KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1432/1433)

Solution Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50 Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a, b, c, d\}$	d	b	a	c	b	a	b	d	a	c	d	c	a	a	a	b	a	c	a	b

Q. No: 1 The sum $\sum_{k=1}^{n^2}(k-1)$, is equal to:

(a) $\frac{n^2(n-1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n^2(n^2+1)}{2}$ (d) $\frac{n^2(n^2-1)}{2}$

Q. No: 2 The average value of the function $f(x) = (x+1)^{\frac{1}{3}}$ on $[-2, 0]$ is equal to:

(a) 3 (b) 0 (c) -1 (d) -3

Q. No: 3 The integral $\int 4^x dx$ is equal to:

(a) $\dfrac{2^{2x}}{2\ln 2}+c$ (b) $\dfrac{4^x}{\ln 2}+c$ (c) $\dfrac{4^{x+1}}{\ln 4}+c$ (d) $\dfrac{2^x}{\ln 4}+c$

Q. No: 4 If $F(x) = \int_1^{x^2} \sqrt[3]{t^4+1}\,dt,$ then $F'(x)$ is equal to:

(a) $\sqrt[3]{x^8+1}$ (b) $x^2\sqrt[3]{x^8+1}$ (c) $2x\sqrt[3]{x^8+1}$ (d) $2x\sqrt[3]{x^4+1}$

Q. No: 5 The integral $\int 2^{\sin x}\cos x dx$ is equal to:

(a) $2^{\sin x}+c$ (b) $\frac{2^{\sin x}}{\ln 2}+c$ (c) $(\ln 2)\,2^{\sin x}+c$ (d) $\frac{-2^{\sin x}}{\ln 2}+c$

Q. No: 6 If $f(x) = x^{\ln x}$ then $f'(e)$ is equal to:

(a) 2 (b) $2e$ (c) 0 (d) e

Q. No: 7 $\lim\limits_{x\to 0}\left(\frac{\sin x - x}{x^3}\right)$ is equal to:

(a) ∞ (b) $-\frac{1}{6}$ (c) $\frac{1}{6}$ (d) 0

Q. No: 8 The integral $\int \dfrac{1}{(x+1)\sqrt{x^2+2x}}dx$ is equal to:

(a) $\ln|x^2+2x|+c$ (b) $\sin^{-1}(x+1)+c$ (c) $sech^{-1}(x+1)+c$ (d) $\sec^{-1}(x+1)+c$

Q. No: 9 The improper integral $\int_0^2 \frac{2x}{\sqrt{16-x^4}}dx$

(a) converges to $\frac{\pi}{2}$ (b) converges to $\frac{\pi}{4}$ (c) converges to π (d) Diverges

Q. No: 10 To evaluate the integral $\int \frac{1}{x\sqrt{x^6-1}}dx$, we use the substitution:

$(a)\ \ u = x^6 \qquad (b)\ \ u = x^2 \qquad (c)\ \ u = x^3 \qquad (d)\ \ u = x^6 - 1$

Q. No: 11 The partial fraction decomposition of $\dfrac{2x^3}{x\left(x^2-1\right)}$ takes the form:

$(a)\ 2 + \frac{A}{x} + \frac{B}{(x^2-1)} \quad (b)\ 2 + \frac{A}{x} + \frac{Bx+C}{(x^2-1)} \quad (c)\ \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+1)} \quad (d)\ 2 + \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+1)}$

Q. No: 12 If $\int \frac{x^{\frac{1}{2}}}{6\left(x^{\frac{1}{3}}-1\right)}dx = \int \frac{u^8}{u^2-1}\,du$ then

$(a)\ x = u^2 \qquad (b)\ x = u^3 \qquad (c)\ x = u^6 \qquad (d)\ x = u^8$

Q. No: 13 The area of the region bounded by the graphs of the functions $y = 2x$, $y = x$, $0 \le x \le 1$ is equal to:

$(a)\ \ \frac{1}{2} \qquad (b)\ \ 2 \qquad (c)\ \ \frac{1}{4} \qquad (d)\ \ \frac{1}{3}$

Q. No: 14 The arc length of the graph of $y = 4x$ from $A\left(0,0\right)$ to $B\left(1,4\right)$ is equal to:

$(a)\ \ \sqrt{17} \qquad (b)\ \ \sqrt{5} \qquad (c)\ \ 4\sqrt{17} \qquad (d)\ \ 4\sqrt{5}$

Q. No: 15 The slope of the tangent line at the point corresponding to $t = \frac{\pi}{4}$ on the parametric curve given by the equations, $x = \sin t$, $y = \cos t$, $0 \le t \le 2\pi$ is:

