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STAT – 324

Probability and Statistics for Engineers

EXERCISES

A Collection of Questions Selected from
Midterm and Final Examinations' Papers
for the Years from 1422 to 1427

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1. COMBINATIONS

$\binom{n}{r}$ =The number of combinations of n distinct objects taken r at a time (r objects in each combination)

= The number of different selections of r objects from n distinct objects.
= The number of different ways to select r objects from n distinct objects.
= The number of different ways to divide a set of n distinct objects into 2 subsets; one subset contains r objects and the other subset contains the rest.

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

$$0! = 1$$

Q1. Compute:

(a) $\binom{6}{2}$ (b) $\binom{6}{4}$.

Q2. Show that $\binom{n}{x} = \binom{n}{n-x}$.

Q3. Compute:

(a) $\binom{n}{0}$, (b) $\binom{n}{1}$, (c) $\binom{n}{n}$

Q4. A man wants to paint his house in 3 colors. If he can choose 3 colors out of 6 colors, how many different color settings can he make?

(A) 216 (B) 20 (C) 18 (D) 120

Q5. The number of ways in which we can select two students among a group of 5 students is

(A) 120 (B) 10 (C) 60 (D) 20 (E) 110

Q6. The number of ways in which we can select a president and a secretary among a group of 5 students is

(A) 120 (B) 10 (C) 60 (D) 20

2. PROBABILITY, CONDITIONAL PROBABILITY, AND INDEPENDENCE

Q1. Let A, B, and C be three events such that: $P(A)=0.5$, $P(B)=0.4$, $P(C\cap A^c)=0.6$, $P(C\cap A)=0.2$, and $P(A\cup B)=0.9$. Then

(a) $P(C)$ =

(A) 0.1 (B) 0.6 (C) 0.8 (D) 0.2 (E) 0.5

(b) $P(B\cap A)$ =

(A) 0.0 (B) 0.9 (C) 0.1 (D) 1.0 (E) 0.3

(c) $P(C|A)$ =

(A) 0.4 (B) 0.8 (C) 0.1 (D) 1.0 (E) 0.7

(d) $P(B^c\cap A^c)$ =

(A) 0.3 (B) 0.1 (C) 0.2 (D) 1.1 (E) 0.8

Q2. Consider the experiment of flipping a balanced coin three times independently.

(a) The number of points in the sample space is

(A) 2 (B) 6 (C) 8 (D) 3 (E) 9

(b) The probability of getting exactly two heads is

(A) 0.125 (B) 0.375 (C) 0.667 (D) 0.333 (E) 0.451

(c) The events 'exactly two heads' and 'exactly three heads' are

(A) Independent (B) disjoint (C) equally likely (D) identical (E) None

(d) The events 'the first coin is head' and 'the second and the third coins are tails' are

(A) Independent (B) disjoint (C) equally likely (D) identical (E) None

Q3. Suppose that a fair die is thrown twice independently, then

1. the probability that the sum of numbers of the two dice is less than or equal to ٤ is;
   (A) 0.1667 (B) 0.6667 (C) 0.8333 (D) 0.1389
2. the probability that at least one of the die shows ٤ is;
   (A) 0.6667 (B) 0.3056 (C) 0.8333 (D) 0.1389
3. the probability that one die shows one and the sum of the two dice is four is;
   (A) 0.0556 (B) 0.6667 (C) 0.3056 (D) 0.1389
4. the event A={the sum of two dice is 4} and the event B={exactly one die shows two} are,
   (A) Independent (B) Dependent (C) Joint (D) None of these.

Q4. Assume that $P(A)=0.3$, $P(B)=0.4$, $P(A\cap B\cap C)=0.03$, and $P(\overline{A\cap B})=0.88$, then

1. the events A and B are,
   (A) Independent (B) Dependent (C) Disjoint (D) None of these.
2. $P(C|A\cap B)$ is equal to,
   (A) 0.65 (B) 0.25 (C) 0. 35 (D) 0.14

Q5. If the probability that it will rain tomorrow is 0.23, then the probability that it will not rain tomorrow is:

(A) −0.23 (B) 0.77 (C) −0.77 (D) 0.23

Q6. The probability that a factory will open a branch in Riyadh is 0.7, the probability that it will open a branch in Jeddah is 0.4, and the probability that it will open a branch in either Riyadh or Jeddah or both is 0.8. Then, the probability that it will open a branch:

1) in both cities is:

(A) 0.1 (B) 0.9 (C) 0.3 (D) 0.8

2) in neither city is:

(A) 0.4 (B) 0.7 (C) 0.3 (D) 0.2

Q7. The probability that a lab specimen is contaminated is 0.10. Three independent specimen are checked.

1) the probability that none is contaminated is:

(A) 0.0475 (B) 0.001 (C) 0.729 (D) 0. 3

2) the probability that exactly one sample is contaminated is:

(A) 0.٢٤٣ (B) 0.0٨١ (C) 0.7٥٧ (D) 0. 3

Q8. 200 adults are classified according to sex and their level of education in the following table:

Sex / Education	Male (M)	Female (F)
Elementary (E)	28	50
Secondary (S)	38	45
College (C)	22	17

If a person is selected at random from this group, then:

1) the probability that he is a male is:

(A) 0.3182 (B) 0.44 (C) 0.28 (D) 78

2) The probability that the person is male given that the person has a secondary education is:

(A) 0.4318 (B) 0.4578 (C) 0.19 (D) 0.44

3) The probability that the person does not have a college degree given that the person is a female is:

(A) 0.8482 (B) 0.1518 (C) 0.475 (D) 0.085

4) Are the events M and E independent? Why? [$P(M)=0.44 \neq P(M|E)=0.359 \Rightarrow$ dependent]

Q9. 1000 individuals are classified below by sex and smoking habit.

		SEX	
		Male (M)	Female (F)
SMOKING HABIT	Daily (D)	300	50
	Occasionally (O)	200	50
	Not at all (N)	100	300

A person is selected randomly from this group.

1. Find the probability that the person is female. [$P(F)=0.4$]
2. Find the probability that the person is female and smokes daily. [$P(F\cap D)=0.05$]
3. Find the probability that the person is female, given that the person smokes daily. [$P(F|D)=0.1429$]
4. Are the events F and D independent? Why? [$P(F)=0.4 \neq P(F|D)=0.1429 \Rightarrow$ dependent]

Q10. Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that the other engine will start is 0.6, then the probability that both will start is:

(A) 1 (B) 0.24 (C) 0.2 (D) 0.5

Q11. If $P(B) = 0.3$ and $P(A|B) = 0.4$, then $P(A \cap B)$ equals to;

(A) 0.67 (B) 0.12 (C) 0.75 (D) 0.3

Q12. The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.10, then the probability that the computer system has the electrical failure or the virus is:

(A) 1.15 (B) 0.2 (C) 0.15 (D) 0.30

Q13. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:

(A) 6/27 (B) 2/27 (C) 12/27 (D) 4/27

Q14. 80 students are enrolled in STAT-324 class. 60 students are from engineering college and the rest are from computer science college. 10% of the engineering college students have taken this course before, and 5% of the computer science college students have taken this course before. If one student from this class is randomly selected, then:

1) the probability that he has taken this course before is:

(A) 0.25 (B) 0.0875 (C) 0.80٢١ (D) 0. 75

2) If the selected student has taken this course before then the probability that he is from the computer science college is:

(A) 0.14٢٩ (B) 0.375 (C) 0.80 (D) 0. 25

Q15. Two machines A and B make 80% and 20%, respectively, of the products in a certain factory. It is known that 5% and 10% of the products made by each machine, respectively, are defective. A finished product is randomly selected.

1. Find the probability that the product is defective. [P(D)=0.06]
2. If the product were found to be defective, what is the probability that it was made by machine B. [P(B|D)=0.3333]

Q16. If $P(A_1)=0.4$, $P(A_1 \cap A_2)=0.2$, and $P(A_3|A_1 \cap A_2)=0.75$, then

(1) $P(A_2|A_1)$ equals to

(A) 0.00 (B) 0.20 (C) 0.08 (D) 0.50

(2) $P(A_1 \cap A_2 \cap A_3)$ equals to

(A) 0.06 (B) 0.35 (C) 0.15 (D) 0.08

Q17. If P(A)=0.9, P(B)=0.6, and $P(A \cap B)=0.5$, then:

(1) $P(A \cap B^C)$ equals to

(A) 0.4 (B) 0.1 (C) 0.5 (D) 0.3

(2) $P(A^C \cap B^C)$ equals to

(A) 0.2 (B) 0.6 (C) 0.0 (D) 0.5

(3) P(B|A) equals to

(A) 0.5556 (B) 0.8333 (C) 0.6000 (D) 0.0

(4) The events A and B are

(A) independent (B) disjoint (C) joint (D) none

(5) The events A and B are

(A) disjoint (B) dependent (C) independent (D) none

Q18. Suppose that the experiment is to randomly select with replacement 2 children and register their gender (B=boy, G=girl) from a family having 2 boys and 6 girls.

(1) The number of outcomes (elements of the sample space) of this experiment equals to

(A) 4 (B) 6 (C) 5 (D) 125

(2) The event that represents registering at most one boy is

(A) {GG, GB, BG} (B) {GB, BG} (C) $\{GB\}^C$ (D) {GB, BG, BB}

(3) The probability of registering no girls equals to

(A) 0.2500 (B) 0.0625 (C) 0.4219 (D) 0.1780

(4) The probability of registering exactly one boy equals to

(A) 0.1406 (B) 0.3750 (C) 0.0141 (D) 0.0423

(5) The probability of registering at most one boy equals to

(A) 0.0156 (B) 0.5000 (C) 0.4219 (D) 0.9375

3. BAYES RULE:

Q1. 80 students are enrolled in STAT-324 class. 60 students are from engineering college and the rest are from computer science college. 10% of the engineering college students have taken this course before, and 5% of the computer science college students have taken this course before. If one student from this class is randomly selected, then:

1) the probability that he has taken this course before is:

(A) 0.25 (B) 0.0875 (C) 0.80٢١ (D) 0. 75

2) If the selected student has taken this course before then the probability that he is from the computer science college is:

(A) 0.14٢٩ (B) 0.375 (C) 0.80 (D) 0. 25

Q2. Two machines A and B make 80% and 20%, respectively, of the products in a certain factory. It is known that 5% and 10% of the products made by each machine, respectively, are defective. A finished product is randomly selected.

