OPER 441: Modeling and Simulation
Exercises Sheet #6

Question 1:

Consider the triangular distribution:

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{(x-a)^2}{(b-a)(c-a)} & a \le x \le c \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)} & c < x \le b \\ 1 & b < x \end{cases}$$

a) Use a spreadsheet to generate 1000 observations of the triangular distribution with $a = 2$, $c = 5$, $b = 10$.

b) Use your favorite statistical software to make a histogram of 1000 observations from your implementation of the triangular distribution with
$a = 2$, $c = 5$, $b = 10$.

Question 2:

The times to failure for an automated production process have been found to be randomly distributed according to a Rayleigh distribution:

$$f(x) = \begin{cases} 2\beta^{-2} x e^{(-(x/\beta)^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Setup a spreadsheet to generate 5 random numbers from your algorithm with = 2.

Question 3:

The times to failure for an automated production process have been found to be randomly distributed according to a Rayleigh distribution:

$$f(x) = \begin{cases} 2\beta^{-2} x e^{(-(x/\beta)^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Setup a spreadsheet to generate 5 random numbers from your algorithm with = 2.

Question 4:

The times to failure for an automated production process have been found to be randomly distributed according to a Rayleigh distribution:

$$f(x) = \begin{cases} 2\beta^{-2} x e^{(-(x/\beta)^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Setup a spreadsheet to generate 5 random numbers from your algorithm with = 2.

Question 5:

Setup a spreadsheet to generate 5 random numbers from the negative binomial distribution with parameters ($r = 4$, $p = 0.4$) using:

a) The convolution method
b) The number of Bernoulli trials to get 4 successes.

Question 6:

Setup a spreadsheet that will generate 30 observations from the following probability density function using the Acceptance-Rejection algorithm for generating random variates.

$$f(x) = \begin{cases} \frac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Question 7:

Parts arrive to a machine center with three drill presses according to a Poisson distribution with mean λ. The arriving customers are assigned to one of the three drill presses randomly according to the respective probabilities p_1, p_2, and p_3 where $p_1 + p_2 + p_3 = 1$ and $p_i > 0$ for i = 1, 2, 3. What is the distribution of the inter-arrival times to each drill press? Specify the parameters of the distribution.

a) Suppose that p_1, p_2, and p_3 equal to 0.25, 0.45, and 0.3 respectively and that λ is equal to 12 per minute. Setup a spreadsheet to generate the first 3 arrival times.

This question demonstrates the splitting property of a Poisson distribution. Each machine experience a Poisson process with mean $\lambda \times p_i$. Thus, the distribution of the inter-arrival times to each drill press will be exponential with mean $1/(\lambda \times p_i)$

Because of the splitting rule for Poisson processes, the drill presses each see arrivals according to the following three Poisson processes:

$$\lambda_1 = \lambda p_1 = 12 * 0.25 = 3$$

$$\lambda_2 = \lambda p_2 = 12 * 0.45 = 5.4$$

$$\lambda_3 = \lambda p_3 = 12 * 0.3 = 3.6$$