\right] = -\nabla_{p^l}g \qquad (1)$$

$$\sum_{m=o,w,g}\left[\left(\nabla_{S_w^l}\left[f_m^l-A_m^l-q_m^l\right]^T\right)\lambda_m^l + \nabla_{S_w^l}\left[A_m^l\right]^T\lambda_m^{l+1}\right] = -\nabla_{S_w^l}g \qquad (2)$$

$$\sum_{m=o,w,g}\left[\left(\nabla_{S_g^l}\left[f_m^l-A_m^l-q_m^l\right]^T\right)\lambda_m^l + \nabla_{S_g^l}\left[A_m^l\right]^T\lambda_m^{l+1}\right] = -\nabla_{S_g^l}g \qquad (3)$$

A complete derivation of the above adjoint system of equations can be found elsewhere (Daoud, 2004; Daoud and Vega, 2006; Li, 2001; Wu et al., 1999). The above adjoint system of equations is solved backward in time for the adjoint variable λ^l per each timestep using the condition,

$$\lambda_o^L = \lambda_w^L = \lambda_g^L = 0 \qquad (4)$$

Where L is the last timestep used in the forward simulation run.

In Eqs. (1)–(3), f_m and A_m are the flow and accumulation term, respectively, in the partial differential equation governing flow in porous media, and they are given in the discretized form (neglecting the capillary pressure) as follows:

$$\begin{aligned}
f_{m,i,j,k}^l = &\ T_{mx,i+1/2,j,k}^l\left(P_{i+1,j,k}^l-P_{i,j,k}^l-\gamma_{m,i+1/2,j,k}^l\left(D_{i+1,j,k}-D_{i,j,k}\right)\right)\\
&+ T_{mx,i-1/2,j,k}^l\left(P_{i-1,j,k}^l-P_{i,j,k}^l-\gamma_{m,i-1/2,j,k}^l\left(D_{i-1,j,k}-D_{i,j,k}\right)\right)\\
&+ T_{my,i,j+1/2,k}^l\left(P_{i,j+1,k}^l-P_{i,j,k}^l-\gamma_{m,i,j+1/2,k}^l\left(D_{i,j+1,k}-D_{i,j,k}\right)\right)\\
&+ T_{my,i,j-1/2,k}^l\left(P_{i,j-1,k}^l-P_{i,j,k}^l-\gamma_{m,i,j-1/2,k}^l\left(D_{i,j-1,k}-D_{i,j,k}\right)\right)\\
&+ T_{mz,i,j,k+1/2}^l\left(P_{i,j,k+1}^l-P_{i,j,k}^l-\gamma_{m,i,j,k+1/2}^l\left(D_{i,j,k+1}-D_{i,j,k}\right)\right)\\
&+ T_{mz,i,j,k-1/2}^l\left(P_{i,j,k-1}^l-P_{i,j,k}^l-\gamma_{m,i,j,k-1/2}^l\left(D_{i,j,k-1}-D_{i,j,k}\right)\right)
\end{aligned} \qquad (5)$$

$$\begin{aligned}
f_{g,i,j,k}^l = &\ T_{gx,i+1/2,j,k}^l\left(P_{i+1,j,k}^l-P_{i,j,k}^l-\gamma_{g,i+1/2,j,k}^l\left(D_{i+1,j,k}-D_{i,j,k}\right)\right)\\
&+ T_{gx,i-1/2,j,k}^l\left(P_{i-1,j,k}^l-P_{i,j,k}^l-\gamma_{g,i-1/2,j,k}^l\left(D_{i-1,j,k}-D_{i,j,k}\right)\right)\\
&+ T_{gy,i,j+1/2,k}^l\left(P_{i,j+1,k}^l-P_{i,j,k}^l-\gamma_{g,i,j+1/2,k}^l\left(D_{i,j+1,k}-D_{i,j,k}\right)\right)\\
&+ T_{gy,i,j-1/2,k}^l\left(P_{i,j-1,k}^l-P_{i,j,k}^l-\gamma_{g,i,j-1/2,k}^l\left(D_{i,j-1,k}-D_{i,j,k}\right)\right)\\
&+ T_{gz,i,j,k+1/2}^l\left(P_{i,j,k+1}^l-P_{i,j,k}^l-\gamma_{g,i,j,k+1/2}^l\left(D_{i,j,k+1}-D_{i,j,k}\right)\right)\\
&+ T_{gz,i,j,k-1/2}^l\left(P_{i,j,k-1}^l-P_{i,j,k}^l-\gamma_{g,i,j,k-1/2}^l\left(D_{i,j,k-1}-D_{i,j,k}\right)\right)
\end{aligned} \qquad (6)$$

$$A_{m,i,j,k}^l = \frac{V_{b,i,j,k}\phi_{i,j,k}S_{m,i,j,k}^l}{5.615\Delta t^l B_{m,i,j,k}^l} \qquad (7)$$

$$A_{g,i,j,k}^l = \frac{V_{b,i,j,k}\phi_{i,j,k}}{5.615\Delta t^l}\left[\frac{S_{g,i,j,k}^l}{B_{g,i,j,k}^l} + R_{so}^l\frac{S_{o,i,j,k}^l}{B_{o,i,j,k}^l}\right] \qquad (8)$$

Where, m stands for the flowing phase oil and water, and g for gas. It is important to mention that prior to solving the adjoint equations, we need to run the flow simulator and save the pressure and saturation for each grid block at each time step to compute the coefficients of the adjoint equations.

In our work, the source term in the adjoint system of equations, which is (g) in Eqs. (1)–(3), is the generalized travel time shift at the well j that corresponds to the production response required to be matched. In our case the required production response are the water cut and gas–oil ratio per each well. Fig. 1 illustrates the methodology used to calculate the generalized travel time shift per each well per iteration and Eq. (9) gives the formulation used to estimate the generalized travel time shift per each well. Fig. 1 shows the water cut as the production response. However the same methodology can be also applied for gas–oil ratio and any other response. This formulation assumes that we are shifting the observed response towards the calculated response with all the data points are shifted with an equal time shift. Thus, the generalized travel time shift at well j takes into consideration all the data points (Daoud and Vega, 2006), and is given by the following equation,

$$g = \Delta\tilde{t}_j = \frac{1}{n_{dj}}\sum_{i=1}^{n_{dj}}\left(t_{shift,j}^i-t_{obs,j}^i\right) \cong \frac{1}{n_{dj}}\sum_{i=1}^{n_{dj}}\left(t_{cal,j}^i-t_{obs,j}^i\right) \qquad (9)$$

Where, n_{dj} is the number of data points for well j, (i) is the index for the data point at time t^i. Accordingly, in solving the adjoint system of equations backward in time, the gradient of the scalar quantity (g) with respect to pressure, water and gas saturations, as given in the right hand side of Eqs. (1)–(3), will be null vectors except at the time corresponding to observed data point time, where the non-zero elements in those vectors will be corresponding to the grid blocks containing producing wells only. It is important to mention that at each time step the adjoint system of equations, Eqs. (1), (2) and (3), is solved only n_w times with the same matrix but with different right hand side for each production response to get the adjoint variables (λ). Thus, in case of matching two production responses which are, in our case, the water cut and gas–oil ratio, it is required to solve the adjoint system of equations ($2n_w$) times.

