



KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1431/1432)

Solution Final Exam

Programmable Calculators are Not Authorized

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50

Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a, b, c, d\}$	d	a	b	c	d	b	c	d	b	a	a	c	a	a	b	c	c	d	a	b

Q. No: 1 $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n^2} \right)$ is equal to:

- (a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

Q. No: 2 The average value of the function $f(x) = e^x$ on $[0, \ln 2]$ is equal to:

- (a) $\frac{1}{\ln 2}$ (b) $\frac{2}{\ln 2}$ (c) 1 (d) 2

Q. No: 3 The value of the integral $\int_0^2 |x - 1| dx$ is equal to:

- (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

Q. No: 4 If $F(x) = x \int_{\sqrt{\pi}}^x \cos(t^2) dt$, then $F'(\sqrt{\pi})$ is equal to:

- (a) 0 (b) $\sqrt{\pi}$ (c) $-\sqrt{\pi}$ (d) 1

Q. No: 5 The integral $\int \frac{\tan^2 x}{\sec x} dx$ is equal to:

- (a) $\ln |\sec x + \tan x| + \sin x + c$ (b) $\ln |\sec x| - \sin x + c$
(c) $\ln |\sec x + \tan x| - \cos x + c$ (d) $\ln |\sec x + \tan x| - \sin x + c$

Q. No: 6 If $f(x) = \ln(\ln x)$ then $f'(x)$ is equal to:

- (a) $\frac{1}{\ln x}$ (b) $\frac{1}{x \ln x}$ (c) $\frac{-1}{(\ln x)^2}$ (d) $\frac{-1}{x \ln x}$

Q. No: 7 $\lim_{x \rightarrow \infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1} \right)$ is equal to:

- (a) ∞ (b) 1 (c) 2 (d) 0

Q. No: 8 To evaluate the integral $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx$, we use the substitution:

- (a) $u = \sqrt{x}$ (b) $u = \sqrt[4]{x}$ (c) $x = \sqrt[6]{u}$ (d) $u = \sqrt[6]{x}$

Q. No: 9 The integral $\int \frac{\cosh x}{1+\sinh^2(x)} dx$ is equal to:

- (a) $-\tan^{-1}(\sinh x) + c$ (b) $\tan^{-1}(\sinh x) + c$ (c) $\frac{1}{1+\cosh x} + c$ (d) $\tanh^{-1}(\sinh x) + c$

Q. No: 10 To evaluate the integral $\int \frac{\sqrt{x^2 - 25}}{x} dx$, we use the substitution:

(a) $x = 5 \sec \theta$ (b) $x = 25 \sec \theta$ (c) $x = 5 \tan \theta$ (d) $x = 25 \tan \theta$

Q. No: 11 The partial fraction decomposition of $\frac{x^3+2}{(x^2-1)(x^2+2)}$ takes the form:

(a) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+2}$ (b) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+2}$ (c) $\frac{A}{x^2-1} + \frac{B}{x^2+2}$ (d) $\frac{Ax+B}{x^2-1} + \frac{Cx+D}{x^2+2}$

Q. No: 12 The improper integral $\int_0^1 x^\alpha dx$ converges **if and only if**

(a) $\alpha > 1$ (b) $-2 < \alpha < -1$ (c) $\alpha > -1$ (d) $\alpha < -2$

Q. No: 13 The area of the region bounded by the graphs of the functions $x = -y^2$ and $x = -1$ is equal to:

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{8}{3}$ (d) $\frac{3}{4}$

Q. No: 14 Let C be the curve given parametrically by: $x = t^2 + t$, $y = t^2 + 3$ $t \in \mathbb{R}$.

The point on C at which the slope of the tangent line equal to 2 is given by:

(a) $(0, 4)$ (b) $(2, 4)$ (c) $(\frac{3}{4}, \frac{13}{4})$ (d) $(4, 4)$

Q. No: 15 If a graph has polar equation $r = \csc \theta$, then its equation in xy -system is:

(a) $x = 1$ (b) $y = 1$ (c) $x + 1 = 0$ (d) $y + 1 = 0$

Q. No: 16 The length of the curve $C : x = \cos(4t)$, $y = \sin(4t)$, $0 \leq t \leq \frac{\pi}{4}$ is equal to:

(a) $\frac{\pi}{2}$ (b) 2π (c) π (d) 4π

Q. No: 17 The integral for finding the surface of revolution obtained by revolving the curve $y = \sqrt{4 - x^2}$, $x \in [-2, 2]$ about the x -axis is:

(a) $\int_{-2}^2 2\pi (\sqrt{4 - x^2})^2 dx$ (b) $\int_{-2}^2 2\pi \sqrt{4 - x^2} dx$ (c) $\int_{-2}^2 4\pi dx$ (d) $\int_{-2}^2 \pi \sqrt{4 - x^2} dx$