(a) Find the probability that the product is defective. [P(D)=0.06]

(b) If the product were found to be defective, what is the probability that it was made by machine B. [P(B|D)=0.3333]

Q3. Dates' factory has three assembly lines, A, B, and C. Suppose that the assembly lines A, B, and C account for 50%, 30%, and 20% of the total product of the factory. Quality control records show that 4% of the dates packed by line A, 6% of the dates packed by line B, and 12% of the dates packed by line C are improperly sealed. If a pack is randomly selected, then:

(a) the probability that the pack is from line B and it is improperly sealed is

(A) 0.018 (B) 0.30 (C) 0.06 (D) 0.36 (E) 0.53

(b) the probability that the pack is improperly sealed is

(A) 0.62 (B) 0.022 (C) 0.062 (D) 0.22 (E) 0.25

(c) if it is found that the pack is improperly sealed, what is the probability that it is from line B?

(A) 0.0623 (B) 0.0223 (C) 0.6203 (D) 0.2203 (E) 0.2903

Q4. Two brothers, Mohammad and Ahmad own and operate a small restaurant. Mohammad washes 50% of the dishes and Ahmad washes 50% of the dishes. When Mohammad washes a dish, he might break it with probability 0.40. On the other hand, when Ahmad washes a dish, he might break it with probability 0.10. Then,

(a) the probability that a dish will be broken during washing is:

(A) 0.667 (B) 0.25 (C) 0.8 (D) 0.5

(b) If a broken dish was found in the washing machine, the probability that it was washed by Mohammad is:

(A) 0.667 (B) 0.25 (C) 0.8 (D) 0.5

Q5. A vocational institute offers two training programs (A) and (B). In the last semester, 100 and 300 trainees were enrolled for programs (A) and (B), respectively. From the past experience it is known that the passing probabilities are 0.9 for program (A) and 0.7 for program (B). Suppose that at the end of the last semester, we selected a trainee at random from this institute.

(1) The probability that the selected trainee passed the program equals to

(A) 0.80 (B) 0.75 (C) 0.85 (D) 0.79

(2) If it is known that the selected trainee passed the program, then the probability that he has been enrolled in program (A) equals to

(A) 0.8 (B) 0.9 (C) 0.3 (D) 0.7

4. RANDOM VARIABLES, DISTRIBUTIONS, EXPECTATIONS AND CHEBYSHEV's THEOREM:

4.1. DISCRETE DISTRIBUTIONS:

Q1. Consider the experiment of flipping a balanced coin three times independently.
Let X= Number of heads – Number of tails.

(a) List the elements of the sample space S.
(b) Assign a value x of X to each sample point.
(c) Find the probability distribution function of X.
(d) Find $P(X \leq 1)$
(e) Find $P(X < 1)$
(f) Find $\mu=E(X)$
(g) Find $\sigma^2=Var(X)$

Q2. It is known that 20% of the people in a certain human population are female. The experiment is to select a committee consisting of two individuals at random. Let X be a random variable giving the number of females in the committee.

1. List the elements of the sample space S.
2. Assign a value x of X to each sample point.
3. Find the probability distribution function of X.
4. Find the probability that there will be at least one female in the committee.
5. Find the probability that there will be at most one female in the committee.
6. Find $\mu=E(X)$
7. Find $\sigma^2=Var(X)$

Q3. A box contains 100 cards; 40 of which are labeled with the number 5 and the other cards are labeled with the number 10. Two cards were selected randomly with replacement and the number appeared on each card was observed. Let X be a random variable giving the total sum of the two numbers.

(i) List the elements of the sample space *S*.
(ii) To each element of *S* assign a value x of X.
(iii) Find the probability mass function (probability distribution function) of X.
(iv) Find P(X=0).
(v) Find P(X>10).
(vi) Find $\mu=E(X)$
(vii) Find $\sigma^2=Var(X)$

Q4. Let X be a random variable with the following probability distribution:

x	–3	6	9
f(x)	0.1	0.5	0.4

1) Find the mean (expected value) of X, $\mu=E(X)$.
2) Find $E(X^2)$.
3) Find the variance of X, Var (X) = σ_X^2.
4) Find the mean of 2X+1, $E(2X+1)=\mu_{2X+1}$.

5) Find the variance of 2X+1, Var(2X+1)= σ^2_{2X+1}.

Q5. Which of the following is a probability distribution function:

(A) $f(x)=\frac{x+1}{10}$; x=0,1,2,3,4 (B) $f(x)=\frac{x-1}{5}$; x=0,1,2,3,4

(C) $f(x)=\frac{1}{5}$; x=0,1,2,3,4 (D) $f(x)=\frac{5-x^2}{6}$; x=0,1,2,3

Q6. Let the random variable X have a discrete uniform with parameter *k*=3 and with values 0,1, and 2. The probability distribution function is:

f(x)=P(X=x)=1/3 ; x=0, 1, 2.

(1) The mean of X is
 (A) 1.0 (B) 2.0 (C) 1.5 (D) 0.0

(2) The variance of X is
 (A) 0.0 (B) 1.0 (C) 0.67 (D) 1.33

Q7. Let X be a discrete random variable with the probability distribution function:
f(x) = *k*x for x=1, 2, and 3.

(i) Find the value of *k*.
(ii) Find the cumulative distribution function (CDF), *F*(x).
(iii) Using the CDF, *F*(x), find P (0.5 < X ≤ 2.5).

Q8. Let X be a random variable with cumulative distribution function (CDF) given by:

$$F(x)=\begin{cases}0\ , & x<0\\ 0.25, & 0\le x<1\\ 0.6\ , & 1\le x<2\\ 1\ , & x\ge 2\end{cases}$$

(a) Find the probability distribution function of X, f(x).
(b) Find P(1≤X<2). (using both f(x) and F(x))
(c) Find P(X>2). (using both f(x) and F(x))

Q9. Consider the random variable X with the following probability distribution function:

X	0	1	2	3
f(x)	0.4	*c*	0.3	0.1

The value of *c* is
 (A) 0.125 (B) 0.2 (C) 0.1 (D) 0.125 (E) − 0.2

Q10. Consider the random variable X with the following probability distribution function:

X	−1	0	1	2
f(x)	0.2	0.3	0.2	*c*

Find the following:
(a) The value of *c*.
(b) P(0 < X ≤ 2)
(c) μ = E(X)
(d) E(X^2)

(e) $\sigma^2 = \text{Var}(X)$

Q11. Find the value of k that makes the function

$$f(x) = k\binom{2}{x}\binom{3}{3-x} \quad \text{for } x=0,1,2$$

serve as a probability distribution function of the discrete random variable X.

Q12. The cumulative distribution function (CDF) of a discrete random variable, X, is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/16 & \text{for } 0 \le x < 1 \\ 5/16 & \text{for } 1 \le x < 2 \\ 11/16 & \text{for } 2 \le x < 3 \\ 15/16 & \text{for } 3 \le x < 4 \\ 1 & \text{for } x \ge 4. \end{cases}$$

(a) the $P(X = 2)$ is equal to:

(A) 3/8 (B) 11/16 (C) 10/16 (D) 5/16

(b) the $P(2 \le X < 4)$ is equal to:

(A) 20/16 (B) 11/16 (C) 10/16 (D) 5/16

Q13. If a random variable X has a mean of 10 and a variance of 4, then, the random variable $Y = 2X - 2$,

(a) has a mean of:

(A) 10 (B) 18 (C) 20 (D) 22

(b) and a standard deviation of:

(A) 6 (B) 2 (C) 4 (D) 16

Q14. Let X be the number of typing errors per page committed by a particular typist. The probability distribution function of X is given by:

x	0	1	2	3	4
$f(x)$	3k	3k	2k	k	k

(1) Find the numerical value of k.
(2) Find the average (mean) number of errors for this typist.
(3) Find the variance of the number of errors for this typist.
(4) Find the cumulative distribution function (CDF) of X.
(5) Find the probability that this typist will commit at least 2 errors per page.

Q15. Suppose that the discrete random variable X has the following probability function: f(−1)=0.05, f(0)=0.25, f(1)=0.25, f(2)=0.45, then:

(1) P(X<1) equals to

(A) 0.30 (B) 0.05 (C) 0.55 (D) 0.50

(2) P(X≤1) equals to

(A) 0.05 (B) 0.55 (C) 0.30 (D) 0.45

(3) The mean μ=E(X) equals to

(A) 1.1 (B) 0.0 (C) 1.2 (D) 0.5

(4) $E(X^2)$ equals to

(A) 2.00 (B) 2.10 (C) 1.50 (D) 0.75

(5) The variance σ^2=Var(X) equals to

(A) 1.00 (B) 3.31 (C) 0.89 (D) 2.10

(6) If F(x) is the cumulative distribution function (CDF) of X, then F(1) equals to

(A) 0.50 (B) 0.25 (C) 0.45 (D) 0.55

4.2. CONTINUOUS DISTRIBUTIONS:

Q1. If the continuous random variable X has mean μ=16 and variance σ^2=5, then $P(X=16)$ is

(A) 0.0625 (B) 0.5 (C) 0.0 (D) None of these.

Q2. Consider the probability density function:

$$f(x)=\begin{cases} k\sqrt{x}\,, & 0<x<1 \\ 0, & \text{elsewhere.} \end{cases}$$

1) The value of k is:

(A) 1 (B) 0.5 (C) 1.5 (D) 0.667

2) The probability $P(0.3<X\le 0.6)$ is,

(A) 0.4647 (B) 0.3004 (C) 0.1643 (D) 0.4500

3) The expected value of X, $E(X)$ is,

(A) 0.6 (B) 1.5 (C) 1 (D) 0.667

[Hint: $\int\sqrt{x}\,dx=\dfrac{x^{3/2}}{(3/2)}+c$]

Q3. Let X be a continuous random variable with the probability density function $f(x)=k(x+1)$ for $0<x<2$.

(i) Find the value of k.

(ii) Find $P(0 < X \le 1)$.

(iii) Find the cumulative distribution function (CDF) of X, $F(x)$.

(iv) Using F(x), find $P(0 < X \le 1)$.

Q4. Let X be a continuous random variable with the probability density function $f(x)=3x^2/2$ for $-1<x<1$.

1. P(0<X<1) = …………..
2. E(X) = …………………
3. Var(X) = ………………
4. E(2X+3)= ………………
5. Var(2X+3)= ……………

Q5. Suppose that the random variable X has the probability density function:

$$f(x)=\begin{cases} k\,x; & 0<x<2 \\ 0; & \textit{elsewhere.} \end{cases}$$

1. Evaluate k.
2. Find the cumulative distribution function (CDF) of X, F(x).
3. Find $P(0 < X < 1)$.
4. Find $P(X = 1)$ and $P(2 < X < 3)$.

Q6. Let X be a random variable with the probability density function:

$$f(x) = \begin{cases} 6x(1-x); & 0<x<1 \\ 0; & elsewhere. \end{cases}$$

1. Find μ = E(X).
2. Find σ^2 = Var(X).
3. Find E (4X+5).
4. Find Var(4X+5).

Q7. If the random variable X has a uniform distribution on the interval (0,10) with the probability density function given by:

$$f(x) = \begin{cases} \frac{1}{10}; & 0<x<10 \\ 0; & elsewhere. \end{cases},$$

1. Find P(X<6).
2. Find the mean of X.
3. Find E(X^2).
4. Find the variance of X.
5. Find the cumulative distribution function (CDF) of X, F(x).
6. Use the cumulative distribution function, F(x), to find P(1<X≤5).

