**4.Random Variables ( Pages 8-16)**

Q1: (a) S ={HHH ,HHT,HTH,THH,TTH,HTT,THT,TTT}

 (b)

|  |  |
| --- | --- |
|  S | X=number of heads- number of tails |
| HHH |  X =3 - 0 = 3 |
| HHT |  X = 2 - 1 = 1 |
| HTH |  X = 2 - 1 = 1 |
| THH |  X = 2 - 1 = 1 |
| TTH |  X = 1 - 2 = -1 |
| HTT |  X = 1 - 2 = -1 |
| THT |  X = 1 - 2 = -1 |
| TTT |  X = 0 - 3 = -3 |

Values of x = -3,-1,1,3

(c)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | -3 | -1 | 1 | 3 | total |
| .f(x) | 1/8 | 3/8 | 3/8 | 1/8 | 1 |

(d)P(X≤1) = 1/8+3/8+3/8 =7/8 (e)P(X<1) = 1/8+3/8 =4/8=1/2

(f)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | -3 | -1 | 1 | 3 | total |
| .f(x) | 1/8 | 3/8 | 3/8 | 1/8 | 1 |
| Xf(x) | -3/8 | -3/8 | 3/8 | 3/8 | 0 =E(x) |
| X2 f(x) | 9/8 | 3/8 | 3/8 | 6/8 | 21/8=E(X2) |

(F) mean =E(x) = 0

(g) V(x) = E(X2) – [E(X)]2 = 21/8 -02 = 21/8

Q2:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | total |
| .f(x) | 0.64 | 0.32 | 0.04 | 1 |

4.P( at least one ) = P( X ≥ 1) = 0.32+0.04 =0.36

5. P( at most one ) = P( X ≤ 1) = 0.64+0.32 =0.96

6.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | total |
| .f(x) | 0.64 | 0.32 | 0.04 | 1 |
| Xf(x) | 0 | 0.32 | 0.08 | 0.40=E(x) |
| X2 f(x) | 0 | 0.32 | 0.16 | 0.48=E(X2) |

6. Mean = E(X) = 0.40

7. V(x) = E(X2) – [E(X)]2 = 0.48- 0.402 = 0.32

8.Standard deviation =

Q4.

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| --- | --- | --- | --- | --- |
| X | -3 | 6 | 9 | total |
| .f(x) | 0.1 | 0.5 | 0.4 | 1 |
| Xf(x) | -0.3 | 3 | 3.6 | 6.3=E(x) |
| X2 f(x) | 0.9 | 18 | 32.4 | 51.3=E(X2) |

1. Mean = E(X) = 6.3
2. E(X2) = 51.3
3. V(x) = E(X2) – [E(X)]2 = 51.3- 6.32 = 11.61

Standard deviation =

1. E(2X +1) = 2E(X) + 1 = 2x(6.3) + 1 =13.6
2. V(2X +1) = 2 2V(X) + V(1) = 4x 11.61 +0 = 46.44

 Q5 : (A)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | total |
| .f(x) | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 1 |

(B)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | total |
| .f(x) | -1/5 | 0 | 1/5 | 2/5 | 3/5 | 1 |

(C)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | total |
| .f(x) | 1/5 | 1/5 | 1/5 | 1/5 | 1/5 | 1 |

(D)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | total |
| .f(x) | 5/6 | 4/6 | 1/6 | -1/6 | 1 |

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**Q7: (1)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | total |
| .f(x) | k | 2k | 3k | 1 |

(2) F(x) =P(X ≤ x) =

 (3) P(0.5 < X ≤ 2.5 ) =1/6 + 2/6 = 3/6

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Q9:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | total |
| .f(x) | 0.4 | C | 0.3 | 0.1 | 1 |

 0.4+ c+ 0.3 +0.1 =1

 0.8+ c = 1 Then , c = 0.2

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**Q12:** F(x) =P(X ≤ x) =

1. Find f(x) :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X |  |  |  |  |  | total |
| .f(x) |  |  |  |  |  | 1 |

1. P(X=2) = 6/16 =3/8
2. P(2 ≤ X ≤ 4 ) = F(4) – F(1) = 1 – 5/16 = 11/16

Q13:

 Mean of X = E(x) = 10

 V(x) =4

1. Mean of Y : E( Y ) = E( 2X -2 ) = E(2X) – 2 = 2x10 -2 = 18
2. Variance of Y : V( Y ) = V( 2X -2 ) = V(2X) – V(2) = 2 2x4 -0 = 4x4 = 16

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**Q15:**  f(-1) =0.05 , f(0) =0.25 , f(1) = 0.25 , f(2) = 0.45

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | -1 | 0 | 1 | 2 | total |
| .f(x) | 0.05 | 0.25 | 0.25 | 0.45 | 1 |

1. P(X <1) = 0.05+ 0.25 = 0.30 (A)
2. P(X ≤1) = 0.05+ 0.25 + 0.25 = 0.55 (B)
3. E(X) = 1.1 (A)
4. E(X 2) = 2.10 (B)
5. Var(X) = 0.89
6. F(1) = P(X ≤1) = 0.05+ 0.25 + 0.25 = 0.55 (D)

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**4.2 Continouse Distribution**

**Q1:** µ = 16 , σ2 = 5 , then P(X =16 ) = 0 (C)

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**Q2:** f(x)

1.

 k = 3/2 =1.5

1. P(0.3 < x <0.6) =

 Q3:

 f(x)

1.

 k(4) = 1 , Then k = 1/4

1. P( 0<X ≤ 1) =
2. F(x) = P(X ≤ x) =
3. P( 0 < X ≤ 1) = F(1) – F(0) =

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**Q9 :**

1. P( 0 < X < 2) = F(2) – F(0) = =
2. P( X ≤ k) = F(k) = 0.5

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**4.3 CHYBYSHEV’S THEORM**

**Q2:** µ =12 , σ2 =9 , σ =3

 P( 3 < X < 21 )

 P( µ - kσ <X < µ + kσ ) ≥ 1 -

 To find value of k : µ + kσ = 21

 12+ 3k =21 Then 3k = 21-12 = 9 , So k = 9/3=3

Then lower bound is

 1 - = 1 -

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**Q4:** µ = 5 , σ = 2.89 , f(x) = 1/10 0<x<10

1. P( µ - 1.5σ <X < µ + 1.5σ )= P( 5 - 1.5(2.89) <X < 5 + 1.5(2.89) )=

P( 5 – 4.335 <X < 5 + 4.335)=

1. P( µ - 1.5σ <X < µ + 1.5σ )=

P( µ - kσ <X < µ + kσ ) ≥ 1 -

 To find value of k : µ + kσ = 9.335

 5+ 2.8k = 9.335 Then 2.89k = 9.335-5 , So k = 4.335/3 = 1.45

Then lower bound is

 1 - = 1 -

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Q1:

**3.2: From book :**

B:Blemished , N Non-Blemished

 (a) S ={BBB ,BBN,BNB,NBB,NNB,NBN,BNN,NNN}

|  |  |
| --- | --- |
|  S | X=number of Blemished |
| BBB |  X = 3 |
| BBN | X =2 |
| BNB | X =2 |
| NBB | X =2 |
| NNB | X =1 |
| NBN | X =1 |
| BNN | X =1 |
| NNN | X =0 |

Values of x = 0 ,1 ,2 ,3

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 F(x)=

P( less than 12) = P(X ≤ 12 ) = F(12) =

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**3.12:**

Average = mean =

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