Q. No: 18 If a point has xy -coordinates $(x, y) = (1, 1)$ then one of its (r, θ) -coordinates is:

(a) $(1, \frac{\pi}{4})$ (b) $(2, \frac{\pi}{4})$ (c) $(\sqrt{2}, \frac{-\pi}{4})$ (d) $(-\sqrt{2}, \frac{5\pi}{4})$

Q. No: 19 The equation in polar coordinates for the line $y = x - 1$ is:

(a) $r = \frac{1}{\cos \theta - \sin \theta}$ (b) $r = \frac{1}{\cos \theta + \sin \theta}$ (c) $r = \frac{1}{\cos \theta} + \frac{1}{\sin \theta}$ (d) $r = \cos \theta + \sin \theta$

Q. No: 20 The parametric equations of the circle centered at the origin and with radius 5 is given by

(a) $\begin{cases} x = \cos(5\theta) \\ y = \sin(5\theta) \end{cases}$ (b) $\begin{cases} x = 5 \cos(\theta) \\ y = 5 \sin(\theta) \end{cases}$ (c) $\begin{cases} 5x = \cos(\theta) \\ 5y = \sin(\theta) \end{cases}$ (d) $\begin{cases} x = \cos(\theta) \\ y = \sin(\theta) \end{cases}$

Full Questions

Question No: 21 Approximate the integral $\int_0^2 \frac{x}{\sqrt{x+1}} dx$ using the **Simpson's rule**

with $n = 4$. [5]

Solution:

$$\text{Let } f(x) = \frac{x}{\sqrt{x+1}}.$$

$$\Delta x = \frac{2}{4} = 0.5$$

$$x_0 = 0, \quad x_1 = 0.5, \quad x_2 = 1, \quad x_3 = 1.5 \quad \text{and} \quad x_4 = 2. \quad (1)$$

$$\int_0^2 \frac{x}{\sqrt{x+1}} dx \approx \frac{2-0}{3 \times 4} \{f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + f(2)\} \quad (1)$$

$$= \frac{1}{6} \{0 + 4(0.40825) + 2(0.70711) + 4(0.94868) + 1.1547\} \quad (1)$$

$$= \frac{1}{6} \{0 + 1.633 + 1.4142 + 3.7947 + 1.1547\}$$

$$= \frac{1}{6} \{7.9966\} \approx 1.3328 \quad (2)$$

Question No: 22 **Evaluate** $\int \frac{x}{\sqrt{3-2x-x^2}} dx$ [5]

Solution:

$$\text{Let } u = x + 1 \Rightarrow du = dx \quad (0.5)$$

$$\int \frac{x}{\sqrt{3-2x-x^2}} dx = \int \frac{x}{\sqrt{4-(x+1)^2}} dx \quad (0.5)$$

$$= \int \frac{u-1}{\sqrt{4-u^2}} du \quad (1)$$

$$= -\sqrt{4-u^2} - \sin^{-1}\left(\frac{u}{2}\right) + c \quad (1+1)$$

$$= -\sqrt{4-(x+1)^2} - \sin^{-1}\left(\frac{x+1}{2}\right) + c \quad (1)$$

Question No: 23 **Evaluate** $\int \frac{x^2+1}{x(x-2)} dx$ [5]

Solution:

$$\frac{x^2+1}{x(x-2)} = 1 - \frac{1}{2x} + \frac{5}{2(x-2)} \quad (0.5+1+1)$$

So

$$\int \frac{x^2+1}{x(x-2)} dx = x - \frac{1}{2} \ln|x| + \frac{5}{2} \ln|x-2| + c \quad (0.5+1+1)$$

Question No: 24 Determine whether the improper integral $\int_{-\infty}^0 xe^{2x} dx$ converges or diverges and if it converges find its value [5]

Solution:

$$\begin{aligned} \int_{-\infty}^0 xe^{2x} dx &= \lim_{t \rightarrow -\infty} \int_t^0 xe^{2x} dx \\ &= \lim_{t \rightarrow -\infty} \left(\left[\frac{1}{2} xe^{2x} \right]_t^0 - \frac{1}{2} \int_t^0 e^{2x} dx \right) \quad (2) \\ &= \lim_{t \rightarrow -\infty} \left[-\frac{1}{2} te^{2t} - \frac{1}{4} + \frac{1}{4} e^{2t} \right] \quad (1) \\ &= -\frac{1}{4} \quad (1) \end{aligned}$$