Q8. Suppose that the failure time (in months) of a specific type of electrical device is distributed with the probability density function:

$$f(x) = \begin{cases} \frac{1}{50}x & ,0<x<10 \\ 0 & ,otherwise \end{cases}$$

(a) the average failure time of such device is:
(A) 6.667 (B) 1.00 (C) 2.00 (D) 5.00

(b) the variance of the failure time of such device is:
(A) 0 (B) 50 (C) 5.55 (D) 10

(c) P(X>5) =
(A) 0.25 (B) 0.55 (C) 0.65 (D) 0.75

Q9. If the cumulative distribution function of the random variable X having the form:

$$P(X \le x) = F(x) = \begin{cases} 0 & ; x<0 \\ x/(x+1) & ; x \ge 0 \end{cases}$$

Then

(1) $P(0 < X < 2)$ equals to
(a) 0.555 (b) 0.333 (c) 0.667 (d) none of these.

(2) If $P(X \le k) = 0.5$, then k equals to
(a) 5 (b) 0.5 (c) 1 (d) 1.5

Q10. If the diameter of a certain electrical cable is a continuous random variable X (in cm) having the probability density function :

$$f(x) = \begin{cases} 20x^3(1-x) & 0<x\le 1 \\ 0 & otherwise \end{cases}$$

(1) $P(X > 0.5)$ is:

a) 0.8125 b) 0.1875 c) 0.9844 d) 0.4445

(2) $P(0.25 < X < 1.75)$ is:

a) 0.8125 b) 0.1875 c) 0.9844 d) 0.4445

(3) μ=E(X) is:

a) 0.667 b) 0.333 c) 0.555 d) none of these.

(4) σ^2 =Var(X) is:

a)0.3175 b) 3.175 c) 0.0317 d) 2.3175

(5) For this random variable, $P(\mu - 2\sigma \le X \le \mu + 2\sigma)$ will have an exact value equals to:

a) 0.3175 b) 0.750 c) 0.965 d) 0.250

(6) For this random variable, $P(\mu - 2\sigma \le X \le \mu + 2\sigma)$ will have a lower bound value according to chebyshev's theory equals to:

a) 0.3175 b) 0.750 c) 0.965 d) 0.250

(7) If Y= 3X−1.5, then E(Y) is:

a) – 0.5 b) – 0.3335 c) 0.5 d) none of these.

(8) If Y= 3X−1.5, then Var(Y) is:

a) 2.8575 b) 0.951 c) 0.2853 d) 6.9525

4.3. CHEBYSHEV'S THEOREM :

Q1. According to Chebyshev's theorem, for any random variable X with mean μ and variance σ^2, a lower bound for $P(\mu - 2\sigma < X < \mu + 2\sigma)$ is,

(A) 0.3175 (B) 0.750 (C) 0.965 (D) 0.250

Note: $P(\mu - 2\sigma < X < \mu + 2\sigma) = P(|X - \mu| < 2\sigma)$

Q2. Suppose that X is a random variable with mean μ=12, variance σ^2=9, and unknown probability distribution. Using Chebyshev's theorem, $P(3 < X < 21)$ is at least equal to,

(A) 8/9 (B) 3/4 (C) 1/4 (D) 1/16

Q3. Suppose that E(X)=5 and Var(X)=4. Using Chebyshev's Theorem,

(i) find an approximated value of P(1<X<9).

(ii) find some constants a and b (a<b) such that P(a<X<b) ≈ 15/16.

Q4. Suppose that the random variable X is distributed according to the probability density function given by:

$$f(x) = \begin{cases} \frac{1}{10}; & 0 < x < 10 \\ 0; & elsewhere. \end{cases},$$

Assuming μ=5 and σ = 2.89,

1. Find the exact value of P(μ−1.5σ < X <μ+1.5σ).
2. Using Chebyshev's Theorem, find an approximate value of P(μ−1.5σ < X <μ+1.5σ).
3. Compare the values in (1) and (2).

Q5. Suppose that X and Y are two independent random variables with E(X)=30, Var(X)=4, E(Y)=10, and Var(Y)=2. Then:

(1) E(2X−3Y−10) equals to

(A) 40 (B) 20 (C) 30 (D) 90

(2) $Var(2X-3Y-10)$ equals to

(A) 34 (B) 24 (C) 2.0 (D) 14

(3) Using Chebyshev's theorem, a lower bound of $P(24<X<36)$ equals to

(A) 0.3333 (B) 0.6666 (C) 0.8888 (D) 0.1111

DISCRETE UNIFORM DISTRIBUTION:

Q1. Let the random variable X have a discrete uniform with parameter *k*=3 and with values 0,1, and 2. Then:

(a) P(X=1) is
  (A) 1.0 (B) 1/3 (C) 0.3 (D) 0.1 (E) None

(b) The mean of X is:
  (A) 1.0 (B) 2.0 (C) 1.5 (D) 0.0 (E) None

(c) The variance of X is:
  (A) 0/3=0.0 (B) 3/3=1.0 (C) 2/3=0.67 (D) 4/3=1.33 (E) None

5. BINOMIAL DISTRIBUTION:

Q1. Suppose that 4 out of 12 buildings in a certain city violate the building code. A building engineer randomly inspects a sample of 3 new buildings in the city.

(a) Find the probability distribution function of the random variable X representing the number of buildings that violate the building code in the sample.

(b) Find the probability that:
  (i) none of the buildings in the sample violating the building code.
  (ii) one building in the sample violating the building code.
  (iii) at lease one building in the sample violating the building code.

(c) Find the expected number of buildings in the sample that violate the building code (E(X)).

(d) Find σ^2=Var(X).

Q2. A missile detection system has a probability of 0.90 of detecting a missile attack. If 4 detection systems are installed in the same area and operate independently, then

(a) The probability that at least two systems detect an attack is
  (A) 0.9963 (B) 0.9477 (C) 0.0037 (D) 0.0523 (E) 0.5477

(b) The average (mean) number of systems detect an attack is
  (A) 3.6 (B) 2.0 (C) 0.36 (D) 2.5 (E) 4.0

Q3. Suppose that the probability that a person dies when he or she contracts a certain disease is 0.4. A sample of 10 persons who contracted this disease is randomly chosen.

(1) What is the expected number of persons who will die in this sample?
(2) What is the variance of the number of persons who will die in this sample?
(3) What is the probability that exactly 4 persons will die among this sample?
(4) What is the probability that less than 3 persons will die among this sample?
(5) What is the probability that more than 8 persons will die among this sample?

Q4. Suppose that the percentage of females in a certain population is 50%. A sample of 3 people is selected randomly from this population.

(a) The probability that no females are selected is
  (A) 0.000 (B) 0.500 (C) 0.375 (D) 0.125

(b) The probability that at most two females are selected is
  (A) 0.000 (B) 0.500 (C) 0.875 (D) 0.125

(c) The expected number of females in the sample is
  (A) 3.0 (B) 1.5 (C) 0.0 (D) 0.50

(d) The variance of the number of females in the sample is
(A) 3.75 (B) 2.75 (C) 1.75 (D) 0.75

Q5. 20% of the trainees in a certain program fail to complete the program. If 5 trainees of this program are selected randomly,

(i) Find the probability distribution function of the random variable X, where:
X = number of the trainees who fail to complete the program.
(ii) Find the probability that all trainees fail to complete the program.
(iii) Find the probability that at least one trainee will fail to complete the program.
(iv) How many trainees are expected to fail completing the program?
(v) Find the variance of the number of trainees who fail completing the program.

Q6. In a certain industrial factory, there are 7 workers working independently. The probability of accruing accidents for any worker on a given day is 0.2, and accidents are independent from worker to worker.

(a) The probability that at most two workers will have accidents during the day is
(A) 0.7865 (B) 0.4233 (C) 0.5767 (D) 0.6647
(b) The probability that at least three workers will have accidents during the day is:
(A) 0.7865 (B) 0.2135 (C) 0.5767 (D) 0.1039
(c) The expected number workers who will have accidents during the day is
(A) 1.4 (B) 0.2135 (C) 2.57 (D) 0. 59

Q7. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:
(A) 6/27 (B) 2/27 (C) 12/27 (D) 4/27

Q8. The probability that a lab specimen is contaminated is 0.10. Three independent samples are checked.

1) the probability that none is contaminated is:
(A) 0.0475 (B) 0.001 (C) 0.729 (D) 0. 3
2) the probability that exactly one sample is contaminated is:
(A) 0.٢٤٣ (B) 0.0٨١ (C) 0.7٥٧ (D) 0. 3

Q9. If X~Binomial(n,p), E(X)=1, and Var(X)=0.75, find P(X=1).

Q10. Suppose that X~Binomial(3,0.2). Find the cumulative distribution function (CDF) of X.

Q11. A traffic control engineer reports that 75% of the cars passing through a checkpoint are from Riyadh city. If at this checkpoint, five cars are selected at random.

(1) The probability that none of them is from Riyadh city equals to:
(A) 0.00098 (B) 0.9990 (C) 0.2373 (D) 0.7627
(2) The probability that four of them are from Riyadh city equals to:
(A) 0.3955 (B) 0.6045 (C) 0 (D) 0.1249
(3) The probability that at least four of them are from Riyadh city equals to:
(A) 0.3627 (B) 0.6328 (C) 0.3955 (D) 0.2763
(4) The expected number of cars that are from Riyadh city equals to:
(A) 1 (B) 3.75 (C) 3 (D) 0

6. HYPERGEOMETRIC DISTRIBUTION:

Q1. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets.

(i) Find the probability distribution function of the random variable X representing the number of defective sets purchased by the hotel.
(ii) Find the probability that the hotel purchased no defective television sets.
(iii) What is the expected number of defective television sets purchased by the hotel?
(iv) Find the variance of X.

Q2. Suppose that a family has 5 children, 3 of them are girls and the rest are boys. A sample of 2 children is selected randomly and without replacement.

(a) The probability that no girls are selected is
(A) 0.0 (B) 0.3 (C) 0.6 (D) 0.1
(b) The probability that at most one girls are selected is
(A) 0.7 (B) 0.3 (C) 0.6 (D) 0.1
(c) The expected number of girls in the sample is
(A) 2.2 (B) 1.2 (C) 0.2 (D) 3.2
(d) The variance of the number of girls in the sample is
(A) 36.0 (B) 3.6 (C) 0.36 (D) 0.63

Q3. A random committee of size 4 is selected from 2 chemical engineers and 8 industrial engineers.

