

Math106 Midterm I

Question 1(2+3+3)

a) Find the value of  $F'(0)$  if  $F(x) = (\cos x) \cdot \int_0^{\tan x} \sqrt{1+t^2} dt$

b) Evaluate the indefinite integral  $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

c) Approximate the integral  $\int_0^5 \frac{2^x dx}{1+x}$  using Trapezoid rule with  $n=5$ .

Question 2(3+2+3)

a) Evaluate the integral  $\int (\ln x + 1) 3^{x \ln x} dx$

b) If  $y = \frac{x(x^2+1)^3}{\sqrt[4]{2x-1}}$  find  $y'$

c) Compute  $\int \frac{\sec^2 x \cdot dx}{\sqrt{9-(\tan x)^2}}$

Question 3(3+3+3)

a) Find  $\int \frac{dx}{x\sqrt{16x^4-1}}$

b) Evaluate the integral  $\int \frac{x^2 dx}{\sqrt{x^6-25}}$

c) Compute  $\int \frac{dx}{x \ln x \sqrt{1-(\ln x)^4}}$

Math106 Midterm2

Question 1(2+3+2)

a) Find the  $\lim_{x \rightarrow 0} \frac{\int_0^x \sin(t^2) dt}{x^3}$

b) Evaluate the indefinite integral  $\int \cosh^{-1}(x) dx$

c) Compute the integral  $\int (\sin x)^3 (\cos x)^7 dx$

Question 2(3+3+3)

a) Evaluate the integral  $\int \frac{dx}{(x^2+4)^2}$

b) Find  $\int \frac{dx}{x^4+x^2}$

c) Compute  $\int \frac{dx}{x^{1/6}+x^{1/3}}$

Question 3(3+3+3)

a) Find  $\int \frac{dx}{3+\cos x+2 \sin x}$

b) Show that the integral  $\int_1^{\infty} \frac{\ln x}{x^2} dx$  converges and find its value.

c) Sketch the region bounded by the curves  $x = y^2$ ,  $x = 8 - y^2$ , and find its area.

**Question 1(2+2)**

- a) Approximate the integral  $\int_0^6 \sqrt{1+x^2} dx$  using Simpson's Rule with  $n=6$
- b) Find the number  $c$  in the mean value theorem for  $f(x) = \frac{8}{x^2}$  on  $[2, 4]$

**Question 2(3+3+3)**

- a) Evaluate the integral  $\int \frac{dx}{\sqrt{e^{6x}-25}}$
- b) Compute the integral  $\int \frac{dx}{x\sqrt{1-x^8}}$
- c) Find  $\int x \tan^{-1}(x) dx$

**Question 3(3+3+3)**

- a) Compute the following integral  $\int (\tan x)^5 (\sec x)^3 dx$
- b) Find the integral  $\int \cos(7x) \cos(5x) dx$
- c) Evaluate the integral  $\int \frac{dx}{(x^2-1)^{3/2}}$

**Question 4(3+3+3)**

- a) Evaluate the integral  $\int \frac{2x-1}{x^2+4x+20} dx$
- b) Sketch the region bounded by the curves:  $y = 2 - x^2$ ,  $y = x$ ,  
 $x = 0$ ,  $x = 2$  and find its area.

$x = 0$ ,  $x = 2$  and find its area.

b) Set up an integral for the volume obtained by revolving the region bounded by the curves  $y = x^2$ ,  $y = 4$  about the line of equation

i)  $y = 6$

ii)  $x = -3$ .

**Question5(3+3+3)**

a) Sketch the region R that lies inside the curve  $r = 2\sin\theta$  and outside the curve  $r = 2 - 2\sin\theta$ , and find its area.

b) Find the area of the surface obtained by revolving the curve  $r = 4\cos\theta$   $0 \leq \theta \leq \pi/2$  about the y-axis.

c) Find the length of the curve given by the equations  $x = \frac{t^4}{4}$ ,  $y = \frac{t^6}{6}$   $0 \leq t \leq 1$ .