1 Exercises on swaps

1. Companies A and B have been offered the following rates per annum on a \$20 million 5-year loan :

	Fixed rate	Floating rate
Company A	5.0%	LIBOR+0.1%
Company B	6.4%	LIBOR $+0.6\%$

Company A requires a floating–rate loan; company B requires a fixed–rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

Solution :

A has an apparent comparative advantage in fixed-rate markets but wants to borrow floating. B has an apparent comparative advantage in floating-rate markets but wants to borrow fixed. This provides the basis for the swap. There is a 1.4% per annum differential between the fixed rates offered to the two companies and a 0.5% per annum differential between the floating rates offered to the two companies. The total gain to all parties from the swap is therefore 1.4 - 0.5 = 0.9% per annum. Because the bank gets 0.1% per annum of this gain, the swap should make each of A and B 0.4% per annum better off. This means that it should lead to A borrowing at LIBOR -0.3% and to B borrowing at 6%. The appropriate arrangement is therefore as shown in the following figure

<	Comp. A	5.3% <	F.I.	5.4%	Comp. B	LIBOR+0.6%
5 %						
		LIBOR		LIBOR		

2. A \$100 million interest rate swap has a remaining life of 10 months. Under the terms of the swap; 6-month LIBOR is exchanged for 7% per annum (compounded semiannually). The average of the bid-offer rate being exchanged for 6-month LIBOR in swaps of all maturities is currently 5% per annum with continuous compounding. The 6-month LIBOR rate was 4.6% per annum 2 months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed? Solution :

In four months \$6 million (= $0.5 \times 0.12 \times 100 million) will be received and \$4.8 million (= $0.5 \times 0.096 \times 100 million) will be paid. (We ignore day count issues.) In 10 months \$6 million will be received, and the LIBOR rate prevailing in four months' time will be paid. The value of the fixed-rate bond underlying the swap is

$$6 \exp(-0.1 \times 4/12) + 6 \exp(-0.1 \times 10/12) =$$
\$103.328 million

The value of the floating-rate bond underlying the swap is

$$(100 + 4.8) \exp(-0.1 \times 4/12) =$$
\$101.364 million

The value of the swap to the party paying floating is 103.328 - 101.364 = 1.964 million. The value of the swap to the party paying fixed is -1.964 million. These

results can also be derived by decomposing the swap into forward contracts.

Consider the party paying floating. The first forward contract involves paying \$4.8 million and receiving \$6 million in four months. It has a value of $1.2e^{-0.1 \times 4/12} = 1.161 million.

To value the second forward contract, we note that the forward interest rate is 10% per annum with continuous compounding, or 10.254% per annum with semiannual compounding. The value of the forward contract is

$$100 \times (0.12 \times 0.5 - 0.10254 \times 0.5)e^{-0.1 \times 10/12} =$$
\$0,803 million.

The total value of the forward contract is therefore 1.161 + 0/803 = 1,964 million.

3. Companies A and B have been offered the following rates per annum on a \$5 million 10–year investment :

	Fixed rate	Floating rate
Company A	8%	LIBOR
Company B	8.8%	LIBOR

Company A requires a fixed-rate investment; company B requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to A and B.

Solution :

The spread between the interest rates offered to A and B is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments. This means that the total apparent benefit to all parties from the swap is 0.8% per annum. Of this 0.2% per annum will go to the bank. This leaves 0.3% per annum for each of A and B. In other words, company A should be able to get a fixed-rate return of 8.3% per annum while company B should be able to get a floating-rate return LIBOR +0.3% per annum. The bank earns 0.2%, company A earns 8.3%, and company B earns LIBOR +0.3%.



4. A financial institution has entered into an interest rate swap with company A. Under the terms of the swap, it receives 10% per annum and pays 6-month LIBOR on a principal of \$10 million for 5 years. Payments are made every 6 months. Suppose that company A defaults on the sixth payment date (at the end of year 3) when the interest rate (with semiannual compounding) is 8% per annum for all maturities. What is the loss to the financial institution? Assume that 6-month LIBOR was 9% per annum halfway through year 3. Use LIBOR discounting

Solution :

At the end of year 3 the financial institution was due to receive \$500,000 (% of \$10 million) and pay \$450,000 (% of \$10 million). The immediate loss is therefore \$50,000. To value the remaining swap we assume than forward rates are realized. All forward

rates are 8% per annum. The remaining cash flows are therefore valued on the assumption that the floating payment is and the net payment that would be received is . The total cost of default is therefore the cost of foregoing the following cash flows :

3 year :	\$50,000
3.5 year :	\$100,000
4 year :	\$100,000
4.5 year :	\$100,000
5 year :	\$100,000

Discounting these cash flows to year 3 at 4% per six months, we obtain the cost of the default as \$413,000.

