

$$\begin{aligned}
& \langle U + V, W \rangle \\
&= \langle (u_1, u_2) + (v_1, v_2), (w_1, w_2) \rangle \\
&= \langle (u_1 + v_1, u_2 + v_2), (w_1, w_2) \rangle \\
&= 3(u_1 + v_1)w_1 + 2(u_2 + v_2)w_2 \\
&= 3u_1w_1 + 3v_1w_1 + 2u_2w_2 + 2v_2w_2 \\
&= 3u_1w_1 + 2u_2w_2 + 3v_1w_1 + 2v_2w_2 \\
&= (3u_1w_1 + 2u_2w_2) + (3v_1w_1 + 2v_2w_2) \\
&= \langle (u_1, u_2), (w_1, w_2) \rangle + \langle (v_1, v_2), (w_1, w_2) \rangle \\
&= \langle U, W \rangle + \langle V, W \rangle
\end{aligned}$$