Finally, the sensitivity coefficients, which are the sensitivity of the production response at each well with respect to change in history
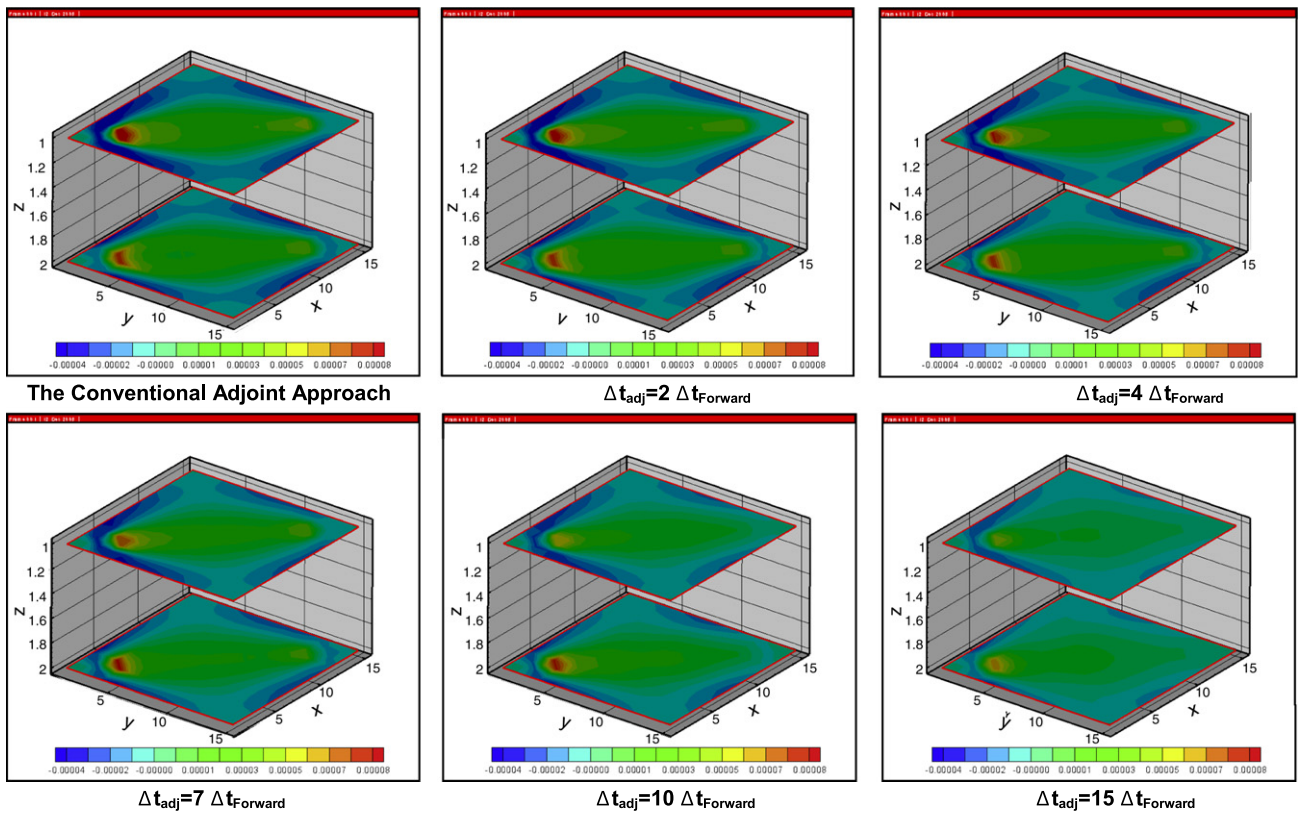
Fig. 3. Comparison of sensitivity for the water cut with respect to horizontal permeability for case 1 between the conventional approach and the proposed approach.

Table 2
Data for case 2.

Grid blocks	$13 \times 13 \times 1$
Grid size	$\Delta x = 40$ ft, $\Delta y = 40$ ft, $\Delta z = 30$ ft
Porosity	0.22
Permeability	100 md
Oil and water viscosity	$\mu_o = 0.82$ cp, $\mu_w = 1.0$ cp
Oil formation volume factor, B_o at P_i	1.24 bbl/stb
Water formation volume factor, B_w at P_i	1.0 bbl/stb
Initial water saturation	0.2
Initial pressure	5500 psi
Wells	Producer at grid 3, 3, 1:2 (controlled by Liquid rate of 50 stb/d for 500 days, and then by 500 stb/d for the next 500 days) Injector at grid 11, 11, 1 (controlled by BHP of 6500 psi)

match parameter, are calculated by Eq. (10) using the adjoint variables obtained from the previous step. In our work, the production responses are the water cut and gas–oil ratio at each well, and the history match parameter is the permeability at each grid block.

$$\nabla_{K_x} \Delta \tilde{t}_j = \left(\sum_{m=o,w,g} \sum_{l=0}^{L-1} \nabla_{K_x} \left[f_m^{l+1} - q_m^{l+1} \right]^T \lambda_m^{l+1} \right) \qquad (10)$$

Eq. (10) gives the gradient of the generalized travel time for the selected production response at well (j) with respect to the permeability at each grid block, which represents a vector of order ($M \times 1$). The combination of all the vectors representing the generalized travel time sensitivity for all the production response at all the wells is represented by a sensitivity matrix (G) of order ($n_w \times M$). In our case, as we are
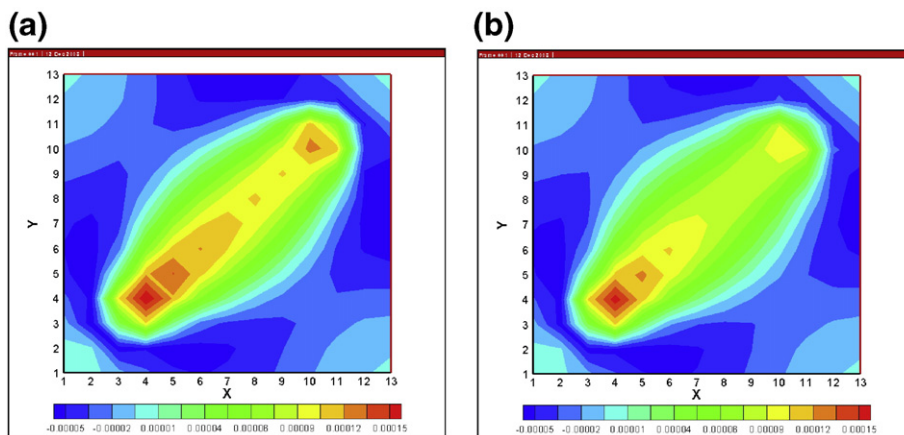

Fig. 4. Comparison of sensitivity for water cut with respect to horizontal permeability for case 2, (a) Perturbation, (b) the conventional Adjoint method.
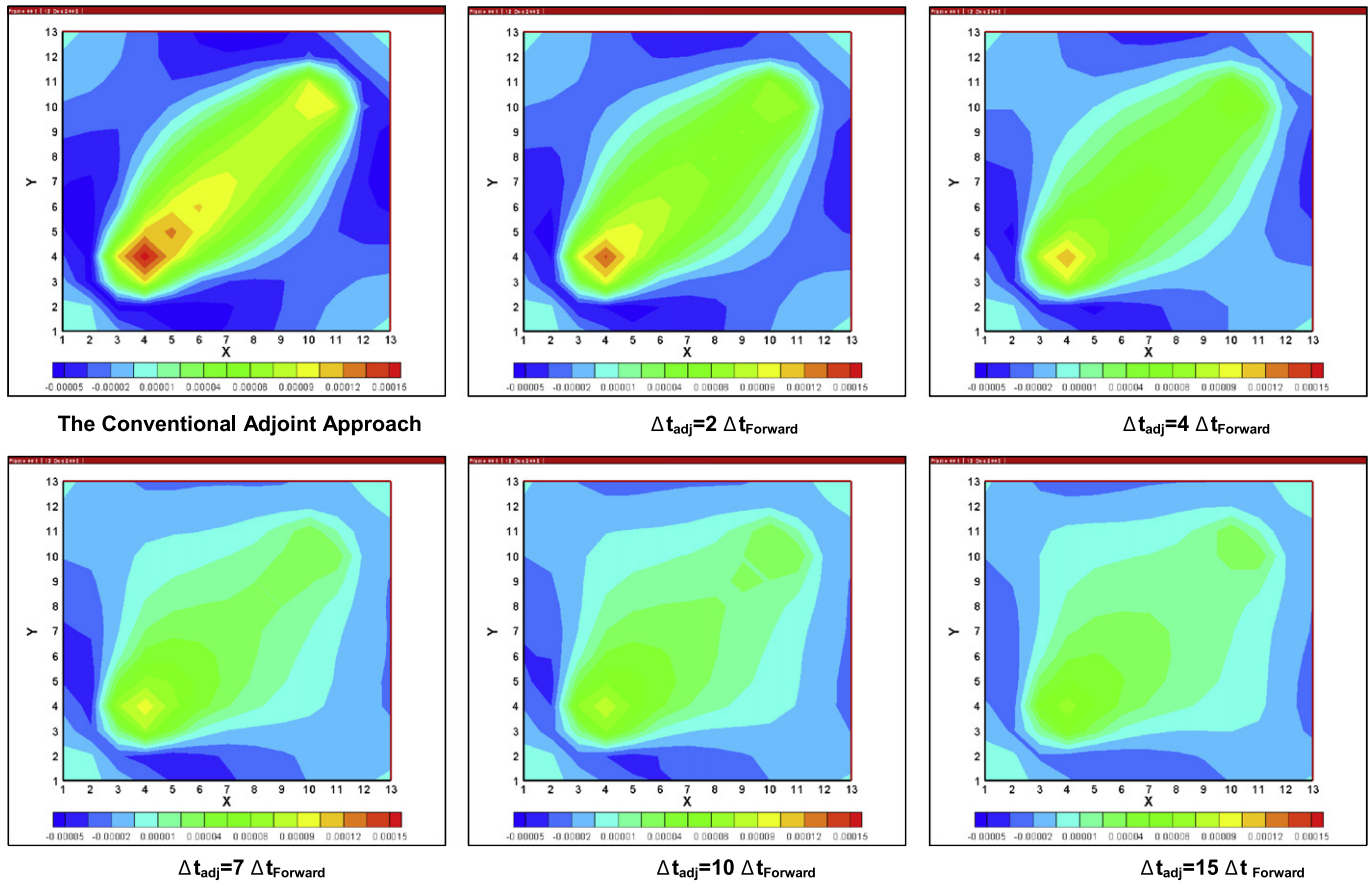
Fig. 5. Comparison of sensitivity for water cut with respect to horizontal permeability for case 2 between the conventional approach and the proposed approach.

