Sampling Distribution for Sample Proportion:

Example:

Suppose that the proportion of all college students who have used sport facilities in the past 6 months is 0.40. For a random sample of 200 students, what is the probability that the proportion of students who have used sport facilities in the past 6 months is less than 0.32?

Solution:

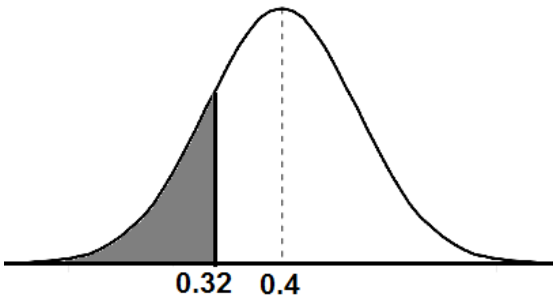
Let $\hat{p}$ be the sample proportion of students who have used sportfacilities in the past 6 months.

We are given that:

p=0.40

n=200 (large)

We want to find $P(\hat{p}<0.32)$.



Note that:

$$\hat{p} \sim N\left(p\,,\sqrt{\frac{p(1-p)}{n}}\right)$$

$$\hat{p} \sim \mathrm{N}\left(0.4\,,\sqrt{\frac{0.4 \times 0.6}{200}}\right)$$

Now,

$$\mathrm{P}(\hat{p} < 0.32) = \mathrm{P}\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} < \frac{0.32-p}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$$= \mathrm{P}\left(Z < \frac{0.32-0.4}{\sqrt{\frac{0.4 \times 0.6}{200}}}\right)$$

$$= \mathrm{P}(\mathrm{Z} < -2.31) = 0.0104$$

Sampling Distribution ofthe Difference between Proportions of Two Samples:

Example:

Suppose that 25% of the male students and 20% of the female students in a certain university are left-handed. A random sample of 35 male students is taken from this university. Another random sample of 30 female students is independently taken from this university. Let $\hat{p}_1$ and $\hat{p}_2$ be the proportions of left-handed students in the two samples, respectively.

(1) Find $E(\hat{p}_1 - \hat{p}_2) = \mu_{\hat{p}_1 - \hat{p}_2}$, the mean of $\hat{p}_1 - \hat{p}_2$.

(2) Find $Var(\hat{p}_1 - \hat{p}_2) = \sigma^2_{\hat{p}_1 - \hat{p}_2}$, the variance of $\hat{p}_1 - \hat{p}_2$.

(3) Find $\sigma_{\hat{p}_1 - \hat{p}_2}$, the standard deviation of $\hat{p}_1 - \hat{p}_2$.

(4) Find an approximate distribution of $\hat{p}_1 - \hat{p}_2$.

(5) Find P(0.10< $\hat{p}_1 - \hat{p}_2$ <0.2).

Solution:

We are given that:

$$p_1 = 0.25 \qquad p_2 = 0.20$$

$$n_1 = 35 \text{ (large)} n_2 = 30 \text{(large)}$$

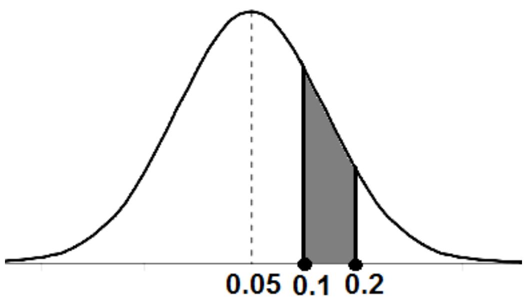
(1) $E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 = 0.25 - 0.20 = 0.05$

(2) $Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_1} = \frac{0.25 \times 0.75}{35} + \frac{0.2 \times 0.8}{30} = 0.01069$

(3) $\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{Var(\hat{p}_1 - \hat{p}_2)} = \sqrt{0.01069} = 0.1034$

(4) $\hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_1}}\right) = N(0.05, 0.1034)$

(5) P$(0.1 < \hat{p}_1 - \hat{p}_2 < 0.2)$



$\mathrm{P}(0.1 < \hat{p}_1 - \hat{p}_2 < 0.2) =$

$$= P\left(\frac{0.1 - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_1}}} < \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_1}}} < \frac{0.2 - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_1}}}\right)$$

$$= P\left(\frac{0.1 - 0.05}{\sqrt{0.01069}} < \quad Z \quad < \frac{0.2 - 0.05}{\sqrt{0.01069}}\right)$$

$$= P\left(\frac{0.1 - 0.05}{0.1034} < \quad Z \quad < \frac{0.2 - 0.05}{0.1034}\right)$$

$$= P(0.48 < \quad Z \quad < 1.45)$$

$$= P(Z < 1.45) - \ P(\ Z \ < 0.48)$$

$$= 0.9265 - \ 0.6844$$

$$= 0.2421$$