$(a)\ \ -1 \qquad (b)\ \ 1 \qquad (c)\ \ 0 \qquad (d)\ \ \frac{1}{3}$

Q. No: 16 If a graph has polar equation $r = 2\csc\theta$, then its equation in $xy-$system is:

$(a)\ \ x = 2 \qquad (b)\ \ y = 2 \qquad (c)\ \ x = \frac{1}{2} \qquad (d)\ \ y = \frac{1}{2}$

Q. No: 17 The length of the curve $C : x = \cos\left(2t\right), y = \sin\left(2t\right),\ 0 \le t \le \frac{\pi}{2}$ is equal to:

$(a)\ \ \pi \qquad (b)\ \ \frac{\pi}{2} \qquad (c)\ \ 2\pi \qquad (d)\ \ \frac{\pi}{4}$

Q. No: 18 To evaluate the integral $\int \tan^5\left(x\right)\sec^5\left(x\right)dx$ we use the substitution:

$(a)\ \ u = \tan^2 x \qquad (b)\ \ u = \tan x \qquad (c)\ \ u = \sec x \qquad (d)\ \ u = \sin x$

Q. No: 19 If a point has $\left(r,\theta\right)-$coordinates $\left(r,\theta\right) = \left(1, \frac{\pi}{6}\right)$ then its $\left(x,y\right)-$coordinates is:

$(a)\ \ \left(\frac{\sqrt{3}}{2}, \dfrac{1}{2}\right) \qquad (b)\ \ \left(\dfrac{1}{2}, \frac{\sqrt{3}}{2}\right) \qquad (c)\ \ \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \qquad (d)\ \ \left(1,0\right)$

Q. No: 20 The slope of the tangent line to the curve: $r = \cos\theta$ at $\theta = \dfrac{\pi}{4}$ is:

$(a)\ \ \frac{\pi}{2} \qquad (b)\ \ 0 \qquad (c)\ \ \frac{\pi}{4} \qquad (d)\ \ 1$

Full Questions

Question No: 21 **Evaluate** $\int \sin^2(x)\cos^5(x)\,dx$. [**4**]

Solution:

Let $u = \sin\theta \qquad (du = \cos\theta d\theta)$ (0.5)

So

$$\begin{aligned}\int \sin^2(x)\cos^5(x)\,dx &= \int u^2\left(1-u^2\right)^2 du && (1)\\ &= \tfrac{1}{7}u^7 - \tfrac{2}{5}u^5 + \tfrac{1}{3}u^3 + c && (2)\\ &= \tfrac{1}{7}(\sin\theta)^7 - \tfrac{2}{5}(\sin\theta)^5 + \tfrac{1}{3}(\sin\theta)^3 + c && (0.5)\end{aligned}$$

Question No: 22 Find the area of the surface generated by revolving $y = \sqrt{x}$, $1 \le x \le 4$ about the x-axis.. [**4**]

Solution:

$$\begin{aligned}S &= \int_1^4 2\pi\sqrt{x}\sqrt{1+\left(\tfrac{1}{2\sqrt{x}}\right)^2}dx && (1)\\ &= \int_1^4 \pi\sqrt{4x+1}dx && (1)\\ &= \left[\frac{\pi}{6}(4x+1)^{\frac{3}{2}}\right]_1^4 && (1)\\ &= \frac{\pi}{6}\left((17)^{\frac{3}{2}} - (5)^{\frac{3}{2}}\right) && (1)\end{aligned}$$

Question No: 23 **Evaluate** $\int \frac{x^3}{x^2(x^2+1)}dx$ [**6**]

Solution:

$$\frac{x^3}{x^2(x^2+1)} = \frac{x}{x^2+1} \qquad (3)$$

So

$$\begin{aligned}\int \frac{x^3}{x^2(x^2+1)}dx &= \int \tfrac{x}{x^2+1}dx\\ &= \tfrac{1}{2}\ln\left(x^2+1\right) && (3)\end{aligned}$$

Question No: 24 **Evaluate** $\int \frac{\ln x}{\sqrt{x}} dx$ [**5**]

Solution: Let $\begin{cases} u = \ln x \\ v' = \dfrac{1}{\sqrt{x}} \end{cases}$, then $\begin{cases} u' = \frac{1}{x} \\ v = 2\sqrt{x} \end{cases}$ (1)