dealing with two production responses which are water cut and gas–oil ratio, the sensitivity matrix (G) is of order ($2n_w$ X M). This sensitivity matrix is required by the optimization algorithm used to update the permeability field per iteration using Gauss-Newton.

It is proved that the majority of the CPU time per iteration is spent in calculating the sensitivity matrix (G) using adjoint method and not in the optimization algorithm (Daoud, 2004). Thus, any reduction in the CPU time taken in the sensitivity matrix calculation will have a strong impact in the automatic history matching of the production response by the automatic change of the permeability at each grid block. The next section presents a proposed approach to speed up the calculation of the adjoint method based sensitivity.

3. Proposed approach for fast and efficient sensitivity calculation

As mentioned previously, most of CPU time for the automatic history matching process is consumed in calculating the adjoint method based sensitivity, which depends mainly on the iterative solver used to solve the adjoint system of equations at each time step, and also on the number of time steps used in the forward simulation run. The CPU time can be reduced whether by decreasing the number of time steps used of solving the adjoint system of equations backward in time or by modifying the solver. We used LSQR (Paige and Saunders, 1982) as an iterative matrix solver which is robust and well suited for field-scale applications (Daoud and Vega, 2006; Nolet, 1987). Therefore, we propose a new methodology to reduce the number of time steps used to solve the adjoint system of equations.

All the previous works of adjoint method based sensitivity used timesteps for solving the adjoint system of equations identical to the

number of time steps used in the forward model run, that is due to the constraint imposed in the formulation of the adjoint system of equations, which requires that the flow equations should be satisfied at each time step so that the sensitivity of the augmented function (J) with respect to permeability at each gridblock is equivalent to the

Table 3
Data for case 3.

|	
Grid blocks	5×5×2
Grid size	$\Delta x = 75$ ft, $\Delta y = 75$ ft, $\Delta z = 30$ ft
Porosity	0.22
Permeability	Upper layer of 10 md and lower layer of 15 md
Oil and water viscosity	$\mu_o = 0.82$ cp, $\mu_w = 1.0$ cp
Gas viscosity at Pi	$\mu_g = 0.02$ cp
Oil formation volume factor, B_o at P_i	1.24 bbl/stb
Water formation volume factor, B_w at P_i	1.0 bbl/stb
Gas formation volume factor, B_g at P_b	0.614 bbl/Mscf
Solution gas ratio at bubble point pressure	0.9971 Mscf/stb
Gravity	$\rho_o = 50$ lb/cu-ft, $\rho_w = 62.5$ lb/cu-ft, $\rho_g = 0.07$ lb/cu-ft
Initial water saturation	0.2
Initial pressure	5500 psi
Bubble point pressure	5000 psi
Wells	Producer at grid 3, 3, 1:2 (controlled by Liquid rate of 50 stb/day)

(a)   **(b)**
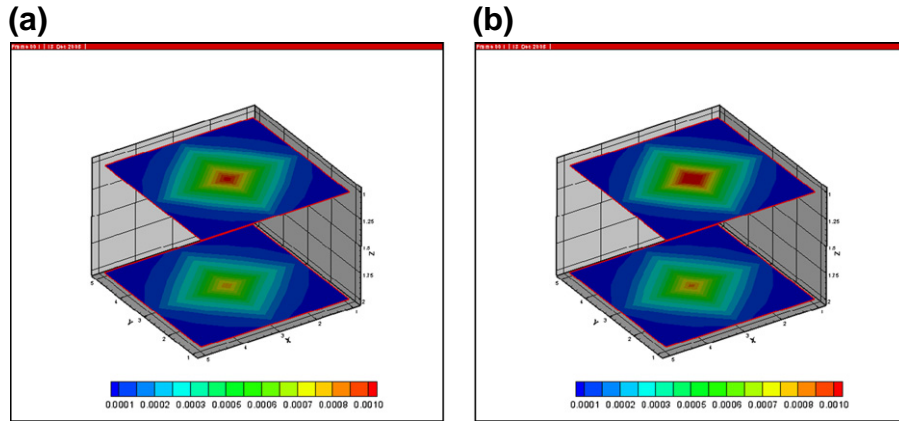
Fig. 6. Comparison of sensitivity for gas–oil ratio with respect to horizontal permeability for case 3, (a) Perturbation, (b) the conventional Adjoint method.

sensitivity of the generalized travel time (g) with respect to the permeability at each gridblock according to the following equations,

$$J = g + \sum_{m=o,w,g} \sum_{l=0}^{L-1} \left(\lambda_m^{l+1}\right)^T \left[f_m^{l+1} - q_m^{l+1} - A_m^{l+1} + A_m^l\right] \tag{11}$$

Where,

$$\left[f_m^{l+1} - q_m^{l+1} - A_m^{l+1} + A_m^l\right] = 0 \tag{12}$$

Eq. (12) stands for the oil, water, and gas flow equation at each timestep for $l = 0$ to L.

