Chapter 4

9.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of **40** hours. If a sample of **30** bulbs has an average life of **780** hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

Population normal and σ = 40 "known", $n = 30, \bar{X} = 780$

a 96% confidence interval

$\alpha = 1 - 0.96 = 0.04$

$Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 2.055$

From Z table

96% C.I for μ is: $\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$$780 \pm 2.055 \frac{40}{\sqrt{30}}$$

$$780 \pm 15.007$$

$$(764.99, 795.008)$$

9.6 How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within **10** hours of the true mean?

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}} . \sigma}{e}\right)^2 = \left(\frac{2.055\ (40)}{10}\right)^2 = 67.24 \approx 68$$

"we always rounded the number up"

9.4 The heights of a random sample of **50** college students showed a mean of **174.5** centimeters and a standard deviation of **6.9** centimeters.

(a) Construct a 98% confidence interval for the mean height of all college students.

$n = 50$ **(n ≥ 30)**, $\bar{X} = 174.5$, $S = 6.9$ (*σ unknown*)

98% confidence interval

$\alpha = 1 - 0.98 = 0.02$

$Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.325$

98% C.I for μ is: $\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$$174.5 \pm 2.325 \frac{6.9}{\sqrt{50}}$$

$$174.5 \pm 2.2736$$

$$98\%\ C.I\ for\ \mu \in (172.23, 176.77)$$

(b) What can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be **174.5** centimeters?

The error will not exceed $Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 2.2736$

H.W **9.5** A random sample of **100** automobile owners in the state of Virginia shows that an automobile is driven on average **23,500** kilometers per year with a standard deviation of **3900** kilometers. Assume the distribution of measurements to be approximately normal.

(a) Construct a 99% confidence interval for the average number of kilometers an automobile is driven annually in Virginia.

(b) What can we assert with **99%** confidence about the possible size of our error if we estimate the average number of kilometers driven by car owners in Virginia to be **23,500** kilometers per year?

$n = 100$ (**n large**) , $\bar{X} = 23500$, $S = 3900$ (**σ unknown**)

a)

$$99\%\ C.I\ for\ \mu \text{ is: } \bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$$23500 \pm 2.575 \frac{3900}{\sqrt{100}}$$

$$23500 \pm 1004.25$$

$$99\%\ C.I\ for\ \mu \in (22495.75, 24504.25)$$

99% confidence interval

$\alpha = 1 - 0.99 = 0.01$

$Z_{1-\frac{\alpha}{2}} = Z_{0.995} = 2.575$

From Z table

b) The error will not exceed $Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 1004.25$

Q. A group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

7.25 , 8.5 , 5.0 , 6.75 , 8.0 , 5.25 , 10.5 , 8.5 , 6.75 , 9.25

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2. Let μ the college student spend doing his/her homework during the weekend. be the mean of the number of hours that

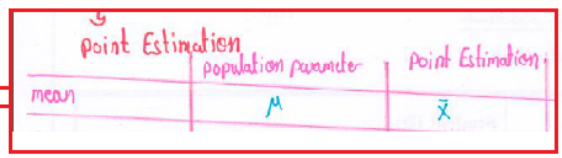
$\bar{X}$ S^2

(a) Find the sample mean and the sample variance.

$\bar{X} = 7.575$, $S^2 = (1.724)^2$ (σ^2 unknown)

(b) Find a point estimate for μ

$\bar{X} = 7.575$



(a) Construct a 80% confidence interval for μ.

$\bar{X} = 7.575$, $S = 1.724$ (σ unknown) , $df = n - 1 = 9$

$$80\%\ C.I \text{ for is: } \bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$$7.575 \pm 1.383 \frac{1.724}{\sqrt{10}}$$

$$7.575 \pm 0.754$$

$$(6.821, 8.329)$$

(error=e=0.754)

80% confidence interval

$\alpha = 1 - 0.80 = 0.20$

$t_{\frac{\alpha}{2}} = t_{0.1} = 1.383$

From T table

9.35 A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5$, has a mean $\bar{X}_1 = 80$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3$, has a mean $\bar{X}_2 = 75$.

