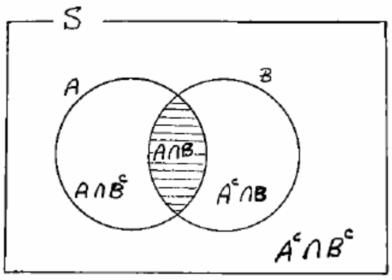
Review of Calculus and Probability

بعض المصطلحات الهامة

- $\text{not } (A) \xrightarrow{\text{النفي بكلمة}} A^c$
- at least A, B / A or B $\Big\} \longrightarrow A \cup B$

بعض القوانين تفهم من خلال الرسم



- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(A^c \cap B)$
- $P(A \cup B) = P(B) + P(A \cap B^c)$
- $P(A \cap B^c) = P(A) - P(A \cap B)$
- $P(A^c \cap B) = P(B) - P(A \cap B)$
- $P(A^c \cap B^c) = 1 - P(A \cup B)$ $\quad P(A \cup B)^c$
- $P(A - B) = P(A \cap B^c) = P(A) - P(A \cap B)$
- $P(A^c) = 1 - P(A)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

↳ <u>for disjoint (or mutually exclusive) events</u>

- $A \cap B = \Phi \quad \therefore P(\Phi) = 0$
- $P(A \cup B) = P(A) + P(B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Conditional probability

الإحتمال الشرطي : هو إحتمال حدوث حدث ما ، إذا علم حدوث حدث آخر .

$p(A/B) \rightarrow$ probability of (A) given (B)

يعني أوجد إحتمال (A) إذا كانت (B) قد حدثت

الأولى   الثاني

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}} = \frac{n(A \cap B)/n(S)}{n(B)/n(S)} = \frac{n(A \cap B)}{n(B)}$$

for equally likely outcomes

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

Suppose we draw a single card from a deck of 52 cards.

1-What is the probability that a heart or spade is drawn?
2-What is the probability that the drawn card is not a 2?
3-Given that a red card has been drawn, what is the probability that it is a diamond? Are the eventsindependent events?
*E*1 = red card is drawn
*E*2 = diamond is drawn
4-Show that the events are independent events?
E1 = spade is drawn
E2 =2 is drawn

Solution:

1-What is the probability that a heart or spade is drawn?

1 Define the events

$$E_1 = \text{heart is drawn}$$
$$E_2 = \text{spade is drawn}$$

E_1 and E_2 are mutually exclusive events with $P(E_1) = P(E_2) = \frac{1}{4}$. We seek $P(E_1 \cup E_2)$. From probability rule 3,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) = (\tfrac{1}{4}) + (\tfrac{1}{4}) = \tfrac{1}{2}$$

2-What is the probability that the drawn card is not a 2?

2 Define event E = a 2 is drawn. Then $P(E) = \frac{4}{52} = \frac{1}{13}$. We seek $P(\bar{E})$. From probability rule 4, $P(\bar{E}) = 1 - \frac{1}{13} = \frac{12}{13}$.

3- Given that a red card has been drawn, what is the probability that it is a diamond? Are the events independent events?
E1 = red card is drawn
E2 = diamond is drawn

3 From (1),

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$
$$P(E_1 \cap E_2) = P(E_2) = \frac{13}{52} = \frac{1}{4}$$
$$P(E_1) = \frac{26}{52} = \frac{1}{2}$$

Thus,

$$P(E_2|E_1) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Since $P(E_2) = \frac{1}{4}$, we see that $P(E_2|E_1) \neq P(E_2)$. Thus, E_1 and E_2 are not independent events. (This is because knowing that a red card was drawn increases the probability that a diamond was drawn.)

4-Show that the events are independent events?

E1 = spade is drawn

E2 =2 is drawn

4 $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{4}{52} = \frac{1}{13}$, and $P(E_1 \cap E_2) = \frac{1}{52}$. Since $P(E_1)\,P(E_2) = P(E_1 \cap E_2)$, E_1 and E_2 are independent events. Intuitively, since $\frac{1}{4}$ of all cards in the deck are spades and $\frac{1}{4}$ of all 2's in the deck are spades, knowing that a 2 has been drawn does not change the probability that the card drawn was a spade.

Baye's Rule

Baye's Rule:.