As seen from Eqs. (11) and (12), there are no constraints imposed on the size of the timestep used which means that the time steps used in the adjoint solution do not need to be identical to the time steps used in the forward reservoir simulator. Accordingly, we proposed an approach to solve the adjoint system of equations backward in time using larger time step compared to that used during the forward simulation run. However, to do that, one may need to interpolate between the pressure and saturation values generated in a simulation run to obtain pressure and saturation values at time steps used in the

The Conventional Adjoint Method $\Delta t_{adj} = 2\ \Delta t_{Forward}$ $\Delta t_{adj} = 4\ \Delta t_{Forward}$

$\Delta t_{adj} = 7\ \Delta t_{Forward}$ $\Delta t_{adj} = 10\ \Delta t_{Forward}$ $\Delta t_{adj} = 15\ \Delta t_{Forward}$
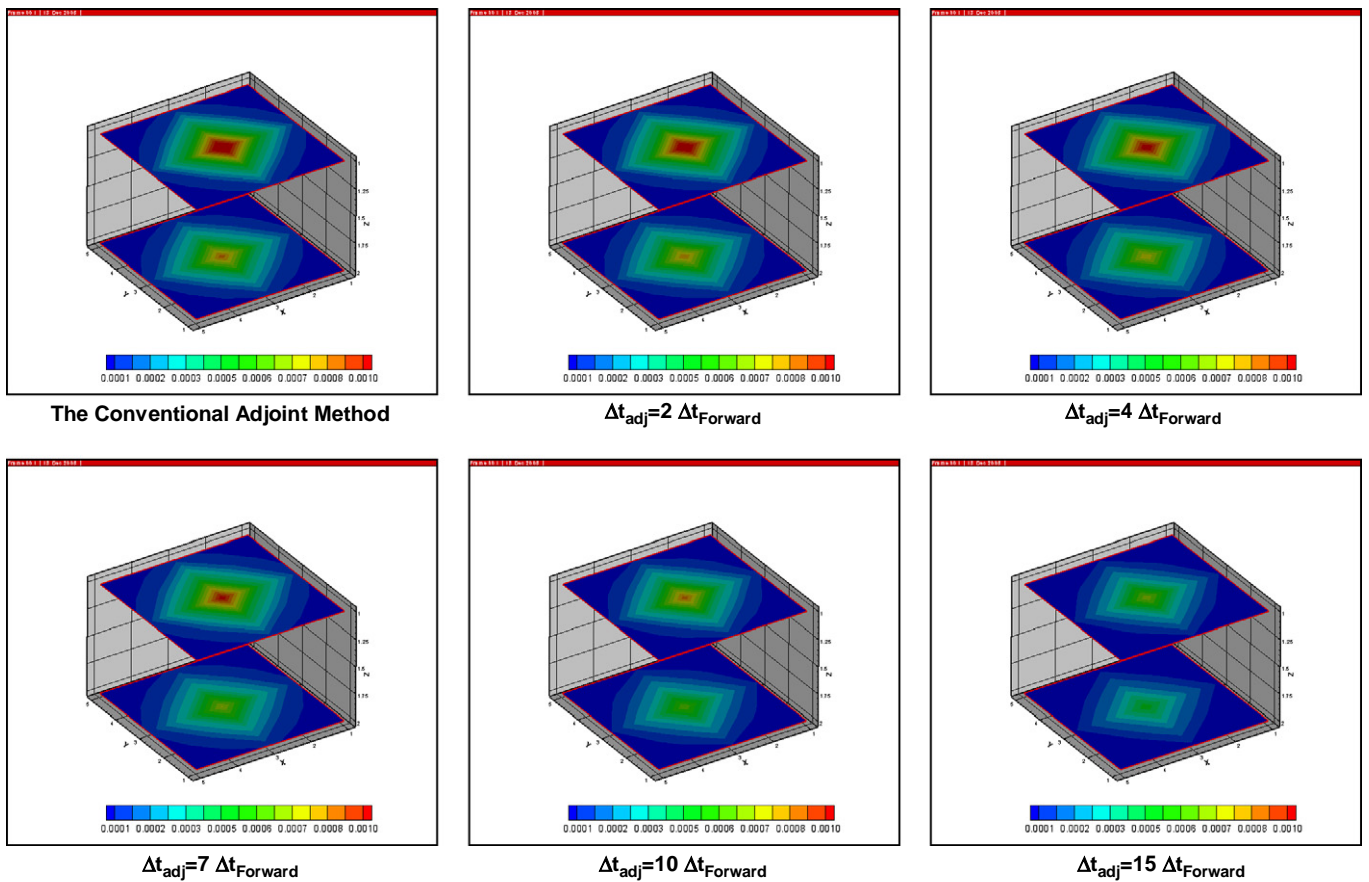
Fig. 7. Comparison of gas–oil ratio sensitivity with respect to horizontal permeability for case 3 between the conventional approach and the new proposed approach.

adjoint solution. Due to the fact that the simulator problem is highly nonlinear, so interpolation of pressure and saturation at the timestep used in the adjoint solution may be inaccurate. Sudden change in the pressure and saturation during simulation run may take place due to existence of infill wells or change of well condition. To avoid interpolation of pressure and saturation that may cause inaccuracy in the solution of adjoint system of equations, we force the simulation to report pressure and saturation every certain fixed timestep (Δt) and solve the adjoint system of equations at this fixed timestep or its multiplications in order to assure that we will have exact pressure and saturation reported at the selected timestep. Solving the adjoint system of equations at larger time steps saves a considerable amount of CPU time and memory; however this will reduce the accuracy of the sensitivity calculation. Thus, selecting the optimum timestep to be used during the adjoint solution is a tradeoff between accuracy and fast calculation with less memory allocation; it may require different sensitivity runs to come up with the best timestep used for the adjoint solution.

Different cases are given to compare the sensitivity from perturbation with that obtained using the conventional and our proposed approach for sensitivity calculation based on the adjoint method. The objective is to examine the accuracy of our proposed approach in the sensitivity calculation compared to the conventional approach with special reference to the amount of CPU time and memory saved using our proposed approach.

Case 1. Two-phase, three-dimensional problem.

The model dimension of this case is $15 \times 15 \times 2$; the data used for this case is given in Table 1. This reservoir contains 1 producer and 1 water injector; it is a two phase oil and water flowing in the model. First, we calculated the sensitivity coefficient of water cut with respect to change in the horizontal permeability after 600 days of production using the conventional approach with solving the adjoint system of equations backward in time with the same timestep used in the forward simulation run. Second, we compared the adjoint method based sensitivity with that obtained from the perturbation to examine its accuracy. Fig. 2 shows this comparison which reflects the high accuracy of the adjoint method based sensitivity in estimating the sensitivity of water cut compared to perturbation. As the permeability increases in the area between the injector and producer the water moves faster towards the producer, and hence the water cut at the producer increases at a given time which shows positive values and the opposite is true in case of the permeability away from the area between the producer and the injector.

We applied our proposed approach on this case by using time step of adjoint system of equations larger than the forward model time step. We used steps 2, 4, 7, 10, and 15 times the forward model time step which is 20 days in this case; it means that we used adjoint timestep equal to 40, 80, 140, 200, and 300 days. Fig. 3 gives the comparison between the conventional approach and our proposed approach. It can be noticed that our proposed approach shows a reasonable accuracy compared with the sensitivity obtained by the conventional approach. The quality of sensitivity depends on the size of the time step used during the adjoint solution. However, the locations of the positive and negative sensitivity values still show minor changes using larger time step in the adjoint solution which is important during the optimization algorithm. This proves the efficiency of our proposed approach in the estimation of the sensitivity with a good accuracy, and high reductions in CPU time and memory as shown in Fig. 8a and b.

Case 2. Two-phase, two-dimensional problem with changing well conditions.

This case is $13 \times 13 \times 1$; Table 2 gives the data used for this reservoir model. There are 1 producer and 1 water injector in this model. The objective of this case is to test the accuracy of our proposed approach for sensitivity estimation under changing well conditions or introducing infill wells. We changed the producing well condition from producing with constant liquid rate of 50 STB/day to 500 STB/day after 500 days of production. We obtained the sensitivity of water cut at 620 days using the perturbation, the conventional approach of the adjoint method, and our proposed approach. Fig. 4 shows the comparison between the adjoint method and the perturbation; Fig. 5 shows the comparison between the conventional and our new approach. It is clear that changing well conditions reduces the accuracy of the sensitivity estimate using larger timestep compared to the case of no change in well condition, Fig. 3. However, the locations of positive and negative sensitivity values had been changed insignificantly from the conventional adjoint method approach which reflects the efficiency of our proposed approach in estimating the sensitivity with significant reduction in CPU time and memory as shown in Fig. 8a and b.