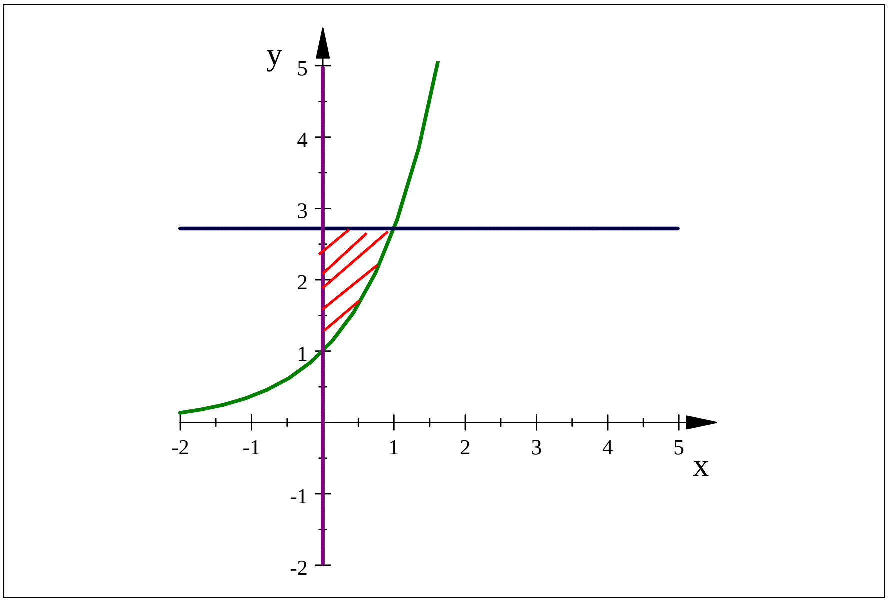
So

$$\begin{aligned} \int \frac{\ln x}{\sqrt{x}} dx &= 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}} dx && (2) \\ &= 2\sqrt{x}\ln x - 4\sqrt{x} + c && (2) \end{aligned}$$

Question No: 25 **Sketch** the region R bounded by the graph of the equations $y = e^x$, $y = e$ and y-axis. **Find** the **volume** of the solid generated by revolving the region R around the x-axis. (Use Washer method) [**6**]

Solution:

Graph: (2)



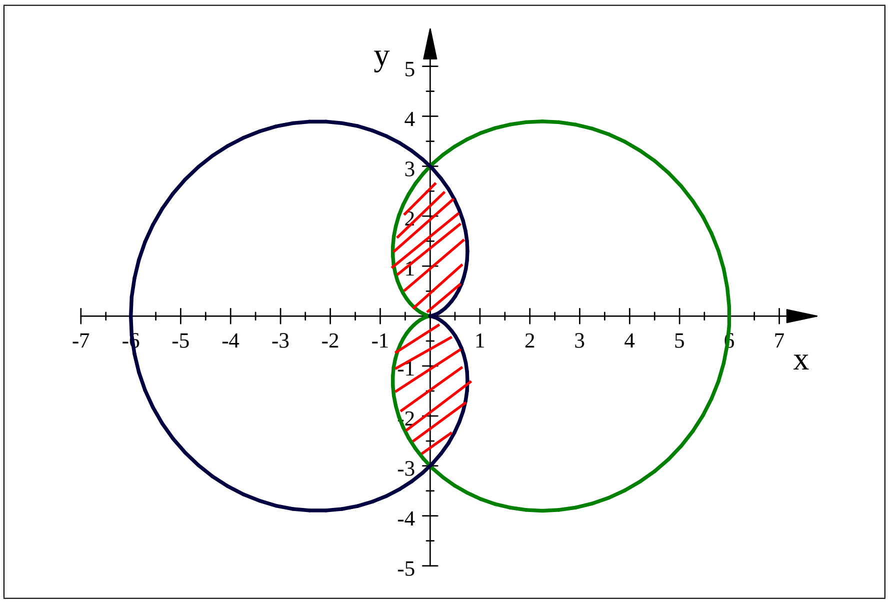
$y = e^x$ and $y = e$

$$\begin{aligned} V &= \int_0^1 \pi \left((e)^2 - (e^x)^2 \right) dx && (2) \\ &= \tfrac{1}{2}\pi \left(e^2 + 1 \right) && (2) \end{aligned}$$

Question No: 26 **Sketch** the region R that lies inside both of graphs of equations $r = 3+3\cos\theta$ and $r = 3-3\cos\theta$. **Set up** (Do not evaluate) an integral that can be used to find its **area**. [**5**]

Solution:

Graph: (3)



$r = 3 - 3\cos\theta$ and $r = 3 + 3\cos\theta$

By symmetry:

$$A = \quad 4 \times \left(\frac{1}{2}\int_0^{\frac{\pi}{2}} (3 - 3\cos\theta)^2 \, d\theta \right) \qquad (2)$$

Or

$$A = \frac{1}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (3 - 3\cos\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3 + 3\cos\theta)^2 \, d\theta$$

$$= \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (3 - 3\cos\theta)^2 \, d\theta = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3 + 3\cos\theta)^2 \, d\theta$$

Or

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} (3 - 3\cos\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} (3 + 3\cos\theta)^2 \, d\theta + \frac{1}{2}\int_{\pi}^{\frac{3\pi}{2}} (3 + 3\cos\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{3\pi}{2}}^{2\pi} (3 - 3\cos\theta)^2 \, d\theta.$$