So improper integral $\int_{-\infty}^0 xe^{2x} dx$ converges and $\int_{-\infty}^0 xe^{2x} dx = -\frac{1}{4}$. (1)

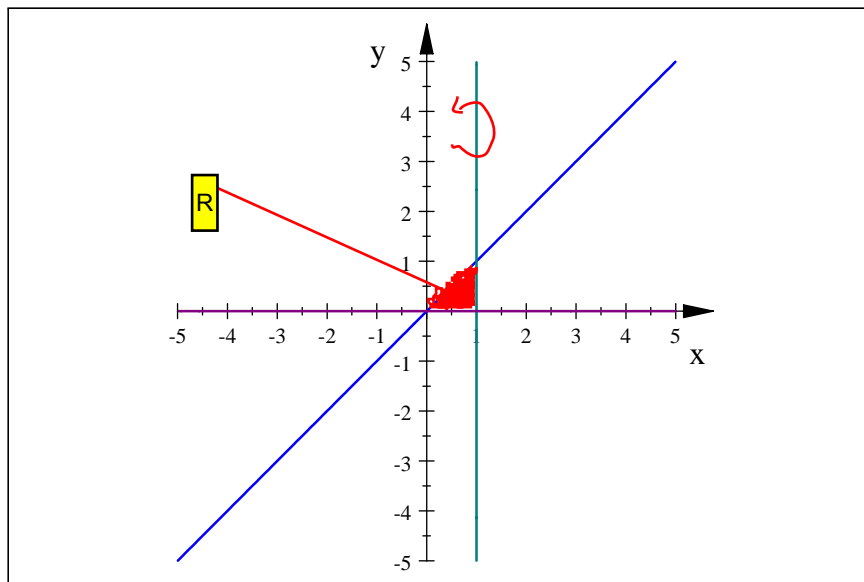
Question No: 25 a) **Sketch** the region R bounded by the graph of the equations:

$y = x$, $y = 0$ and $x = 1$. [2]

b) **Find** the **volume** of the solid generated by revolving the region R around the line $x = 1$. (Use any method) [3]

Solution:

a)



$$y = x \text{ and } x = 1 \quad (2)$$

b)

By cylindrical shell Method

OR

By Disk Method

$$V = 2\pi \int_0^1 (1-x) x dx \quad (2)$$

$$= \frac{\pi}{3} \quad (1)$$

$$V = \pi \int_0^1 (1-x)^2 dx \quad (2)$$

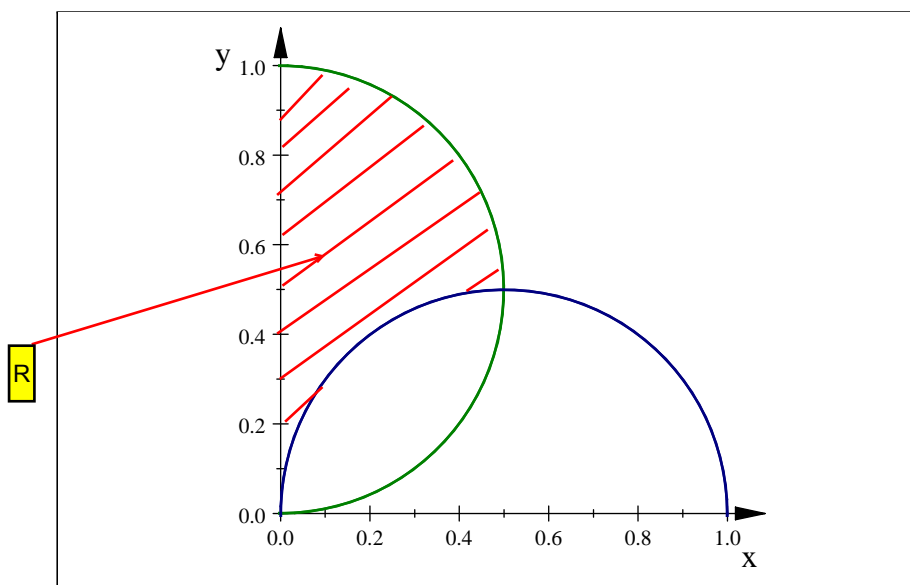
$$= \frac{\pi}{3} \quad (1)$$

Question No: 26 a) **Sketch** the region R that is **inside** $r = \sin \theta$ and **outside** $r = \cos \theta$, for $\theta \in \left[0, \frac{\pi}{2}\right]$. [3]

b) **Set up** (Do not evaluate) an integral that can be used to find the **area** of R . [2]

Solution:

a)



$$r = \sin \theta \quad \text{and} \quad r = \cos \theta \quad (3)$$

b) Area (R) = $\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin^2 \theta - \cos^2 \theta) d\theta \quad (2)$