Case 3. Three-phase, three-dimensional problem

This case is a solution gas drive reservoir discretized into $5 \times 5 \times 2$ with single well at the center completed in the two layers. The data of this example is given in Table 3. We examined the efficiency of our proposed approach for sensitivity of gas–oil ratio at 450 days as seen in Fig. 6 and Fig. 7 which indicates the reasonable accuracy of our proposed approach of the sensitivity estimation at higher timestep for the adjoint solution.
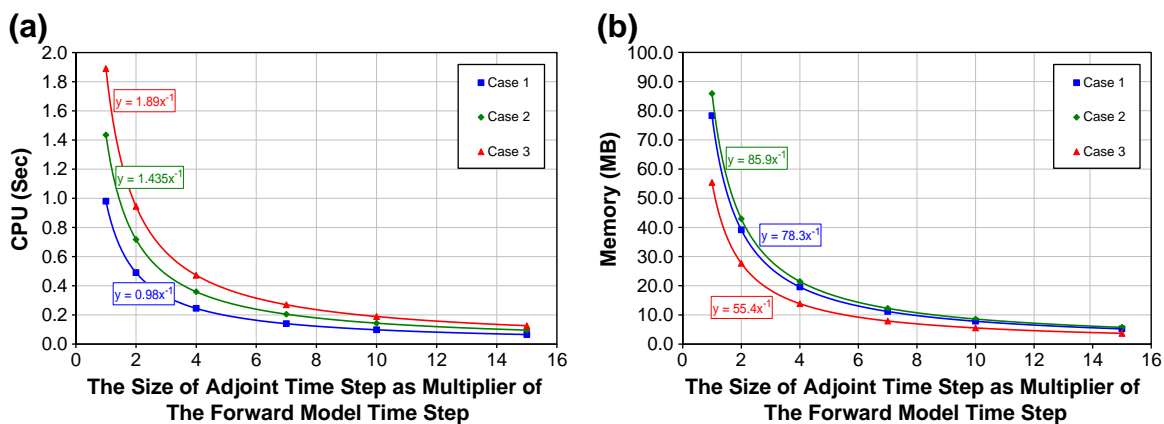

Fig. 8. (a.) Comparison of CPU time required for sensitivity calculation of the three cases at different sizes of Adjoint time step multipliers. (Processor: Core 2 Duo 2.5 GHz CPU). (b.) Comparison of the memory required for sensitivity calculation of the three cases at different sizes of Adjoint time step multipliers.
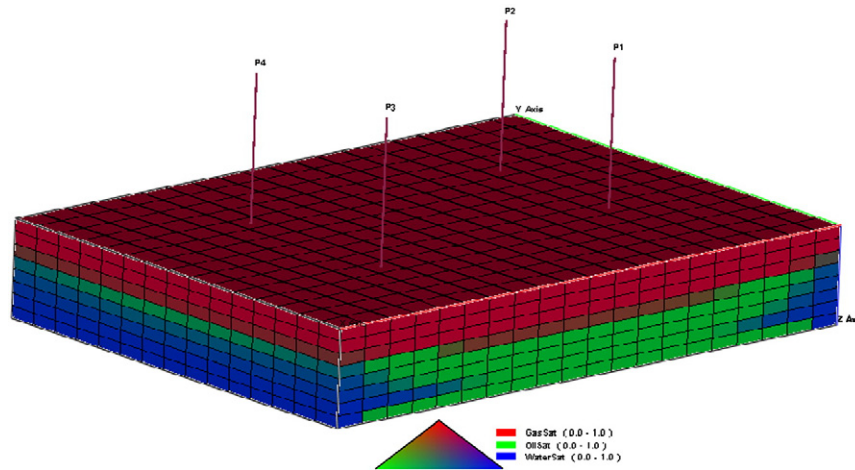
Fig. 9. Location of wells and ternary saturations distribution of pseudo-field case after 3600 days of production.
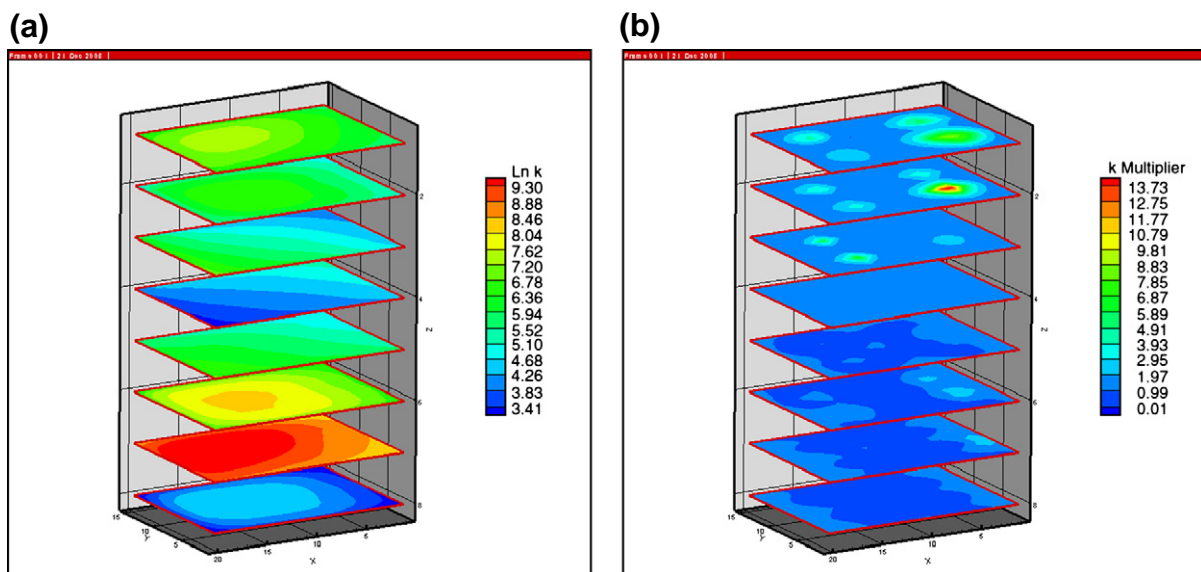
(a)

(b)

Fig. 10. (a) Initial permeability, (b) Permeability multipliers after inversion for the pseudo field case.

Fig. 8a and b shows the reduction of CPU time and memory required for sensitivity calculations compared to the conventional method. From the previous examples, the comparisons between the

Fig. 11. Misfit reduction for the pseudo field case using the proposed approach of $\Delta t_{adj} = 10\ \Delta t_{Forward}$.

conventional adjoint method and our proposed approach demonstrate the feasibility of our approach to reduce the CPU time and memory by a factor equivalent to the amount of the timestep multiplier used in solving the adjoint system of equations backward in time. Significant reductions in CPU time and memory are obtained by using the proposed approach for the three cases; using a time step for the adjoint solution equals ten times the forward model time step reduced CPU time and memory by 90% of those required for the conventional method. However, specifying an exact value to be used for the timestep multiplier in the adjoint solution is a case dependent. Thus, it is recommended to run the sensitivity comparison under different timestep multipliers to select the optimum one for sensitivity calculation before starting the automatic history match process. This is considered an important step as it has the tendency to save more than 90% of the CPU time and memory required for the sensitivity, and hence the automatic history match process.
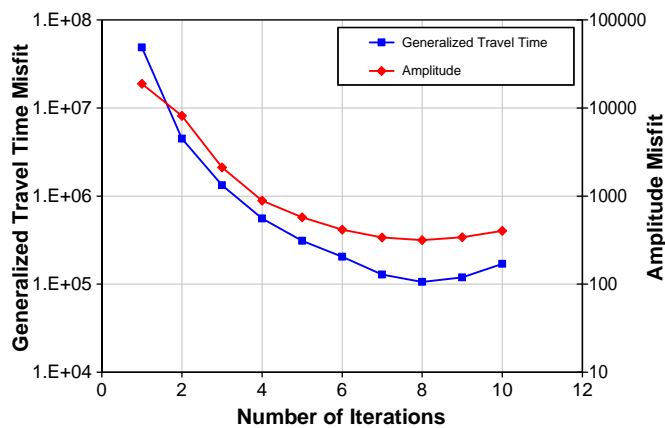
4. Applications