(i) Write a formula for the probability distribution function of the random variable X representing the number of chemical engineers in the committee.
(ii) Find the probability that there will be no chemical engineers in the committee.
(iii) Find the probability that there will be at least one chemical engineer in the committee.
(iv) What is the expected number of chemical engineers in the committee?
(v) What is the variance of the number of chemical engineers in the committee?

Q4. A box contains 2 red balls and 4 black balls. Suppose that a sample of 3 balls were selected randomly and without replacement. Find,

1. The probability that there will be 2 red balls in the sample.
2. The probability that there will be 3 red balls in the sample.
3. The expected number of the red balls in the sample.

Q5. From a lot of 8 missiles, 3 are selected at random and fired. The lot contains 2 defective missiles that will not fire. Let X be a random variable giving the number of defective missiles selected.

1. Find the probability distribution function of X.
2. What is the probability that at most one missile will not fire?
3. Find $\mu = E(X)$ and $\sigma^2 = Var(X)$.

Q6. A particular industrial product is shipped in lots of 20 items. Testing to determine whether an item is defective is costly; hence, the manufacturer samples production rather than using 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to consumers calls for sampling 5 items from each lot and rejecting the lot if more than one defective is observed. (If the lot is rejected, each item in the lot is then tested.) If a lot contains 4 defectives, what is the probability that it will be accepted.

Q7. Suppose that X~h(x;100,2,60); i.e., X has a hypergeometric distribution with parameters N=100, n=2, and K=60. Calculate the probabilities P(X=0), P(X=1), and P(X=2) as follows:

(a) exact probabilities using hypergeometric distribution.

(b) approximated probabilities using binomial distribution.

Q8. A particular industrial product is shipped in lots of 1000 items. Testing to determine whether an item is defective is costly; hence, the manufacturer samples production rather than using 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to consumers calls for sampling 5 items from each lot and rejecting the lot if more than one defective is observed. (If the lot is rejected, each item in the lot is then tested.) If a lot contains 100 defectives, calculate the probability that the lot will be accepted using:

(a) hypergeometric distribution (exact probability.)

(b) binomial distribution (approximated probability.)

Q9. A shipment of 20 digital voice recorders contains 5 that are defective. If 10 of them are randomly chosen (without replacement) for inspection, then:

(1) The probability that 2 will be defective is:

(A) 0.2140 (B) 0.9314 (C) 0.6517 (D) 0.3483

(2) The probability that at most 1 will be defective is:

(A) 0.9998 (B) 0.2614 (C) 0.8483 (D) 0.1517

(3) The expected number of defective recorders in the sample is:

(A) 1 (B) 2 (C) 3.5 (D) 2.5

(4) The variance of the number of defective recorders in the sample is:

(A) 0.9868 (B) 2.5 (C) 0.1875 (D) 1.875

Q10. A box contains 4 red balls and 6 green balls. The experiment is to select 3 balls at random. Find the probability that all balls are red for the following cases:

(1) If selection is without replacement

(A) 0.216 (B) 0.1667 (C) 0.6671 (D) 0.0333

(2) If selection is with replacement

(A) 0.4600 (B) 0.2000 (C) 0.4000 (D) 0.0640

7. POISSON DISTRIBUTION:

Q1. On average, a certain intersection results in 3 traffic accidents per day. Assuming Poisson distribution,

(i) what is the probability that at this intersection:

(1) no accidents will occur in a given day?

(2) More than 3 accidents will occur in a given day?

(3) Exactly 5 accidents will occur in a period of two days?

(ii) what is the average number of traffic accidents in a period of 4 days?

Q2. At a checkout counter, customers arrive at an average of 1.5 per minute. Assuming Poisson distribution, then

(1) The probability of no arrival in two minutes is

(A) 0.0 (B) 0.2231 (C) 0.4463 (D) 0.0498 (E) 0.2498

(2) The variance of the number of arrivals in two minutes is

(A) 1.5 (B) 2.25 (C) 3.0 (D) 9.0 (E) 4.5

Q3. Suppose that the number of telephone calls received per day has a Poisson distribution with mean of 4 calls per day.

(a) The probability that 2 calls will be received in a given day is

(A) 0.546525 (B) 0.646525 (C) 0.146525 (D) 0.746525

(b) The expected number of telephone calls received in a given week is

(A) 4 (B) 7 (C) 28 (D) 14

(c) The probability that at least 2 calls will be received in a period of 12 hours is

(A) 0.59399 (B) 0.19399 (C) 0.09399 (D) 0.29399

Q4. The average number of car accidents at a specific traffic signal is 2 per a week. Assuming Poisson distribution, find the probability that:

(i) there will be no accident in a given week.

(ii) there will be at least two accidents in a period of two weeks.

Q5. The average number of airplane accidents at an airport is two per a year. Assuming Poisson distribution, find

1. the probability that there will be no accident in a year.
2. the average number of airplane accidents at this airport in a period of two years.
3. the probability that there will be at least two accidents in a period of 18 months.

Q6. Suppose that $X \sim Binomial(1000, 0.002)$. By using Poisson approximation, $P(X=3)$ is approximately equal to (choose the nearest number to your answer):

(A) 0.62511 (B) 0.72511 (C) 0.82511 (D) 0.92511 (E) 0.18045

Q7. The probability that a person dies when he or she contracts a certain disease is 0.005. A sample of 1000 persons who contracted this disease is randomly chosen.

(1) What is the expected number of persons who will die in this sample?

(2) What is the probability that exactly 4 persons will die among this sample?

Q8. The number of faults in a fiber optic cable follows a Poisson distribution with an average of 0.6 per 100 feet.

(1) The probability of 2 faults per 100 feet of such cable is:
(A) 0.0988 (B) 0.9012 (C) 0.3210 (D) 0.5

(2) The probability of less than 2 faults per 100 feet of such cable is:
(A) 0.2351 (B) 0.9769 (C) 0.8781 (D) 0.8601

(3) The probability of 4 faults per 200 feet of such cable is:
(A) 0.02602 (B) 0.1976 (C) 0.8024 (D) 0.9739

CONTINUOUS UNIFORM DISTRIBUTION:

Q1. If the random variable X has a uniform distribution on the interval (0,10), then

1. P(X<6) equals to
   (A) 0.4 (B) 0.6 (C) 0.8 (D) 0.2 (E) 0.1
2. The mean of X is
   (A) 5 (B) 10 (C) 2 (D) 8 (E) 6
2. The variance X is
   (A) 33.33 (B) 28.33 (C) 8.33 (D) 25 (E) None

Q2. Suppose that the random variable X has the following uniform distribution:

$$f(x) = \begin{cases} 3 & , \frac{2}{3} < x < 1 \\ 0 & , otherwise \end{cases}$$

(1) $P(0.33 < X < 0.5) =$
   (A) 0.49 (B) 0.51 (C) 0 (D) 3

(2) $P(X > 1.25) =$
   (A) 0 (B) 1 (C) 0.5 (D) 0.33

(3) The variance of X is
   (A) 0.00926 (B) 0.333 (C) 9 (D) 0.6944

Q3. Suppose that the continuous random variable X has the following probability density function (pdf): f(x)=0.2 for 0<x<5. Then

(1) P(X>1) equals to
   (A) 0.4 (B) 0.2 (C) 0.1 (D) 0.8

(2) P(X≥1) equals to
   (A) 0.05 (B) 0.8 (C) 0.15 (D) 0.4

(3) The mean μ=E(X) equals to
   (A) 2.0 (B) 2.5 (C) 3.0 (D) 3.5

(4) $E(X^2)$ equals to
   (A) 8.3333 (B) 7.3333 (C) 9.3333 (D) 6.3333

(5) Var(X) equals to
   (A) 8.3333 (B) 69.444 (C) 5.8333 (D) 2.0833

(6) If F(x) is the cumulative distribution function (CDF) of X, then F(1) equals to
   (A) 0.75 (B) 0.25 (C) 0.8 (D) 0.2

8. NORMAL DISTRIBUTION:

Q1. (A) Suppose that Z is distributed according to the standard normal distribution.

1) the area under the curve to the left of $z = 1.43$ is:
   (A) 0.0764 (B) 0.9236 (C) 0 (D) 0.8133
2) the area under the curve to the left of $z = 1.39$ is:
   (A) 0.7268 (B) 0.9177 (C) .2732 (D) 0.0832
3) the area under the curve to the right of $z = -0.89$ is:
   (A) 0. 7815 (B) 0.8133 (C) 0.1867 (D) 0.0154
4) the area under the curve between $z = -2.16$ and $z = -0.65$ is:
   (A) 0.7576 (B) 0.8665 (C) 0.0154 (D) 0.2424
5) the value of k such that $P(0.93 < Z < k) = 0.0427$ is:
   (A) 0.8665 (B) −1.11 (C) 1.11 (D) 1.00

(B) Suppose that Z is distributed according to the standard normal distribution. Find:

1) $P(Z< -3.9)$
2) $P(Z> 4.5)$
1) $P(Z< 3.7)$
2) $P(Z> -4.1)$

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

1) the proportion of rings that will have inside diameter less than 12.05 centimeters is:
   (A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413
2) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:
   (A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514
3) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:
   (A) 0.905 (B) −0.905 (C) 0.4514 (D) 0.7257

Q3. The average life of a certain type of small motor is 10 years with a standard deviation of 2 years. Assume the live of the motor is normally distributed. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 1.5% of the motors that fail, then he should give a guarantee of :

(A) 10.03 years (B) 8 years (C) 5.66 years (D) 3 years

Q4. A machine makes bolts (that are used in the construction of an electric transformer). It produces bolts with diameters (X) following a normal distribution with a mean of 0.060 inches and a standard deviation of 0.001 inches. Any bolt with diameter less than 0.058 inches or greater than 0.062 inches must be scrapped. Then

(1) The proportion of bolts that must be scrapped is equal to
   (A) 0.0456 (B) 0.0228 (C) 0.9772 (D) 0.3333 (E) 0.1667

(2) If $P(X>a)= 0.1949$, then a equals to:
   (A) 0.0629 (B) 0.0659 (C) 0.0649 (D) 0.0669 (E) 0.0609

Q5. The diameters of ball bearings manufactured by an industrial process are normally distributed with a mean $\mu = 3.0$ cm and a standard deviation $\sigma = 0.005$ cm. All ball bearings with diameters not within the specifications $\mu \pm d$ cm ($d > 0$) will be scrapped.

(1) Determine the value of d such that 90% of ball bearings manufactured by this process will not be scrapped.

(2) If d = 0.005, what is the percentage of manufactured ball bearings that will be scraped?