Find a 94% confidence interval for $\mu_1 - \mu_2$.

$$94\%\ C.I\ for\ \mu_1 - \mu_2\ is: (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(80 - 75) \pm 1.885 \sqrt{\frac{5^2}{25} + \frac{3^2}{36}}$$

$$5 \pm 2.1019$$

$$\mu_1 - \mu_2 \in (2.8981,\ 7.1019)$$

(error=e=2.1019)

a 94% confidence

$$\alpha = 1 - 0.94 = 0.06$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.97} = 1.885$$

From Z table

9.38 Two catalysts(محفز) in a batch chemical process, are being compared for their effect on the output of the process reaction. A sample of 12 batches was prepared using catalyst 1, and a sample of 10 batches was prepared using catalyst 2. The 12 batches for which catalyst 1 was used in the reaction gave an average yield of 85 with a sample standard deviation of 4, and the 10 batches for which catalyst 2 was used gave an average yield of 81 and a sample standard deviation of 5.

Find a 90% confidence interval for the difference between the population means, assuming that the populations are approximately normally distributed with equal variances.

catalyst 2	catalyst 1
$n_2 = 10$	$n_1 = 12$
$\bar{X}_2 = 81$	$\bar{X}_1 = 85$
$s_2 = 5$	$s_1 = 4$

σ_1^2 and σ_2^2 unknown but equal

$$90\%\ C.I\ for\ \mu_1 - \mu_2\ is: (\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(85 - 81) \pm (1.725)(4.4777) \sqrt{\frac{1}{12} + \frac{1}{10}}$$

$$4 \pm 3.3072$$

$$\mu_1 - \mu_2 \in (0.693,\ 7.307)$$

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{11\,(16) + 9(25)}{20}} = 4.4777$$

$$\alpha = 1 - 0.90 = 0.1$$

$$t_{\frac{\alpha}{2}} = t_{0.05} = 1.725$$

df= n1+n2-2

From T table

$(error = e = 3.3072)$

H.W **9.41** The following data represent the length of time, in days, to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication 1	Medication 2
$n_1 = 14$	$n_2 = 16$
$\bar{x}_1 = 17$	$\bar{x}_2 = 19$
$s_1^2 = 1.5$	$s_2^2 = 1.8$

Find a 99% confidence interval for the difference $\boldsymbol{\mu_2 - \mu_1}$ in the mean recovery times for the two medications, assuming normal populations with equal variances.

σ_1^2 *and* σ_2^2 *unknown* but equal + n_1 *and* n_2 *small*

99% confidence interval

$\alpha = 1 - 0.99 = 0.01$

$t_{\frac{\alpha}{2}} = t_{0.005} = 2.763$

df = n1 +n2 - 2 =28

From T table

$$90\%\ C.I\ for\ \mu_2 - \mu_1\ is: (\bar{X}_2 - \bar{X}_1) \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(19 - 17) \pm (2.763)(1.3336)\sqrt{\frac{1}{14} + \frac{1}{16}}$$

$$2 \pm 1.348$$

$$\mu_2 - \mu_1 \in (0.65, 3.35)$$

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{13\,(1.5^2) + 15(1.8^2)}{28}} = 1.3336$$

9.44 A taxi company is trying to decide whether to purchase brand "A" or brand "B" tires for its fleet of taxis (اسطول من سيارات التكسي). The experiment is conducted using 12 of each brand and the tires are run until they wear out.

I) Compute a 99% confidence interval for $\mu_A - \mu_B$, assuming the pop**ulation**s to be approximately normally. You may not assume that the variances are equal.

II) Find a 99% confidence interval for $\mu_A - \mu_B$ if tires of the two brands are assigned at random to the left and right rear wheels of **8** taxis and the following distances, in kilometers, are recorded:

Taxi	**Brand A**	**Brand B**
1	34,400	36,700
2	45,500	46,800
3	36,700	37,700
4	32,000	31,100
5	48,400	47,800
6	32,800	36,400
7	38,100	38,900
8	30,100	31,500

Assume that the differences of the distances are approximately normally distributed.