5. Company A wishes to borrow US dollars at a fixed rate of interest. Company B wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies are subject to the following interest rates, which have been adjusted to reflect the impact of taxes :

	Yen	Dollars
Company A	5.0%	9.6%
Company B	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

Solution :

A has a comparative advantage in yen markets but wants to borrow dollars. B has a comparative advantage in dollar markets but wants to borrow yen. This provides the basis for the swap. There is a 1.5% per annum differential between the yen rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore 1.5 - 0.4 = 1.1% per annum. The bank requires 0.5% per annum, leaving 0.3% per annum for each of A and B. The swap should lead to A borrowing dollars at 9.6 - 0.3 = 9.3% per annum and to B borrowing yen at 6.5 - 0.3 = 6.2% per annum. The appropriate arrangement is therefore as shown in the following diagram. All foreign exchange risk is born by the bank.

A currency swap motivated by comparative advantage

$\begin{vmatrix} < \\ \mathbf{YEN} & 5.0\% \end{vmatrix} \begin{bmatrix} \text{Comp.} \\ A \\ \end{bmatrix} \begin{vmatrix} < \\ \\ \mathbf{US} & 9.3\% \end{vmatrix} \begin{bmatrix} \text{F.I.} \\ \\ \mathbf{US} & 10.0\% \end{bmatrix} \begin{bmatrix} \text{Comp.} \\ B \\ \end{bmatrix} \begin{bmatrix} \text{OS} & 10.0 \\5 \\ B \\ \end{bmatrix}$
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6. A currency swap has a remaining life of 15 months. It involves exchanging interest at 10% on £20 million for interest at 6% on \$30 million once a year. The term structure of interest rates in both the United Kingdom and the United States is currently flat, and if the swap were negotiated today the interest rates exchanged would be 4% in dollars and-7% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (dollars per pound sterling) is 1.8500. What is the value of the

swap to the party paying sterling? What is the value of the swap to the party paying dollars?

Solution :

The swap involves exchanging the sterling interest of $20 \times 0.14 = 2.8$ million for the dollar interest of $30 \times 0.1 = 3 million. The principal amounts are also exchanged at the end of the life of the swap. The value of the sterling bond underlying the swap is

 $2.8(1.11)^{-\frac{1}{4}} + 22.8(1.11)^{-\frac{5}{4}} = 22.739$ million pounds

The value of the dollar bond underlying the swap is

$$3(1.08)^{-\frac{1}{4}} + 33(1.08)^{-\frac{5}{4}} = 32.916$$
 million US\$

The value of the swap to the party paying sterling is therefore

$$32.916 - (22.739 \times 1.65) = -\$4,604$$
 million

The value of the swap to the party paying dollars is +\$4,604 million. The results can also be obtained by viewing the swap as a portfolio of forward contracts. The continuously compounded interest rates in sterling and dollars are 10.436% per annum and 7.696% per annum. The 3-month and 15-month forward exchange rates are

 $1.65e^{-(0.10436-0.07696)\times0.25} = 1.6387$ and $1.65e^{-(0.10436-0.07696)\times1.25} = 1.5944$

The values of the two forward contracts corresponding to the exchange of interest for the party paying sterling are therefore

$$(3 - 2.8 \times 1.6387) e^{-0.07696 \times 0.25} = -2.558$$
 million US\$

and

$$(3 - 2.8 \times 1.5944) e^{-0.07696 \times 1.25} = -1.330$$
 million US\$

The value of the forward contract corresponding to the exchange of principals is

$$(30 - 20 \times 1.5944) e^{-0.07696 \times 1.25} = -1.716$$
 million US\$

The total value of the swap is -\$1.558 - \$1.330 - \$1,716 = -\$4.604 million.