In this section, we used a pseudo field case to demonstrate the power and utility of our proposed sensitivity calculation approach by examining the history match quality using the conventional and our proposed approach in matching the water cut and gas–oil ratio with a special
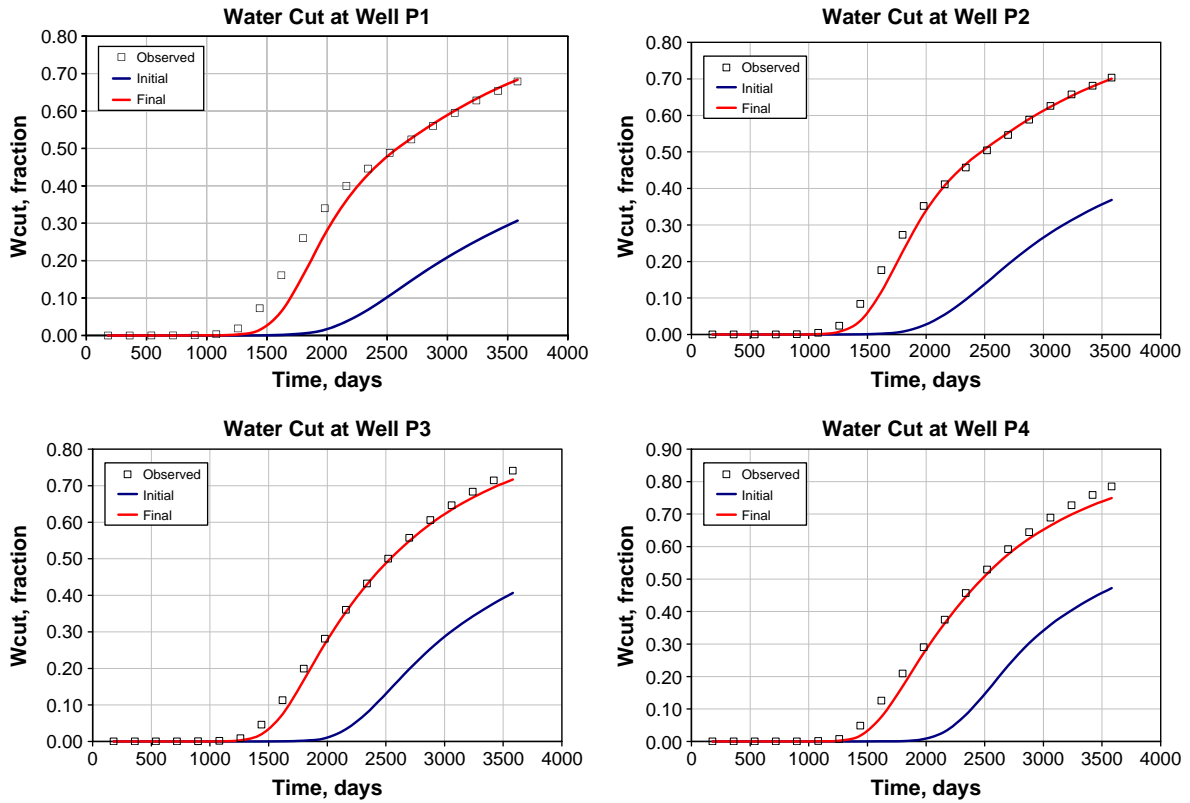
Fig. 12. Water cut match of the four producers for the pseudo field case using the proposed approach of $\Delta t_{adj} = 10 \; \Delta t_{Forward}$.
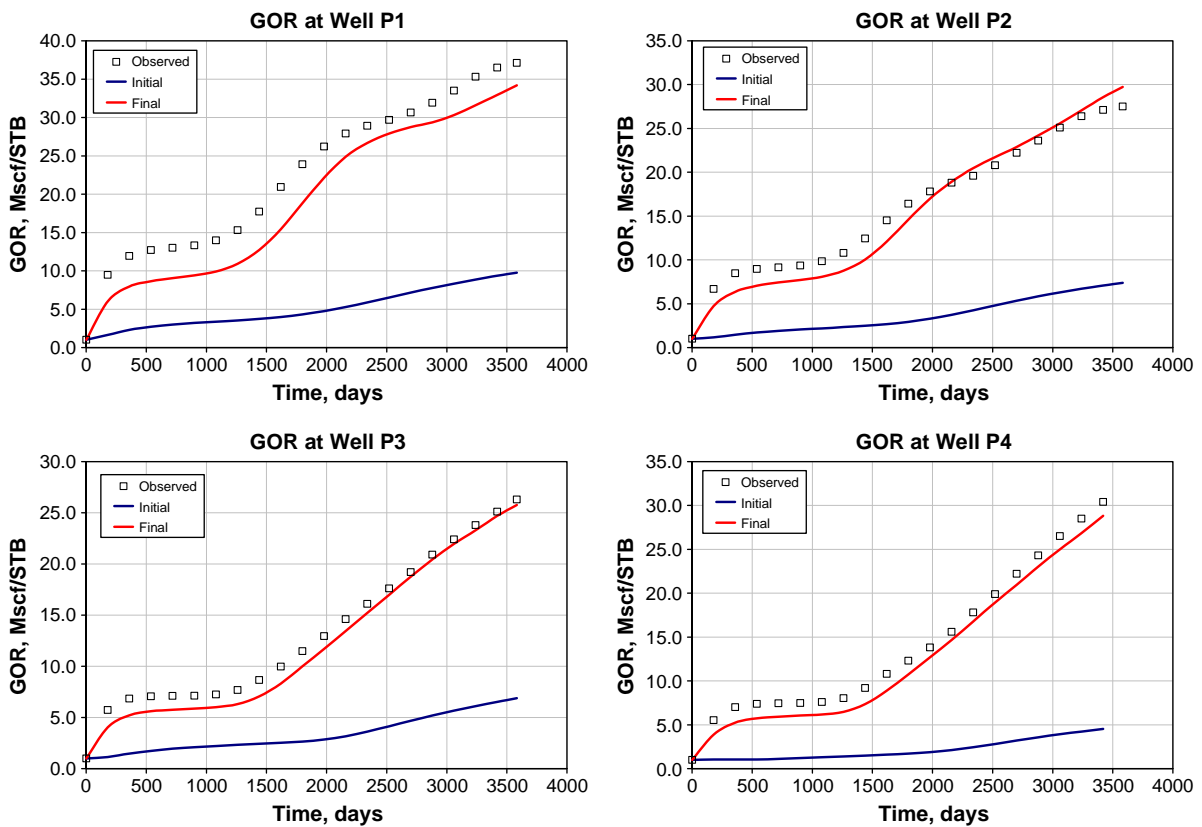

Fig. 13. Gas–oil ratio match of the four producers for the pseudo field case using the proposed approach of $\Delta t_{adj} = 10 \; \Delta t_{Forward}$.
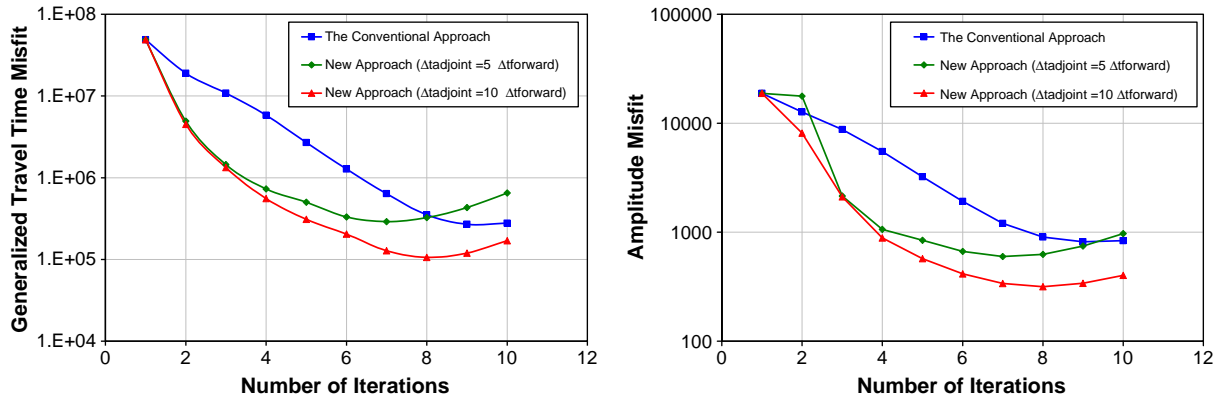
Fig. 14. Comparison of misfit reduction between the conventional approach and the proposed approach at different time steps multipliers.

reference to the amount of CPU time and memory saved using our proposed approach.