I) σ_1^2 *and* σ_2^2 *unknown*, $\sigma_1^2 \neq \sigma_2^2$. n1 and n2 small

Brand A	Brand B
34,400	36,700
45,500	46,800
36,700	37,700
32,000	31,100
48,400	47,800
32,800	36,400
38,100	38,900
30,100	31,500

From the table, we calculate:

$\bar{X}_1 = 37,250, s_1 = 6546.755$

$\bar{X}_2 = 38,362.5, s_2 = 6181.063$

$$99\%\ C.I\ for\ \mu_1 - \mu_2\ is: (\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(37250 - 38362.5) \pm (2.977)\sqrt{\frac{6546.755^2}{8} + \frac{6181.063^2}{8}}$$

$$-1112.5 \pm 9476.587$$

$$\mu_2 - \mu_1 \in (-10589.0873\ ,\ 8364.0873\ )$$

99% confidence interval

$\alpha = 1 - 0.99 = 0.01$

$t_{\frac{\alpha}{2}} = t_{0.005,14} = 2.977$

From T table

df = n1+n2 -2 =8+8-2=14

A	B	D=A-B
34,400	36,700	-2,300
45,500	46,800	-1,300
36,700	37,700	-1,000
32,000	31,100	900
48,400	47,800	600
32,800	36,400	-3,600
38,100	38,900	-800
30,100	31,500	-1,400

$\bar{d} = \frac{\sum d_i}{n} = -1112.5$, $S_d = 1454.488$

II) 99% C.I for μ_d is : $[\ \bar{d} \pm t_{\frac{\alpha}{2}, n-1}\frac{S_d}{\sqrt{n}}\ ]$

$$-1112.5 \pm (3.499)\left(\frac{1454.488}{\sqrt{8}}\right)$$

$$-1112.5 \pm 1799.3$$

$$\mu_d \in [-2911.8\ , 686.8\ ]$$

99% confidence interval

$\alpha = 1 - 0.99 = 0.01$

$t_{\frac{\alpha}{2}, n-1} = t_{0.005,7} = 3.499$

From T table

df = n-1 = 8-1 =7

9.51) In a random sample of **1000** homes in a certain city, it is found that **228** are heated by oil. Find 99% confidence intervals for the proportion of homes in this city that are heated by oil.

$$\hat{p} = \frac{x}{n} = \frac{\text{جزء}}{\text{كل}} = \frac{228}{1000} = 0.228, \quad \hat{q} = 1 - \hat{p}$$

$$\alpha = 1 - 0.99 = 0.01,$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.995} = 2.575$$

From Z table

$$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.228 \pm 2.575 \sqrt{\frac{0.228(1-0.228)}{1000}}$$

$$0.228 \pm 0.034$$

$$\mathbf{P} \in (\mathbf{0.194}\ , \mathbf{0.262})$$

9.65) A certain geneticist is interested in the proportion of males and females in the population who have a minor blood disorder. In a random sample **of 1000 males, 250** are found to be afflicted, whereas **275 of 1000 females** tested appear to have the disorder.
Compute a 95% confidence interval for the difference between the proportions of females and males who have the blood disorder.

Females:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{\text{جزء}}{\text{كل}} = \frac{275}{1000} = 0.275$$

and

males:

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{\text{جزء}}{\text{كل}} = \frac{250}{1000} = 0.250$$

$$\hat{q}_1 = 1 - \hat{p}_1 = 1 - 0.275 = 0.725$$

and

$$\hat{q}_2 = 1 - \hat{p}_2 = 1 - 0.250 = 0.75$$

95% confidence interval

$$\alpha = 1 - 0.95 = 0.05$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

From Z table

$$(\hat{p}_1 - \hat{p}_2) \mp z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$\Rightarrow (0.275 - 0.250) \pm (1.96)\sqrt{\frac{(0.275)(0.725)}{1000} + \frac{(0.250)(0.750)}{1000}}$$

$$= 0.025 \pm 0.039$$

$$\Rightarrow -0.0136 < p_1 - p_2 < 0.0636$$

9.72 A random sample of 20 students yielded a mean of $\bar{X}$= 72 and a variance of S^2= 16 for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2.

