OPER 441: Modeling and Simulation
Exercises Sheet #4

U(0,1) seeds

0.2379	0.7551	0.2989	0.247	0.3237
0.2972	0.8469	0.4566	0.6146	0.6723
0.9496	0.2268	0.8699	0.9084	0.5649
0.3045	0.6964	0.1709	0.3387	0.9804
0.1246	0.842	0.6557	0.9672	0.3356
0.3525	0.8075	0.9462	0.9583	0.3807
0.1489	0.5480	0.9537	0.9376	0.8364
0.5095	0.4047	0.9058	0.3795	0.6242
0.5195	0.6545	0.1117	0.3258	0.8589
0.6536	0.3427	0.6653	0.7864	0.5824

Question1:

The times to failure for an automated production process have been found to be randomly distributed according to a Rayleigh distribution:

$$f(x) = \begin{cases} 2\beta^{-2} x e^{(-(x/\beta)^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Derive an inverse transform algorithm for generating random variables from this distribution.

b) Using the first row of random numbers from the top generate 5 random numbers from your algorithm with β = 2.

$$F(x) = \int_0^x \frac{2}{\beta^2} x e^{\left(-\left(x/\beta\right)^2\right)} dx = -e^{-x^2/\beta^2} + 1$$

Question2:

Using the first two rows of random numbers from the top , generate 5 random numbers from the negative binomial distribution with parameters
(r = 4, p = 0.4) using:

a) the convolution method
b) the number of Bernoulli trials to get 4 successes.

Question3:

Suppose that the processing time for a job consists of two distributions. There is a 30% chance that the processing time is lognormally distributed with a mean of 20 minutes and a standard deviation of 2 minutes, and a 70% chance that the time is uniformly distributed between 10 and 20 minutes. Using the first row of random numbers from the top generate two job processing times.

Hint: $X \sim LN(\mu, \sigma^2)$ if and only if $\ln(X) \sim N(\mu, \sigma^2)$. Also, note that:

$$\mathrm{E}[X] = e^{\mu+\sigma^2/2}$$
$$\mathrm{Var}[X] = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$$

Question 4:

Suppose that the service time for a patient consists of two distributions. There is a 25% chance that the service time is uniformly distributed with minimum of 20 minutes and a maximum of 25 minutes, and a 75% chance that the time is distributed according to a Weibull distribution with shape of 2 and a scale of 4.5. Using the first row of random numbers from the top generate the service time for two patients.

Question 5:

If $Z \sim N(0,1)$ and

$$Y = \sum_{i=1}^{k} Z_i^2 \text{ then } Y \sim \chi_k^2, \text{ where } \chi_k^2 \text{ is a chi-squared}$$

With k degrees of freedom. Using the first row of U(0,1) generate two variables of chi-squeare with degrees of freedom = 5.

Question 6:

In the .... technique for generating random variates, you want the .... function to be as close as possible to the distribution function that you want to generate from in order to ensure that the .... is as high as possible, thereby improving the efficiency of the algorithm.

Question 7:

Consider the following probability density function:

$$f(x) = \begin{cases} \frac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Derive an acceptance-rejection algorithm for this distribution.
b) Using the first row of random numbers from the top, generate 2 random numbers using your algorithm.

Question 8:

Consider

$$f(x) = \beta^{-\alpha} x^{\alpha-1} \frac{e^{-x/\beta}}{\Gamma(\alpha)}$$

where x > 0 and $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. In the case where α is a positive integer, the distribution reduces to the Erlang distribution and $\alpha = 1$ produces the negative exponential distribution.

Acceptance-rejection techniques can be applied to the cases of $0 < \alpha < 1$ and $\alpha > 1$. For the case of $0 < \alpha < 1$ see Ahrens and Dieter (1972). For the case of $\alpha > 1$, Cheng (1977) proposed the following majorizing function:

$$g(x) = \left[\frac{4\alpha^{\alpha} e^{-\alpha}}{a\Gamma(\alpha)}\right] h(x)$$

where $a = \sqrt{(2\alpha - 1)}$, $b = \alpha^{a}$, and $h(x)$ is the resulting probability distribution function when converting $g(x)$ to a density function:

$$h(x) = ab\frac{x^{a-1}}{(b + x^{a})^2} \quad \text{for} x > 0$$

a) Develop an inverse transform algorithm for generating from $h(x)$
b) Using the first two rows of random numbers from Exercise 2.10, generate two random variates from a gamma distribution with parameters $\alpha = 2$ and $\beta = 10$ via the acceptance/rejection method.

Question 9:

Parts arrive to a machine center with three drill presses according to a Poisson distribution with mean λ. The arriving customers are assigned to one of the three drill presses randomly according to the respective probabilities p_1, p_2, and p_3 where $p_1 + p_2 + p_3 = 1$ and $p_i > 0$ for i = 1, 2, 3. What is the distribution of the inter-arrival times to each drill press? Specify the parameters of the distribution.

Suppose that p_1, p_2, and p_3 equal to 0.25, 0.45, and 0.3 respectively and λ that is equal to 12 per minute. Generate the first three arrival times using numbers from the first row of random numbers from the top.