The reservoir for this pseudo field case is discretized into $20 \times 15 \times 8$ (2400 cells) with four producers completed in the bottom 4 layers. Fig. 9 shows the locations of the four producers. The reservoir is initially below the bubble point pressure; it has an active gas cap, and a strong aquifer. The initial field permeability is given in Fig. 10a; it is characterized by high permeability in the layer 7 from where most of the production comes, a moderate permeability for both layer 5 and 6, while layer 8 has a very low permeability. The vertical permeability is kept constant and equals to 10 md.

The inversion is done using the new approach with time step equals to 10 times the forward model which is 18 days, thus the time step used in the adjoint solution is taken as 180 days. The prior covariance matrix of permeability which is used as a regularization term was added to water cut and gas–oil ratio misfit terms, and it is

approximated using $5 \times 5 \times 5$ stencil (Daoud and Vega, 2006). Fig. 10b shows the permeability multipliers after inversion along with the initial permeability in Fig. 10a for comparison purposes. Fig. 11 shows the "generalized travel time" misfit and the "amplitude" misfit of both the water cut and gas–oil ratio from all the wells as a function of the number of iterations. The amplitudes and generalized travel time (GTT) misfit are calculated using Eqs. (13) and (14).

$$Amplitude = \sum_{j=1}^{n_w} \sum_{i=1}^{n_{dj}} \left(Wcut_{cal,j}^i - Wcut_{obs,j}^i \right)^2 + \sum_{j=1}^{n_w} \sum_{i=1}^{n_{dj}} \left(GOR_{cal,j}^i - GOR_{obs,j}^i \right)^2 \tag{13}$$

$$GTT = \sum_{j=1}^{n_w} \left(\Delta \tilde{t}_{j,Wcut} \right)^2 + \sum_{j=1}^{n_w} \left(\Delta \tilde{t}_{j,GOR} \right)^2 \tag{14}$$
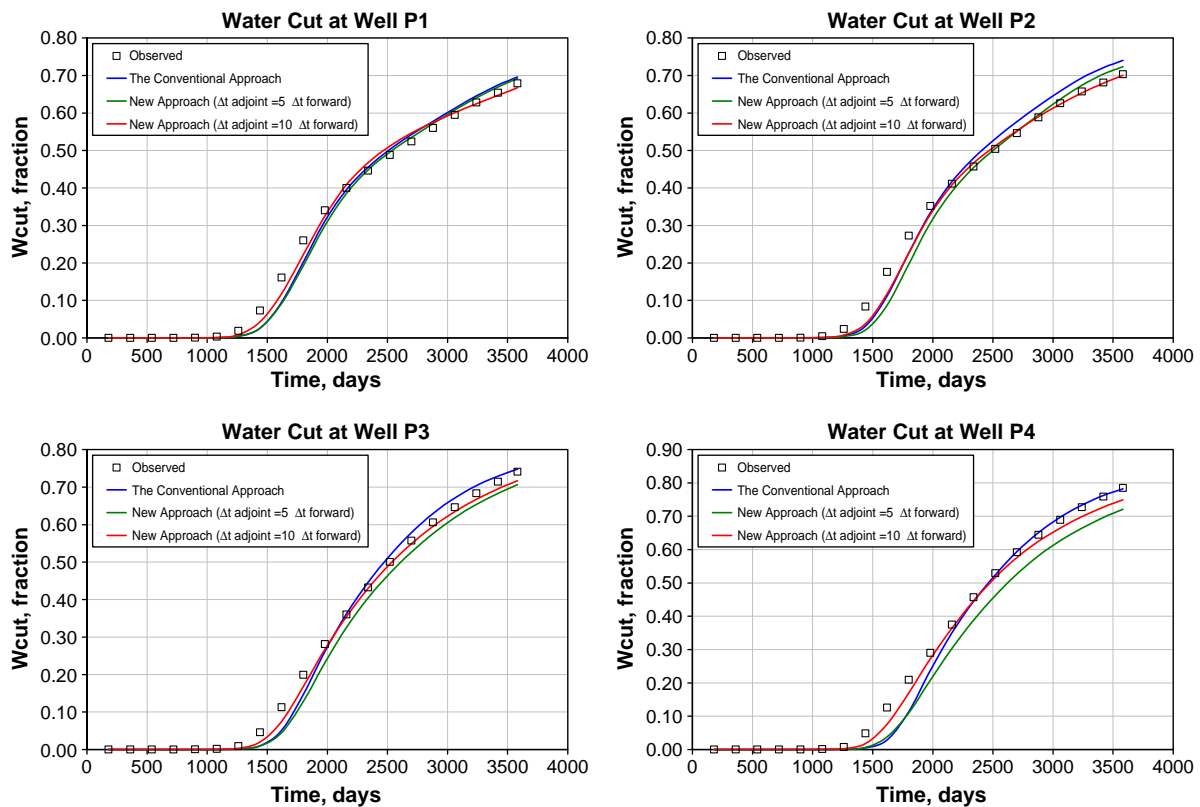

Fig. 15. Comparison of water cut match between the conventional approach and the proposed approach at different time steps multipliers.
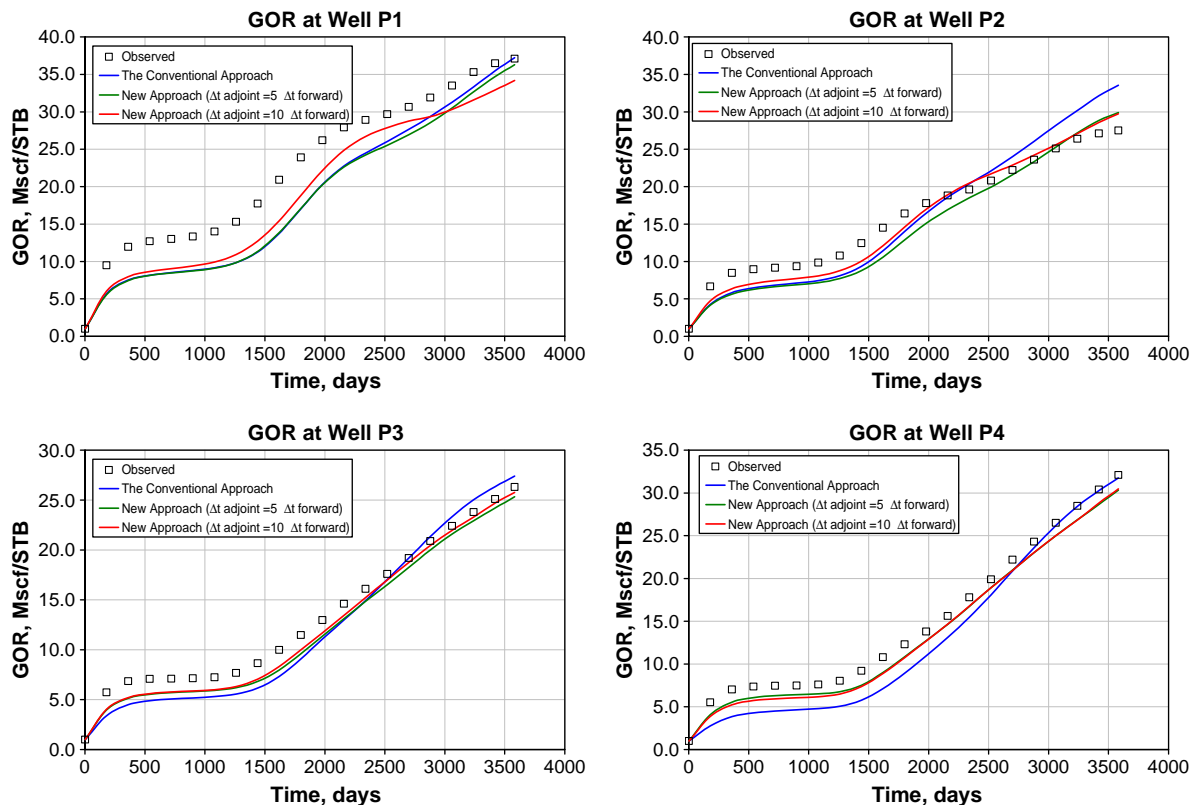
Fig. 16. Comparison of gas–oil ratio match between the conventional approach and the proposed approach at different time steps multipliers.