α=1-0.98 =0.02

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$$\frac{(20-1)16}{36.191} < \sigma^2 < \frac{(20-1)16}{6.844}$$

$$8.4 < \sigma^2 < 39.8$$

df=n-1 =19

$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{0.995} = 6.844$

From Chi -Squared Table

Table A.5 Critical Valu

v	0.995	0.99
1	0.0[illegible]393	0.0^3157
2	0.0[illegible]00	0.0201
3	0.0[illegible]17	0.115
4	0.2[illegible]7	0.297
5	0.4[illegible]2	0.554
16	5.1[illegible]2	0.872
17	5.[illegible]97	1.239
18	6.2[illegible]5	1.647
19	6.844	2.088
20	7.434	
10	2.1[illegible]6	2.558

$\chi^2_{\frac{\alpha}{2}} = \chi^2_{0.01} = 36.191$

From Chi -Squared Table

v	0.02	0.01	0.005
1	5.412	6.6[illegible]5	7.879
2	7.824	9.2[illegible]0	10.597
3	9.837	11.3[illegible]5	12.838
4	11.668	13.2[illegible]7	14.860
5	13.388	15.0[illegible]6	16.750
14	26.873	29.1[illegible]1	31.319
15	28.259	30.5[illegible]8	32.801
16	29.633	32.0[illegible]0	34.267
17	30.995	33.4[illegible]9	35.718
18	32.346	34.8[illegible]5	37.156
19	[illegible]	36.191	38.582
20	35.020	37.566	39.997

Values of Z	
$Z_{0.90}$	**1.285**
$Z_{0.95}$	**1.645**
$Z_{0.97}$	**1.885**
$Z_{0.975}$	**1.96**
$Z_{0.98}$	**2.055**
$Z_{0.99}$	**2.325**
$Z_{0.995}$	**2.575**

H.W

9.52 Compute 95% confidence intervals for the proportion of defective items in a process when it is found that a sample of size 100 yields 8 defectives. $\mathbf{p} \in (\mathbf{0.0268}\,,\mathbf{0.133}\,)$

9.66 Ten engineering schools in the United States were surveyed. The sample contained 250 electrical engineers, 80 being women; 175 chemical engineers, 40 being women.
Compute a 90% confidence interval for the difference between the proportions of women in these two fields of engineering. $\mathbf{P_1} - \mathbf{P_2} \in (\mathbf{0.0201}\,,\mathbf{0.1627})$

H.W

Q. A survey of 500 students from a college of science shows that 275 students own computers. In another independent survey of 400 students from a college of engineering shows that 240 students own computers.

(a) a 99% confidence interval for the true proportion of college of science's student who own computers is $(\,\mathbf{0.4927}, \mathbf{0.6073})$

(b) a 95% confidence interval for the difference between the proportions of students owning computers in the two colleges is $(\,-\mathbf{0.1148}, \mathbf{0.0148})$

H.W

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation σ=2.0. Suppose that a random sample of size *n*=49 from this population gave a sample mean $\overline{X}$ =4.5.

(1) The distribution of $\overline{X}$ is .$\bar{x} \sim N(4.5\,, \frac{\sigma^2}{n} = 0.08\,)$

(2) A good point estimate of μ is = $\overline{\boldsymbol{X}} = \mathbf{4.5}$

(3) The standard error of $\overline{X}$ is = $\mathbf{S.E(\overline{X})} = \frac{\alpha}{\sqrt{n}}$= **0.2875**

(4) A 95% confidence interval for μ is $(\mathbf{3.94}\ \ ,\mathbf{5.06})$

(5) If we use $\overline{X}$ to estimate μ, then we are 95% confident that our estimation error will not exceed. $\boldsymbol{e} = \mathbf{0.56}$

(6) If we want to be 95% confident that the estimation error will not exceed e=0.1 when we use $\overline{X}$ to estimate μ, then the sample size *n* must be equal to
$\boldsymbol{n} = \mathbf{1537}$