Q6. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

(1) The percentage of fat persons with weights at most 110 kg is
(A) 0.09 % (B) 90.3 % (C) 99.82 % (D) 2.28 %

(2) The percentage of fat persons with weights more than 149 kg is
(A) 0.09 % (B) 0.99 % (C) 9.7 % (D) 99.82 %

(3) The weight x above which 86% of those persons will be
(A) 118.28 (B) 128.28 (C) 154.82 (D) 81.28

(4) The weight x below which 50% of those persons will be
(A) 101.18 (B) 128 (C) 154.82 (D) 81

Q7. The random variable X, representing the lifespan of a certain electronic device, is normally distributed with a mean of 40 months and a standard deviation of 2 months. Find

1. $P(X<38)$. (0.1587)
2. $P(38<X<40)$. (0.3413)
3. $P(X=38)$. (0.0000)
4. The value of x such that $P(X<x)=0.7324$. (41.24)

Q8. If the random variable X has a normal distribution with the mean μ and the variance σ^2, then $P(X<\mu+2\sigma)$ equals to
(A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q9. If the random variable X has a normal distribution with the mean μ and the variance 1, and if $P(X<3)=0.877$, then μ equals to
(A) 3.84 (B) 2.84 (C) 1.84 (D) 4.84 (E) 8.84

Q10. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark x is
(A) 67.8 (B) 60.8 (C) 57.8 (D) 50.8 (E) 70.8

Q11. If the random variable X has a normal distribution with the mean 10 and the variance 36, then

1. The value of X above which an area of 0.2296 lie is
(A) 14.44 (B) 16.44 (C) 10.44 (D) 18.44 (E) 11.44
2. The probability that the value of X is greater than 16 is
(A) 0.9587 (B) 0.1587 (C) 0.7587 (D) 0.0587 (E) 0.5587

Q12. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is
(A) 20.56% (B) 90.56% (C) 50.56% (D) 10.56% (E)40.56%

Q13. The average rainfall in a certain city for the month of March is 9.22 centimeters. Assuming a normal distribution with a standard deviation of 2.83 centimeters, then the probability that next March, this city will receive:

(1) less than 11.84 centimeters of rain is:

(A) 0.8238 (B) 0.1762 (C) 0.5 (D) 0.2018

(2) more than 5 centimeters but less than 7 centimeters of rain is:

(A) 0.8504 (B) 0.1496 (C) 0.6502 (D) 0.34221

(3) more than 13.8 centimeters of rain is:

(A) 0.0526 (B) 0.9474 (C) 0.3101 (D) 0.4053

9. EXPONENTIAL DISTRIBUTION

Q1. If the random variable X has an exponential distribution with the mean 4, then:

1. P(X<8) equals to

   (A) 0.2647 (B) 0.4647 (C) 0.8647 (D) 0.6647 (E) 0.0647

2. The variance of X is

   (A) 4 (B) 16 (C) 2 (D) 1/4 (E) 1/2

Q2. Suppose that the failure time (in hours) of a certain electrical device is distributed with a probability density function given by:

$$f(x) = \frac{1}{70} e^{-x/70}, \quad x > 0,$$

1) the probability that a randomly selected device will fail within the first 50 hours is:

   (A) 0.4995 (B) 0.7001 (C) 0.5105 (D) 0.2999

2) the probability that a randomly selected device will last more than 150 hours is:

   (A) 0.8827 (B) 0.2788 (C) 0.1173 (D) 0.8827

3) the average failure time of the electrical device is:

   (A) 1/70 (B) 70 (C) 140 (D) 35

4) the variance of the failure time of the electrical device is:

   (A) 4900 (B) 1/49000 (C) 70 (D) 1225

[Hint: $\int e^{-ax}\, dx = -\frac{1}{a} e^{-ax} + c$]

Q3. The lifetime of a specific battery is a random variable X with probability density function given by:

$$f(x) = \frac{1}{200} e^{-x/200}, \qquad x > 0$$

(1) The mean life time of the battery equals to

   (A) 200 (B) 1/200 (C) 100 (D) 1/100 (E) Non of these

(2) P(X>100) =

   (A) 0.5 (B) 0.6065 (C) 0.3945 (D) 0.3679 (E) 0.6321

(3) P(X=200) =

   (A) 0.5 (B) 0.0 (C) 0.3945 (D) 0.3679 (E) 1.0

Q4. Suppose that the lifetime of a certain electrical device is given by T. The random variable T is modeled nicely by an exponential distribution with mean of 6 years. A random sample of four of these devices are installed in different systems. Assuming that these devices work independently, then:

(1) the variance of the random variable T is

   (A) 136 (B) $(36)^2$ (C) 6 (D) 36

(2) the probability that at most one of the devices in the sample will be functioning more than 6 years is

   (A) 0.4689 (B) 0.6321 (C) 0.5311 (D) 0.3679

(3) the probability that at least two of the devices in the sample will be functioning more than 6 years is

   (A) 0.4689 (B) 0.6321 (C) 0.5311 (D) 0.3679

(4) the expected number of devices in the sample which will be functioning more than 6 years is approximately equal to

   (A) 3.47 (B) 1.47 (C) 4.47 (D) 1.47

Q5. Assume the length (in minutes) of a particular type of a telephone conversation is a random variable with a probability density function of the form:

$$f(x) = \begin{cases} 0.2\ e^{-0.2x} & ; x \geq 0 \\ 0 & ; elsewhere \end{cases}$$

1. $P(3 < x < 10)$ is:

   (a) 0.587 (b) -0.413 (c) 0.413 (d) 0.758
2. For this random variable, $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ will have an exact value equals:

   (a) 0.250 (b) 0.750 (c) 0.950 (d) 0.3175
3. For this random variable, $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ will have a lower bound valued according to chebyshev's theory equals:

   (a) 0.750 (b) 0.250 (c) 0.950 (d) 0.3175

Q6. The length of time for one customer to be served at a bank is a random variable X that follows the exponential distribution with a mean of 4 minutes.

(1) The probability that a customer will be served in less than 2 minutes is:

(A) 0.9534 (B) 0.2123 (C) 0.6065 (D) 0.3935

(2) The probability that a customer will be served in more than 4 minutes is:

(A) 0.6321 (B) 0.3679 (C) 0.4905 (D) 0.0012

(3) The probability that a customer will be served in more than 2 but less than 5 minutes is:

(A) 0.6799 (B) 0.32 (C) 0.4018 (D) 0.5523

(4) The variance of service time at this bank is

(A) 2 (B) 4 (C) 8 (D) 16

10. SAMPLING DISTRIBUTIONS

10.1. Single Mean:

Q1. A machine is producing metal pieces that are cylindrical in shape. A random sample of size 5 is taken and the diameters are 1.70, 2.11, 2.20, 2.31 and 2.28 centimeters. Then,

1) The sample mean is:
   (A) 2.12 (B) 2.32 (C) 2.90 (D) 2.20 (E) 2.22
2) The sample variance is:
   (A) 0.59757 (B) 0.28555 (C) 0.35633 (D) 0.06115 (E) 0.53400

Q2. The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

1) The sample mean $\overline{X}$ of a random sample of 5 batteries selected from this product has a mean
   $E(\overline{X}) = \mu_{\overline{X}}$ equal to:
   (A) 0.2 (B) 5 (C) 3 (D) None of these
2) The variance $Var(\overline{X}) = \sigma^2_{\overline{X}}$ of the sample mean $\overline{X}$ of a random sample of 5 batteries selected from this product is equal to:
   (A) 0.2 (B) 5 (C) 3 (D) None of these
3) The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is:
   (A) 0.1039 (B) 0.2135 (C) 0.7865 (D) 0.9224
4) The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is:
   (A) 0.9772 (B) 0.0228 (C) 0.9223 (D) None of these
5) The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:
   (A) 0.8413 (B) 0.1587 (C) 0.9452 (D) None of these
6) If $P(\ \overline{X} > a\ ) =\ 0.1492$ where $\overline{X}$ represents the sample mean for a random sample of size 9 of such batteries, then the numerical value of a is:
   (A) 4.653 (B) 6.5 (C) 5.347 (D) None of these

Q3. The random variable X, representing the lifespan of a certain light bulb, is distributed normally with a mean of 400 hours and a standard deviation of 10 hours.

1. What is the probability that a particular light bulb will last for more than 380 hours?
2. Light bulbs with lifespan less than 380 hours are rejected. Find the percentage of light bulbs that will be rejected.
3. If 9 light bulbs are selected randomly, find the probability that their average lifespan will be less than 405.

Q4. Suppose that you take a random sample of size n=64 from a distribution with mean μ=55 and standard deviation σ=10. Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the sample mean.

(a) What is the approximated sampling distribution of $\overline{X}$?
(b) What is the mean of $\overline{X}$?
(c) What is the standard error (standard deviation) of $\overline{X}$?

(d) Find the probability that the sample mean $\overline{X}$ exceeds 52.

Q5. The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with the mean 3.0 minutes and the standard deviation of 1.4 minutes. If a random sample of 49 customers is observed, then

(1) the probability that their mean time will be at least 2.8 minutes is

(A) 1.0 (B) 0.8413 (C) 0.3274 (D) 0.4468

(2) the probability that their mean time will be between 2.7 and 3.2 minutes is

(A) 0.7745 (B) 0.2784 (C) 0.9973 (D) 0.0236

Q6. The average life of an industrial machine is 6 years, with a standard deviation of 1 year. Assume the life of such machines follows approximately a normal distribution. A random sample of 4 of such machines is selected. The sample mean life of the machines in the sample is $\overline{X}$.

(1) The sample mean has a mean $\mu_{\overline{X}} = E(\overline{X})$ equals to:

(A) 5 (B) 6 (C) 7 (D) 8

(2) The sample mean has a variance $\sigma^2_{\overline{X}} = Var(\overline{X})$ equals to:

(A) 1 (B) 0.5 (C) 0.25 (D) 0.75

(3) $P(\overline{X} < 5.5) =$

(A) 0.4602 (B) 0.8413 (C) 0.1587 (D) 0.5398

(4) If $P(\overline{X} > a) = 0.1492$, then the numerical value of a is:

(A) 0.8508 (B) 1.04 (C) 6.52 (D) 0.2

10.2. Two Means:

Q1. A random sample of size $n_1 = 36$ is taken from a normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let $\overline{X}_1$ and $\overline{X}_2$ be the averages of the first and second samples, respectively.

a) Find $E(\overline{X}_1)$ and $Var(\overline{X}_1)$.

b) Find $E(\overline{X}_1 - \overline{X}_2)$ and $Var(\overline{X}_1 - \overline{X}_2)$.

c) Find $P(70 < \overline{X}_1 < 71)$.

d) Find $P(\overline{X}_1 - \overline{X}_2 > -16)$.

Q2. A random sample of size 25 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 36 is taken from a different normal population (second population) having a mean of 97 and a standard deviation of 5. Assume that these two samples are independent.