نظرية بايز

نفرض أن هناك عدد من الأسباب المعينة والتي يؤدي وقوع أحدها لحدوث حادثة ما . وهذه الحادثة تقع إذا وقع أحد أسبابها .
ولنفرض أننا نعلم مسبقاً احتمال تحقق كل سبب من هذه الأسباب وكذلك نعلم الاحتمال المشروط لهذه الحادثة عند تحقق كل سبب من أسبابها
وهذه النظرية تعنى بحساب احتمال أن يكون سبباً محدداً من هذه الأسباب هو مصدر حدوث الحادثة .. والتي نعلم مسبقاً بحدوثها

Three machines A_1, A_2, and A_3 make 20%, 30%, and 50%, respectively, of the products. It is known that 1%, 4%, and 7% of the products made by each machine, respectively, are defective. If a finished product is randomly selected, what is the probability that it is defective?

B : The selected product is defective

A_1 : 〃 〃 is made by machine (A_1)

A_2 : 〃 〃 product is made by machine (A_2)

A_3 : the selected product is made by (A_3) .

احتمال إنتاج A_1 : $P(A_1) = \frac{20}{100} = 0.2$, $P(B/A_1) = \frac{1}{100} = 0.01$ احتمال تعيب A_1

A_2 〃 〃 $P(A_2) = \frac{30}{100} = 0.3$, $P(B/A_2) = \frac{4}{100} = 0.04$ A_2 〃 〃

A_3 〃 〃 $P(A_3) = \frac{50}{100} = 0.5$, $P(B/A_3) = \frac{7}{100} = 0.07$ A_3 〃 〃

إنتاج الآلة

A_1 0.2 $B/A_1 \xrightarrow{0.01}$ 0.002

A_2 0.3 $B/A_2 \xrightarrow{0.04}$ 0.012

A_3 0.5 $B/A_3 \xrightarrow{0.07}$ 0.035

المجموع $P(B) = 0.049$

$P(B) = P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)$

$= 0.2 * 0.01 + 0.3 * 0.012 + 0.5 * 0.035 = 0.049$

- If it is Known that the selected product is defective, what is the probability that is made by A_2, A_3 machines?

$$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(B)} = \frac{0.012}{0.049} = 0.2449$$

$$P(A_3/B) = \frac{P(A_3) \cdot P(B/A_3)}{P(B)} = \frac{0.035}{0.049} = 0.2449$$

Suppose that 1% of all children have tuberculosis (TB). When a child who has TB is given the Mantoux test, a positive test result occurs 95% of the time. When a child who does not have TB is given the Mantoux test, a positive test result occurs 1% of the time. Given that a child is tested and a positive test result occurs, what is the probability that the child has TB?

The states of the world are

$$S_1 = \text{child has TB}$$
$$S_2 = \text{child does not have TB}$$

The possible experimental outcomes are

$$O_1 = \text{positive test result}$$
$$O_2 = \text{nonpositive test result}$$

We are given the prior probabilities $P(S_1) = .01$ and $P(S_2) = .99$ and the likelihoods $P(O_1|S_1) = .95$, $P(O_1|S_2) = .01$, $P(O_2|S_1) = .05$, and $P(O_2|S_2) = .99$. We seek $P(S_1|O_1)$. From (7),

$$P(S_1|O_1) = \frac{P(O_1|S_1)P(S_1)}{P(O_1|S_1)P(S_1) + P(O_1|S_2)P(S_2)}$$

$$= \frac{.95(.01)}{.95(.01) + .01(.99)} = \frac{95}{194} = .49$$

Probability Distributions

Binomial Distribution:

$$P\{x = k\} = C_k^n p^k (1 - p)^{n-k}, k = 0, 1, 2, \ldots, n$$

ial distribution with parameters n and p. Its mean

$$E\{x\} = np$$
$$\mathrm{var}\{x\} = np(1 - p)$$

Poisson Distribution:

$$P\{x = k\} = \frac{\lambda^k e^{-\lambda}}{k!}, k = 0, 1, 2, \ldots$$

The mean and variance of the Poisson are

$$E\{x\} = \lambda$$
$$\mathrm{var}\{x\} = \lambda$$

Exponential Distribution:

$$f(x) = \lambda e^{-\lambda x} \quad , \quad x > 0$$

$$E[X] = \frac{1}{\lambda} \quad and \quad Var[X] = \frac{1}{\lambda}$$

Normal Distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)} \quad , \quad -\infty < x < +\infty$$

$$E[X] = \mu \;\; and \quad Var[X] = \sigma^2$$