It is clear that our new approach successfully reduces the misfit of both the generalized travel time and amplitude by more than 95%. Figs. 12 and 13 show the water cut match, and gas oil ratio match respectively from the initial and final permeability model compared to the observed data. The water cut based on the final permeability model is close to the observed water cut, and the gas oil ratio match has been improved from its initial match.

To examine the history match quality of our proposed approach compared to the conventional approach, we applied the inversion for this case using the conventional approach of the adjoint method and our proposed approach using ($\Delta t_{adj} = 5\ \Delta t_{Forward}$) along with the inversion examined before ($\Delta t_{adj} = 10\ \Delta t_{Forward}$). Fig. 14 shows the comparison of the misfit reduction for both generalized travel time and amplitude for the three timestep multipliers used in the adjoint solution. It is clear that the proposed approach has the same trend of convergence compared to the conventional approach which demonstrates the efficiency of our proposed approach in minimizing the

misfit in the production response. The quality of the match after the inversion is given in Fig. 15 for water cut, and Fig. 16 for gas–oil ratio; the proposed approach has the same or even a better match quality for both water cut and gas–oil ratio.

Considering the CPU time and memory savings, Tables 4 and 5 summarize the CPU time and memory required for the sensitivity calculation and for the full history match (HM) process using the conventional approach of the adjoint method and the proposed approach with different timestep multipliers along with the percentage of reduction in CPU time and memory for the different cases. It is easily noticed that by using 10 times the forward model timestep in solving the adjoint system of equations, we were able to reach to about 84% reduction in CPU time with 90% reduction in memory allocated for this case.

5. Summary and conclusions

In this paper we propose an approach to speed up the history match process for field application by minimizing the CPU time required for sensitivity calculation. We demonstrated the power and utility of our

Table 4

CPU time comparison between the conventional and the proposed approach using different time steps multipliers (Processor: Core 2 Duo 2.5 GHz CPU).

Run	Sensitivity calculation		The whole HM process	
	CPU time (min)	CPU time saving (%)	CPU time (min)	CPU time saving (%)
The conventional approach ($\Delta t_{adj} = \Delta t_{Forward}$)	79.2	0	94.3	0
The proposed approach ($\Delta t_{adj} = 5\ \Delta t_{Forward}$)	15.9	80	27.6	71
The proposed approach ($\Delta t_{adj} = 10\ \Delta t_{Forward}$)	8.0	90	14.8	84

Table 5

Memory comparison for the conventional and the proposed approach using different time steps multipliers.

Run	Memory (M byte)	Memory saving (%)
The conventional approach ($\Delta t_{adj} = \Delta t_{Forward}$)	392.6	0
The proposed approach ($\Delta t_{adj} = 5\ \Delta t_{Forward}$)	78.5	80
The proposed approach ($\Delta t_{adj} = 10\ \Delta t_{Forward}$)	39.3	90

proposed approach using synthetic cases and a pseudo field case. Some specific conclusions from this study can be summarized as follows:

(1) A commercial finite difference simulator (via ECLIPSE) is used in this study to model fluid flow in the porous media. The simulator is general and can account for complex physical behavior that dominates most of the field applications.

(2) The data misfit is represented by a single generalized travel time misfit for each production response at each well. The generalized travel time inversion provides a unique advantage over the amplitude and travel time misfit as it depends only on the number of wells by reducing the number of data points into one per well. In addition, it ensures the matching of the entire production history. This saves computation time required during the minimization and makes this approach well-suited for field-scale applications.

(3) We formulated the generalized travel time sensitivity with respect to permeability using adjoint method for 3D, three-phase finite difference flow models. The generalized travel time sensitivity is applicable for matching both water cut and gas oil ratio and has a great impact on reducing CPU time required for history match process.

(4) We proposed a new approach to solve the adjoint system of equations at larger time steps than the forward model time step. The proposed approach can help in saving a considerable amount of CPU time and memory required for calculation of sensitivity by the conventional approach. It makes the history match process using the adjoint method more feasible for large scale applications as the new approach could save up to 90% of CPU time and memory required for the sensitivity calculations.

(5) We used the LSQR method as the sparse matrix solver for updating the model parameters during minimization and also in solving the adjoint system of equations backward in time. The approach is stable, fast, and well-suited for large scale field applications.

(6) We demonstrated the power and utility of our proposed approach for sensitivity calculation using synthetic cases to compare with the conventional approach. The comparisons illustrate the feasibility of our approach to reduce the CPU time and memory by a factor equivalent to the amount of the time step multiplier used in solving the adjoint system of equations. The new approach shows a good accuracy for sensitivity calculation compared to the conventional approach for all the examined cases. However, it is recommended to run the sensitivity comparison under different time step multipliers to select the optimum one for sensitivity calculation before starting the automatic history match process as it is considered a case-dependent.

(7) The full history match process for a pseudo field case proved the feasibility of our approach in history matching the water cut and gas–oil ratio from four wells in much less CPU time and memory with high efficiency compared to the conventional adjoint method based sensitivity. The proposed approach showed a significant reduction of more than 80% in CPU and memory compared to the conventional approach.

Nomenclature

$\Delta\tilde{t}$	Generalized travel time shift
$\Delta\tilde{t}_j$	Generalized travel time shift at well j
$\Delta\tilde{t}^l$	Generalized travel time shift at observed time l
Δt	time step
Δt_{adj}	time step of the adjoint system of equations
$\Delta t_{forward}$	time step of the forward model
A_g	Accumulation term of the flow equation for gas
A_m	Accumulation term of the flow equation for (m) fluid
B_g	Formation volume factor of gas
B_m	Formation volume factor of fluid (m)
D	Depth
f_g	Flow term of the flow equation for gas
f_m	Flow term of the flow equation for (m) fluid
g	Source term in adjoint system of equations
g	Subscript stands for gas
G	Sensitivity matrix
GOR_{cal}	Calculated gas oil ratio
GOR_{obs}	Observed gas oil ratio
GTT	Generalized travel time misfit
J	Augmented Objective function used in the adjoint method
K, K_x	Permeability
l	Time step (l)
L	Last time step
M	Total number of grid block in the reservoir
n_{dj}	Number of data points at well j
n_w, N_w	Number of wells
o	Subscript stands for oil
P	Pressure
q_m	Flow rate of fluid (m)
R_{so}	Solution gas oil ratio
S_g	Gas saturation
S_m	Saturation of fluid (m)
S_o	Oil Saturation
S_w	Water Saturation
t_{cal}	Calculated time
T_{gx}	Transmissibility of gas in the x-direction
T_{gy}	Transmissibility of gas in the y-direction
T_{gz}	Transmissibility of gas in the z-direction
T_{mx}	Transmissibility of fluid m in the x-direction
T_{my}	Transmissibility of fluid m in the y-direction
T_{mz}	Transmissibility of fluid m in the z-direction
t_{obs}	Observed time
t_{shift}	travel time shift
V_b	Bulk volume
w	Subscript stands for water
$Wcut_{cal}$	Calculated water cut
$Wcut_{obs}$	Observed water cut
ϕ	Porosity
γ_g	Specific gravity of gas
γ_m	Specific gravity of fluid (m)
λ_g	Adjoint variables for gas
λ_m	Adjoint variables for fluid (m)
λ_o	Adjoint variables for oil
λ_w	Adjoint variables for water

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