(1) the probability that the sample mean of the first sample will exceed the sample mean of the second sample by at least 6 is

(A) 0.0013 (B) 0.9147 (C) 0.0202 (D) 0.9832

(2) the probability that the difference between the two sample means will be less than 2 is

(A) 0.099 (B) 0.2480 (C) 0.8499 (D) 0.9499

10.3. Single Proportion:

Q1. Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of 5 students is taken from this university. Let $\hat{p}$ be the proportion of smokers in the sample.

(1) Find $E(\hat{p}) = \mu_{\hat{p}}$, the mean $\hat{p}$.

(2) Find $Var(\hat{p}) = \sigma^2_{\hat{p}}$, the variance of $\hat{p}$.

(3) Find an approximate distribution of $\hat{p}$.

(4) Find P($\hat{p}$ >0.25).

Q2: Suppose that you take a random sample of size n=100 from a binomial population with parameter p=0.25 (proportion of successes). Let $\hat{p} = X/n$ be the sample proportion of successes, where X is the number of successes in the sample.

(a) What is the approximated sampling distribution of $\hat{p}$?
(b) What is the mean of $\hat{p}$?
(c) What is the standard error (standard deviation) of $\hat{p}$?
(d) Find the probability that the sample proportion $\hat{p}$ is less than 0.2.

10.4. Two Proportions:

Q1. Suppose that 25% of the male students and 20% of the female students in a certain university smoke cigarettes. A random sample of 5 male students is taken. Another random sample of 10 female students is independently taken from this university. Let $\hat{p}_1$ and $\hat{p}_2$ be the proportions of smokers in the two samples, respectively.

(1) Find $E(\hat{p}_1 - \hat{p}_2) = \mu_{\hat{p}_1 - \hat{p}_2}$, the mean of $\hat{p}_1 - \hat{p}_2$.

(2) Find $Var(\hat{p}_1 - \hat{p}_2) = \sigma^2_{\hat{p}_1 - \hat{p}_2}$, the variance of $\hat{p}_1 - \hat{p}_2$.

(3) Find an approximate distribution of $\hat{p}_1 - \hat{p}_2$.

(4) Find P(0.10< $\hat{p}_1 - \hat{p}_2$ <0.20).

10.5 t-distribution:

Q1. Using t-table with degrees of freedom df=14, find $t_{0.02}$, $t_{0.985}$.

Q2. From the table of t-distribution with degrees of freedom $\nu = 15$, the value of $t_{0.025}$ equals to

(A) 2.131 (B) 1.753 (C) 3.268 (D) 0.0

11. ESTIMATION AND CONFIDENCE INTERVALS:

11.1. Single Mean:

Q1. An electrical firm manufacturing light bulbs that have a length of life that is normally distributed with a standard deviation of 30 hours. A sample of 50 bulbs were selected randomly and found to have an average of 750 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm.

(1) Find a point estimate for μ.
(2) Construct a 94% confidence interval for μ.

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation σ=2.0. Suppose that a random sample of size *n*=49 from this population gave a sample mean $\overline{X}$ =4.5.

(1) The distribution of $\overline{X}$ is
(A) N(0,1) (B) t(48) (C) N(μ, 0.2857) (D) N(μ, 2.0) (E) N(μ, 0.3333)
(2) A good point estimate of μ is
(A) 4.50 (B) 2.00 (C) 2.50 (D) 7.00 (E) 1.125
(3) The standard error of $\overline{X}$ is
(A) 0.0816 (B) 2.0 (C) 0.0408 (D) 0.5714 (E) 0.2857
(4) A 95% confidence interval for μ is
(A) (3.44,5.56) (B) (3.34,5.66) (C) (3.54,5.46) (D) (3.94,5.06) (E) (3.04,5.96)
(5) If the upper confidence limit of a confidence interval is 5.2, then the lower confidence limit is
(A) 3.6 (B) 3.8 (C) 4.0 (D) 3.5 (E) 4.1
(6) The confidence level of the confidence interval (3.88, 5.12) is
(A) 90.74% (B) 95.74% (C) 97.74% (D) 94.74% (E) 92.74%
(7) If we use $\overline{X}$ to estimate μ, then we are 95% confident that our estimation error will not exceed
(A) e=0.50 (B) E=0.59 (C) e=0.58 (D) e=0.56 (E) e=0.51
(8) If we want to be 95% confident that the estimation error will not exceed e=0.1 when we use $\overline{X}$ to estimate μ, then the sample size *n* must be equal to
(A) 1529 (B) 1531 (C) 1537 (D) 1534 (E) 1530

Q3. The following measurements were recorded for lifetime, in years, of certain type of machine: 3.4, 4.8, 3.6, 3.3, 5.6, 3.7, 4.4, 5.2, and 4.8. Assuming that the measurements represent a random sample from a normal population, then a 99% confidence interval for the mean life time of the machine is

(A) $-5.37 \le \mu \le 3.25$ (B) $4.72 \le \mu \le 9.1$
(C) $4.01 \le \mu \le 5.99$ (D) $3.37 \le \mu \le 5.25$

Q4. A researcher wants to estimate the mean lifespan of a certain light bulbs. Suppose that the distribution is normal with standard deviation of 5 hours.

1. Determine the sample size needed on order that the researcher will be 90% confident that the error will not exceed 2 hours when he uses the sample mean as a point estimate for the true mean.
2. Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.
   (i) Find a good point estimate for the true mean μ.
   (ii) Find a 95% confidence interval for the true mean μ.

Q5. The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with a standard deviation of 1.4 minutes. If we wish to estimate the population mean μ by the sample mean $\overline{X}$, and if we want to be 96% confident that the sample mean will be within 0.3 minutes of the population mean, then the sample size needed is

(A) 98 (B) 100 (C) 92 (D) 85

Q6: A random sample of size n=36 from a normal quantitative population produced a mean $\overline{X} = 15.2$ and a variance $S^2 = 9$.

(a) Give a point estimate for the population mean μ.
(b) Find a 95% confidence interval for the population mean μ.

Q7. A group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25

$\sum X = 75.75 , \sum X^2 = 600.563 \}$

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2. Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend.

(a) Find the sample mean and the sample variance.
(b) Find a point estimate for μ.
(c) Construct a 80% confidence interval for μ.

Q8. An electronics company wanted to estimate its monthly operating expenses in thousands riyals (μ). It is assumed that the monthly operating expenses (in thousands riyals) are distributed according to a normal distribution with variance σ^2=0.584.

(I) Suppose that a random sample of 49 months produced a sample mean $\overline{X} = 5.47$.
  (a) Find a point estimate for μ.
  (b) Find the standard error of $\overline{X}$.
  (c) Find a 90% confidence interval for μ.

(II) Suppose that they want to estimate μ by $\overline{X}$. Find the sample size (n) required if they want their estimate to be within 0.15 of the actual mean with probability equals to 0.95.

Q9. The tensile strength of a certain type of thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

(a) A point estimate of the population mean of the tensile strength (μ) is:
  (A) 72.8 (B) 20 (C) 6.8 (D) 46.24 (E) None of these

(b) Suppose that we want to estimate the population mean (μ) by the sample mean $(\overline{X})$. To be 95% confident that the error of our estimate of the mean of tensile strength will be less than 3.4 kilograms, the minimum sample size should be at least:
  (A) 4 (B) 16 (C) 20 (D) 18 (E) None of these

(c) For a 98% confidence interval for the mean of tensile strength, we have the lower bound equal to:
  (A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these

(d) For a 98% confidence interval for the mean of tensile strength, we have the upper bound equal to:

(A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

11.2. Two Means:

Q1.(I) The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

1) To be 95% confident that the error of estimating the mean of tensile strength by the sample mean will be less than 3.4 kilograms, the minimum sample size should be:
   (A) 4 (B) 16 (C) 20 (D) 18 (E) None of these
2) The lower limit of a 98% confidence interval for the mean of tensile strength is
   (A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these
3) The upper limit of a 98% confidence interval for the mean of tensile strength is
   (A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

Q1.(II). The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

1) the lower bound equals to:
   (A) 2.90 (B) 4.21 (C) 3.65 (D) 6.58 (E) None of these
2) the upper bound equals to:
   (A) 13.90 (B) 13.15 (C) 12.59 (D) 10.22 (E) None of these

Q2. Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean ($\overline{X}$)	10.5	10.0
sample variance (S^2)	4	5

Let μ_1 and μ_2 be the true means of the first and second populations, respectively.

1. Find a point estimate for $\mu_1 - \mu_2$..
2. Find 95% confidence interval for $\mu_1 - \mu_2$.

Q3. A researcher was interested in comparing the mean score of female students, μ_f, with the mean score of male students, μ_m, in a certain test. Two independent samples gave the following results:

Sample	Observations							mean	Variance
Scores of Females	89.2	81.6	79.6	80.0	82.8			82.63	15.05
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04	20.79

Assume the populations are normal with equal variances.

(1) The pooled estimate of the variance S_p^2 is
(A) 17.994 (B) 17.794 (C) 18.094 (D) 18.294 (E) 18.494

(2) A point estimate of $\mu_f - \mu_m$ is
(A) 2.63 (B) −2.59 (C) 2.59 (D) 0.00 (E) 0.59

(3) The lower limit of a 90% confidence interval for $\mu_f - \mu_m$ is
(A) −1.97 (B) −1.67 (C) 1.97 (D) 1.67 (E) −1.57

(4) The upper limit of a 90% confidence interval for $\mu_f - \mu_m$ is
(A) 6.95 (B) 7.45 (C) −7.55 (D) 7.15 (E) 7.55

Q4. A study was conducted to compare to brands of tires A and B. 10 tires of brand A and 12 tires of brand B were selected randomly. The tires were run until they wear out. The results are:

Brand A: $\overline{X}_A = 37000$ kilometers $S_A = 5100$

Brand B: $\overline{X}_B = 38000$ kilometers $S_B = 6000$

Assuming the populations are normally distributed with equal variances,

(1) Find a point estimate for $\mu_A - \mu_B$.

(2) Construct a 90% confidence interval for $\mu_A - \mu_B$.