7. Companies A and B face the following interest rates (adjusted for the differential impact of taxes) :

	Canadian dollars (fixed rate)	US dollars (floating rate)
Company A	5%	LIBOR $+0.5\%$
Company B	6.5%	LIBOR $+1.0\%$

Assume that A wants to borrow US dollars at a floating rate of interest and B wants to borrow Canadian dollars at a fixed rate of interest. A financial institution is planning to arrange a swap and requires a 50 basis point spread. If the swap is to appear equally attractive to A and B, what rates of interest will A and B end up paying?. **Solution** :

Company A has a comparative advantage in the Canadian dollar fixed-rate market. Company B has a comparative advantage in the U.S. dollar floating-rate market. (This may be because of their tax positions.) However, company A wants to borrow in the U.S. dollar floating-rate market and company B wants to borrow in the Canadian dollar fixed-rate market. This gives rise to the swap opportunity. The differential between the U.S. dollar floating rates is 0.5% per annum, and the differential between the Canadian dollar fixed rates is 1.5% per annum. The difference between the differentials is 1% per annum. The total potential gain to all parties from the swap is therefore 1% per annum, or 100 basis points. If the financial intermediary requires 50 basis points, each of A and B can be made 25 basis points better off. Thus a swap can be designed so that it provides A with U.S. dollars at LIBOR +0.25% per annum, and B with Canadian dollars at 6.25% per annum.

$\begin{vmatrix} < \\ CAN\$ \ \mathbf{5\%} \end{vmatrix} \begin{array}{c} \mathrm{Comp.} \\ A \end{vmatrix} \begin{array}{c} < \\ \\ \mathbf{5LiBOR+0.25\%} \end{array} > \begin{vmatrix} F.I. \\ F.I. \end{vmatrix}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	> \$LIBOR+1%
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Payoff(F.I.) = (6.25 - 5)% + (0.25 - 1)% = 0.5%

If the notional for **US\$ is 12 million** and notional for **CAN\$ is 15 million**.

The net payoff of the FI for one year is

CAN\$ 15Millions $\times (6.25 - 5)\%$

plus

US 12 Millions $\times (0.25 - 1)\%$

If the payment are settled semi–annually then the net payoff for the F.I. in 3 years and 6 months is

3.5 imes CAN 15Millions imes (6.25 - 5)%

plus

$3.5 imes \mathbf{US}$ **12Millions** imes (0.25 - 1)%

8. A financial institution has entered into a 10-year currency swap with company A. Under the terms of the swap, the financial institution receives interest at 3% per annum in Swiss francs and pays interest at 8% per annum in US dollars. Interest payments are exchanged once a year. The principal amounts are 7 million dollars and 10 million francs. Suppose that company A declares bankruptcy at the end of year 6, when the exchange rate is \$0.80 per franc. What is the cost to the financial institution? Assume that, at the end of year 6, the interest rate is 3% per annum in Swiss francs and 8% per annum in US dollars for all maturities. All interest rates are quoted with annual compounding.

Solution :

When interest rates are compounded annually

$$F_0 = S_0 \left(\frac{1+r}{1+r_f}\right)^T$$

where F_0 is the *T*-year forward rate, S_0 is the spot rate, *r* is the domestic risk-free rate, and r_f is the foreign risk-free rate. As r = 0.08 and $r_f = 0.03$, the spot and forward exchange rates at the end of year 6 are

Spot	0.8000
1 year forward :	0.8388
2 year forward :	0.8796
3 year forward :	0.9223
4 year forward :	0.9670

The value of the swap at the time of the default can be calculated on the assumption that forward rates are realized. The cash flows lost as a result of the default are therefore as follows :

Year	Dollar due	Sw Fr due to	Forward	Dollar Equivalent.	Cash flow
	to be paid	be received	rate	for Sw. Fr Amount	\mathbf{lost}
6	560,000	300,000	0.8000	240,000	(320,000)
7	560,000	300,000	0.8388	251,000	(308, 400)
8	560,000	300,000	0.8796	263,900	(296, 100)
9	560,000	300,000	0.9223	276,700	(283, 300)
10	7,560,000	10,300,000	0.9670	9,960,100	2,400,100

Discounting the numbers in the final column to the end of year 6 at 8% per annum, the cost of the default is \$679,800. Note that, if this were the only contract entered into by company B, it would make no sense for the company to default at the end of year six as the exchange of payments. at that time has a positive value to company B. In practice company B is likely to be defaulting and declaring bankruptcy for reasons unrelated to this particular contract and payments on the contract are likely to stop when bankruptcy is declared.