Q5. The following data show the number of defects of code of particular type of software program made in two different countries (assuming normal populations with unknown equal variances)

Country	observations							mean	standard dev.
A	48	39	42	52	40	48	54	46.143	5.900
B	50	40	43	45	50	38	36	43.143	5.551

(a) A point estimate of $\mu_A - \mu_B$ is

(A) 3.0 (B) −3.0 (C) 2.0 (D) −2.0 (E) None of these

(b) A 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$ is

(A) $-2.46 \le \mu_A - \mu_B \le 8.46$ (B) $1.42 \le \mu_A - \mu_B \le 6.42$

(C) $-1.42 \le \mu_A - \mu_B \le -0.42$ (D) $2.42 \le \mu_A - \mu_B \le 10.42$

Q6. A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the populations are normal with equal unknown variances)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

(a) A 95% confidence interval for the true mean gasoline consumption for brand A is:

(A) $4.462 \le \mu_A \le 6.938$ (B) $2.642 \le \mu_A \le 4.930$

(C) $5.2 \le \mu_A \le 9.7$ (D) $6.154 \le \mu_A \le 6.938$

(b) A 99% confidence interval for the difference between the true means consumption of type (A) and type (B) ($\mu_A - \mu_B$) is:

(A) $-1.939 \le \mu_A - \mu_B \le 2.539$ (B) $-2.939 \le \mu_A - \mu_B \le 1.539$

(C) $0.939 \le \mu_A - \mu_B \le 1.539$ (D) $-1.939 \le \mu_A - \mu_B \le 0.539$

Q7. A geologist collected 20 different ore samples, all of the same weight, and randomly divided them into two groups. The titanium contents of the samples, found using two different methods, are listed in the table:

Method (A)	Method (B)
1.1 1.3 1.3 1.5 1.4	1.1 1.6 1.3 1.2 1.5
1.3 1.0 1.3 1.1 1.2	1.2 1.7 1.3 1.4 1.5
$\overline{X}_1 = 1.25$, $S_1 = 0.1509$	$\overline{X}_2 = 1.38$, $S_2 = 0.1932$

(a) Find a point estimate of $\mu_A - \mu_B$ is

(b) Find a 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$. (Assume two normal populations with equal variances).

11.3. Single Proportion:

Q1. A random sample of 200 students from a certain school showed that 15 students smoke. Let p be the proportion of smokers in the school.

1. Find a point Estimate for p.
2. Find 95% confidence interval for p.

Q2. A researcher was interested in making some statistical inferences about the proportion of females (p) among the students of a certain university. A random sample of 500 students showed that 150 students are female.

(1) A good point estimate for p is
(A) 0.31 (B) 0.30 (C) 0.29 (D) 0.25 (E) 0.27

(2) The lower limit of a 90% confidence interval for p is
(A) 0.2363 (B) 0.2463 (C) 0.2963 (D) 0.2063 (E) 0.2663

(3) The upper limit of a 90% confidence interval for p is
(A) 0.3337 (B) 0.3137 (C) 0.3637 (D) 0.2937 (E) 0.3537

Q3. In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let p be the proportion of homes in this city that are heated by oil.

1) Find a point estimate for p.
2) Construct a 98% confidence interval for p.

Q4. In a study involved 1200 car drivers, it was found that 50 car drivers do not use seat belt.

(1) A point estimate for the proportion of car drivers who do not use seat belt is:
(A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these

(2) The lower limit of a 95% confidence interval of the proportion of car drivers not using seat belt is
(A) 0.0322 (B) 0.0416 (C) 0 .0304 (D) –0.3500 (E) None of these

(3) The upper limit of a 95% confidence interval of the proportion of car drivers not using seat belt is
(A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

Q5. A study was conducted to make some inferences about the proportion of female employees (p) in a certain hospital. A random sample gave the following data:

Sample size	250
Number of females	120

(a) Calculate a point estimate ($\hat{p}$) for the proportion of female employees (p).

(b) Construct a 90% confidence interval for p.

Q6. In a certain city, the traffic police was interested in knowing the proportion of car drivers who do not use seat built. In a study involved 1200 car drivers it was found that 50 car drivers do not use seat belt.

(a) A point estimate for the proportion of car drivers who do not use seat built is:

(A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these

(b) A 95% confidence interval of the proportion of car drivers who do not use seat built has the lower bound equal to:

(A) 0.0322 (B) 0.0416 (C) 0 .0304 (D) –0.3500 (E) None of these

(c) A 95% confidence interval of the proportion of car drivers who do not use seat built has the upper bound equal to:

(A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

11.4. Two Proportions:

Q1. A survey of 500 students from a college of science shows that 275 students own computers. In another independent survey of 400 students from a college of engineering shows that 240 students own computers.

(a) a 99% confidence interval for the true proportion of college of science's student who own computers is

(A) $-0.59 \le p_1 \le 0.71$ (B) $0.49 \le p_1 \le 0.61$

(C) $2.49 \le p_1 \le 6.61$ (D) $0.3 \le p_1 \le 0.7$

(29) a 95% confidence interval for the difference between the proportions of students owning computers in the two colleges is

(A) $0.015 \le p_1 - p_2 \le 0.215$ (B) $-0.515 \le p_1 - p_2 \le 0.215$

(C) $-0.450 \le p_1 - p_2 \le -0.015$ (D) $-0.115 \le p_1 - p_2 \le 0.015$

Q2. A food company distributes "smiley cow" brand of milk. A random sample of 200 consumers in the city (A) showed that 80 consumers prefer the "smiley cow" brand of milk. Another independent random sample of 300 consumers in the city (B) showed that 90 consumers prefer "smiley cow" brand of milk. Define:

p_A = the true proportion of consumers in the city (A) preferring "smiley cow" brand.

p_B = the true proportion of consumers in the city (B) preferring "smiley cow" brand.

(a) A 96% confidence interval for the true proportion of consumers preferring brand (A) is:

(A) $0.328 \le p_A \le 0.375$ (B) $0.228 \le p_A \le 0.675$

(C) $0.328 \le p_A \le 0.475$ (D) $0.518 \le p_A \le 0.875$

(b) A 99% confidence interval for the difference between proportions of consumers preferring brand (A) and (B) is:

(A) $0.0123 \le p_A - p_B \le 0.212$ (B) $-0.2313 \le p_A - p_B \le 0.3612$

(C) $-0.0023 \le p_A - p_B \le 0.012$ (D) $-0.0123 \le p_A - p_B \le 0.212$

Q3. A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B".

(1) Find a point Estimate for $p_1 - p_2$.

(2) Find 95% confidence interval for $p_1 - p_2$.

12. HYPOTHESES TESTING:

12.1. Single Mean:

Q1. Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation $\sigma=2.0$. Suppose that a random sample of size $n=49$ from this population gave a sample mean $\overline{X} = 4.5$.

(1) If we want to test $H_o:\mu=5.0$ against $H_1:\mu\neq5.0$, then the test statistic equals to

(A) Z= −1.75 (B) Z=1.75 (C) T= −1.70 (D) T= 1.70 (E) Z= −1.65

(2) If we want to test H_o: $\mu=5.0$ against H_1: $\mu>5.0$ at $\alpha=0.05$, then the Rejection Region of H_o is

(A) $(1.96,\infty)$ (B) $(2.325,\infty)$ (C) $(-\infty,-1.645)$ (D) $(-\infty,-1.96)$ (E) $(1.645,\infty)$

(3) If we want to test H_o: $\mu=5.0$ against H_1: $\mu>5.0$ at $\alpha=0.05$, then we

(A) Accept H_o (B) Reject H_o (C) (D) (E)

Q2. An electrical firm manufactures light bulbs that have a length of life that is normally distributed with a standard deviation of 30 hours. A sample of 50 bulbs were selected randomly and found to have an average of 750 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm. Test Ho: $\mu = 740$ against H_1: $\mu < 740$? Use a 0.05 level of significance.

Q3. An electrical firm manufactures light bulbs that have a length of life that is normally distributed. A sample of 20 bulbs were selected randomly and found to have an average of 655 hours and a standard deviation of 27 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm. Test Ho: $\mu = 660$ against H_1: $\mu\neq 660$? Use a 0.02 level of significance.

Q4: A random sample of size n=36 from a normal quantitative population produced a mean $\overline{X} = 15.2$ and a variance $S^2 = 9$. Test Ho: $\mu = 15$ against Ha: $\mu \neq 15$, use $\alpha=0.05$.

Q5. A group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25

$\sum X = 75.75$, $\sum X^2 = 600.563$ }

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2. Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend. Test Ho: $\mu = 7.5$ against Ha: $\mu > 7.5$, use $\alpha=0.2$.

Q6. An electronics company wanted to estimate its monthly operating expenses in thousands riyals (μ). It is assumed that the monthly operating expenses (in thousands riyals) are distributed according to a normal distribution with variance $\sigma^2=0.584$. Suppose that a random sample of 49 months produced a sample mean $\overline{X} = 5.47$. Test H_o: $\mu=5.5$ against and H_a: $\mu\neq5.5$. Use $\alpha=0.01$.

Q7. The tensile strength of a certain type of thread is approximately normally distributed with standard deviation of 6.8 kilograms. The manufacturer claims that the mean of the tensile strength of this type of thread equals to 70.0 kilogram. Do you agree with this claim if a sample of 20 pieces of the thread had an average tensile strength of 72.8 kilograms? Use $\alpha=0.05$.

12.2. Two Means:

Q1. Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean ($\overline{X}$)	10.5	10.0
sample variance (S^2)	4	5

Let μ_1 and μ_2 be the true means of the first and second populations, respectively. Test H_o: $\mu_1 = \mu_2$ against H_1: $\mu_1 \neq \mu_2$. (use α=0.05)

Q2. A researcher was interested in comparing the mean score of female students, μ_f, with the mean score of male students, μ_m, in a certain test. Two independent samples gave the following results:

Sample	Observations							mean	variance
Scores of Females	89.2	81.6	79.6	80.0	82.8			82.63	15.05
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04	20.79

Assume that the populations are normal with equal variances.

(1) The pooled estimate of the variance S_p^2 is

(A) 17.994 (B) 17.794 (C) 18.094 (D) 18.294 (E) 18.494

(2) If we want to test H_o: $\mu_f = \mu_m$ against H_1: $\mu_f \neq \mu_m$ then the test statistic equals to

(A) Z= 1.129 (B) T= −1.029 (C) T=1.029 (D) T=1.329 (E) T= −1.329

(3) If we want to test H_o: $\mu_f = \mu_m$ against H_1: $\mu_f \neq \mu_m$ at α=0.1, then the Acceptance Region of H_o is

(A) $(-\infty,1.812)$ (B) $(-1.812,1.812)$ (C) $(-1.372,\infty)$ (D) $(-1.372,1.372)$ (E) $(-1.812,\infty)$

(4) If we want to test H_o: $\mu_f = \mu_m$ against H_1: $\mu_f \neq \mu_m$ at α=0.1, then we

(A) Reject H_o (B) Accept H_o (C) (D) (E)

Q3. A study was conducted to compare to brands of tires A and B. 10 tires of brand A and 12 tires of brand B were selected randomly. The tires were run until they wear out. The results are:

Brand A: $\overline{X}_A$ = 37000 kilometers S_A = 5100

Brand B: $\overline{X}_B$ = 38000 kilometers S_B = 6000

Assuming the populations are normally distributed with equal variances. Test H_o: $\mu_A = \mu_B$ against H_1: $\mu_A < \mu_B$. Use a 0.1 level of significance.

Q4. A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the populations are normal with equal unknown variances)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

(a) Test H_o: $\mu_A \leq 5.6$ against H_1: $\mu_A > 5.6$. Use a 0.1 level of significance.

(b) Test H_o: $\mu_A \geq \mu_B$ against H_1: $\mu_A < \mu_B$. Use a 0.1 level of significance.

Q5. To determine whether car ownership affects a student's academic achievement, two independent random samples of 100 male students were each drawn from the students' body. The first sample is for non-owners of cars and the second sample is for owners of cars. The grade point average for the 100 non-owners of cars had an average equals to 2.70, while the grade point average for the 100 owners of cars had an average equals to 2.54. Do data present sufficient evidence to indicate a difference in the mean achievement between car owners and nonowners of cars? Test using α=0.05. Assume that the two populations have variances $\sigma^2_{non-owner} = 0.36$ and $\sigma^2_{owner} = 0.40$

Q6. A geologist collected 20 different ore samples, all of the same weight, and randomly divided them into two groups. The titanium contents of the samples, found using two different methods, are listed in the table:

Method 1	Method 2
1.1 1.3 1.3 1.5 1.4	1.1 1.6 1.3 1.2 1.5
1.3 1.0 1.3 1.1 1.2	1.2 1.7 1.3 1.4 1.5
$\overline{X}_1 = 1.25$, $S_1 = 0.1509$	$\overline{X}_2 = 1.38$, $S_2 = 0.1932$

Does this data provide sufficient statistical evidence to indicate that there is a difference between the mean titanium contents using the two different methods? Test using α=0.05. (Assume two normal populations with equal variances).

12.3. Single Proportion:

Q1. A researcher was interested in making some statistical inferences about the proportion of smokers (*p*) among the students of a certain university. A random sample of 500 students showed that 150 students smoke.

(1) If we want to test H_o: p=0.25 against H_1:p≠0.25 then the test statistic equals to
(A) z=2.2398 (B) T=−2.2398 (C) z=−2.4398 (D) Z=2.582 (E) T=2.2398

(2) If we want to test H_o: p=0.25 against H_1:p≠0.25 at α=0.1, then the Acceptance Region of H_o is
(A) (−1.645,∞) (B) (−∞,1.645) (C) (−1.645,1.645) (D) (−1.285,∞) (E) (−1.285,1.285)

(3) If we want to test H_o: p=0.25 against H_1:p≠0.25 at α=0.1, then we
(A) Accept H_o (B) Reject H_o (C) (D) (E)

Q2. In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let p be the proportion of homes in this city that are heated by oil. A builder claims that less than 20% of the homes in this city are heated by oil. Would you agree with this claim? Use a 0.01 level of significance.

Q3. The Humane Society in the USA reports that 40% of all U.S. households own at least one dog. In a random sample of 300 households, 114 households said that they owned at least one dog. Does this data provide sufficient evidence to indicate that the proportion of households with at least one dog is different from that reported by the Humane Society? Test using α=0.05.

Q4. A study was conducted to make some inferences about the proportion of female employees (p) in a certain hospital. A random sample gave the following data:

Sample size	250
Number of females	120

Does this data provide sufficient statistical evidence to indicate that the percentage of female employees in this hospital differs form the percentage of male employees? Test using α=0.1. Justify your answer statistically. {Hint: Consider testing H_o: p=0.5 against H_a: p≠0.5}

Q5. In a certain city, the traffic police was interested in knowing the proportion of car drivers who do not use seat built. In a study involved 1200 car drivers it was found that 50 car drivers do not use seat belt. Does this data provide sufficient statistical evidence to indicate that the percentage of car drivers who do not use seat belt is less than 5%. Test using $\alpha=0.1$.

12.4. Two Proportions:

Q1. A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B". Test H_o: $p_1 = p_2$ against H1: $p_1 > p_2$. (use $\alpha=0.05$)

Q2. A food company distributes "smiley cow" brand of milk. A random sample of 200 consumers in the city (A) showed that 80 consumers prefer the "smiley cow" brand of milk. Another independent random sample of 300 consumers in the city (B) showed that 90 consumers prefer "smiley cow" brand of milk. Define:

p_A = the true proportion of consumers in the city (A) preferring "smiley cow" brand.
p_B = the true proportion of consumers in the city (B) preferring "smiley cow" brand.
(a) Test H_o: $p_A = 0.33$ against H_1: $p_A \neq 0.33$. Use a 0.05 level of significance.
(b) Test H_o: $p_A \geq p_B$ against H_1: $p_A < p_B$. Use a 0.05 level of significance.

Q3: Independent random samples of $n_1 = 1500$ and $n_2 = 1000$ observations were selected from binomial populations 1 and 2, and $X_1 = 1200$ and $X_2 = 700$ successes were observed. Let p_1 and p_2 be the population proportions.

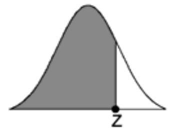
(a) Test H_o: $p_1 = p_2$ against H_1: $p_1 \neq p_2$. Use a 0.1 level of significance.
(b) Find a 90% confidence interval for ($p_1 - p_2$).
(c) Based on the 90% confidence interval in part (b), can you conclude that there is a difference between the two binomial proportions? Explain.

Q4. A study was conducted to compare between the proportions of smokers in two universities. Two independent random samples gave the following data:

	Univ. (1)	Univ. (2)
Sample size	200	300
Number of smokers	80	111

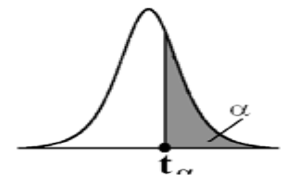
Does this data provide sufficient statistical evidence to indicate that the percentage of students who smoke differs for these two universities? Test using $\alpha=0.01$.

Areas under the Standard Normal Curve



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Percentage Points of the t Distribution; $t_{\nu,\alpha}$ $\{P(T > t_{\nu,\alpha}) = \alpha\}$



	α													
ν	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

Summary of Confidence Interval Procedures

Problem Type	Point Estimate	Two-Sided 100(1-α)% Confidence Interval
Mean μ variance σ^2 known, normal distribution, or any distribution with n>30	$\overline{X}$	$\overline{X}-Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}<\mu<\overline{X}+Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$ or $\overline{X}\pm Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
Mean μ normal distribution, variance σ^2 unknown	$\overline{X}$	$\overline{X}-t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}<\mu<\overline{X}+t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$ or $\overline{X}\pm t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$ (df: $\nu=n-1$)
Difference in two means μ_1 and μ_2 variances σ_1^2 and σ_2^2 are known, normal distributions, or any distributions with n_1, n_2>30	$\overline{X}_1-\overline{X}_2$	$(\overline{X}_1-\overline{X}_2)-Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}<\mu_1-\mu_2<(\overline{X}_1-\overline{X}_2)+Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$ or $(\overline{X}_1-\overline{X}_2)\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$
Difference in means μ_1 and μ_2 normal distributions, variances $\sigma_1^2=\sigma_2^2$ and unknown	$\overline{X}_1-\overline{X}_2$	$(\overline{X}_1-\overline{X}_2)-t_{\frac{\alpha}{2}}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}<\mu_1-\mu_2<(\overline{X}_1-\overline{X}_2)+t_{\frac{\alpha}{2}}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ or $(\overline{X}_1-\overline{X}_2)\pm t_{\frac{\alpha}{2}}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $S_p^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$; (df: $\nu=n_1+n_2-2$)
Proportion p (or parameter of a binomial distribution)	$\hat{p}$	$\hat{p}-Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}<p<\hat{p}+Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ or $\hat{p}\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$; $\hat{q}=1-\hat{p}$
Difference in two proportions p_1-p_2 (or difference in two binomial parameters)	$\hat{p}_1-\hat{p}_2$	$(\hat{p}_1-\hat{p}_2)-Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}<p_1-p_2<(\hat{p}_1-\hat{p}_2)+Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$ or $(\hat{p}_1-\hat{p}_2)\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$

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Summary of Hypotheses Testing Procedures

Null Hypothesis	Test Statistic	Alternative Hypothesis	Critical Region (Rejection Region)
$H_o: \mu = \mu_o$ variance σ^2 is known, Normal distribution, or any distribution with n>30	$Z = \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$	$H_1: \mu \neq \mu_o$	$\lvert Z\rvert > Z_{\alpha/2}$
		$H_1: \mu > \mu_o$	$Z > Z_{\alpha}$
		$H_1: \mu < \mu_o$	$Z < -Z_{\alpha}$
$H_o: \mu = \mu_o$ Normal distribution, variance σ^2 is unknown	$T = \frac{\overline{X} - \mu_o}{S / \sqrt{n}}$; df: $\nu = n-1$	$H_1: \mu \neq \mu_o$	$\lvert T\rvert > t_{\alpha/2}$
		$H_1: \mu > \mu_o$	$T > t_{\alpha}$
		$H_1: \mu < \mu_o$	$T < -t_{\alpha}$
$H_o: \mu_1 = \mu_2$ Variances σ_1^2 and σ_2^2 are known, Normal distributions, or any distributions with n_1, n_2>30	$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$H_1: \mu_1 \neq \mu_2$	$\lvert Z\rvert > Z_{\alpha/2}$
		$H_1: \mu_1 > \mu_2$	$Z > Z_{\alpha}$
		$H_1: \mu_1 < \mu_2$	$Z < -Z_{\alpha}$
$H_o: \mu_1 = \mu_2$ Normal distributions, variances $\sigma_1^2 = \sigma_2^2$ and unknown	$T = \frac{\overline{X}_1 - \overline{X}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$; df: $\nu = n_1 + n_2 - 2$ $S_p^2 = [(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2]/(n_1 + n_2 - 2)$	$H_1: \mu_1 \neq \mu_2$	$\lvert T\rvert > t_{\alpha/2}$
		$H_1: \mu_1 > \mu_2$	$T > t_{\alpha}$
		$H_1: \mu_1 < \mu_2$	$T < -t_{\alpha}$
$H_o: p = p_o$ Proportion or parameter of a binomial distribution p ($q = 1 - p$)	$Z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}} = \frac{X - np_o}{\sqrt{np_o q_o}}$	$H_1: p \neq p_o$	$\lvert Z\rvert > Z_{\alpha/2}$
		$H_1: p > p_o$	$Z > Z_{\alpha}$
		$H_1: p < p_o$	$Z < -Z_{\alpha}$
$H_o: p_1 = p_2$ Difference in two proportions or two binomial parameters $p_1 - p_2$	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ $\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$	$H_1: p_1 \neq p_2$	$\lvert Z\rvert > Z_{\alpha/2}$
		$H_1: p_1 > p_2$	$Z > Z_{\alpha}$
		$H_1: p_1 < p_2$	$Z < -Z_{\alpha}$