Use of Mathematica in Complex Analysis

Example 2: Steiner Circles

Circle of Apollonius is defined by the equation

$$\frac{|z - z_1|}{|z - z_2|} = \rho$$

where $\rho > 0$ and z_1 and z_2 are given fixed points. (Called the limiting points.) As ρ varies one gets a family of circles. The example below gives plots the family of circles for $z_1 = -1$ and $z_2 = 1$. The second example is for $z_1 = i$ and $z_2 = 1$.
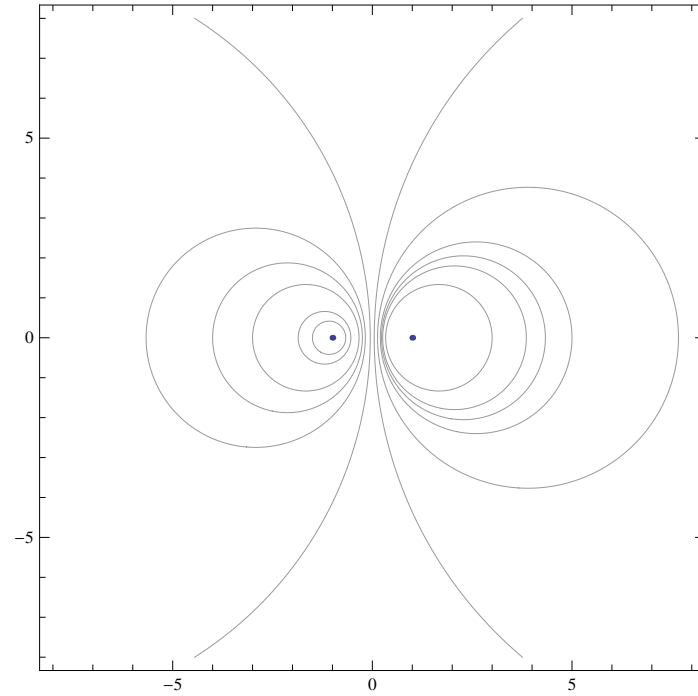
The MATHEMATICA program uses the command `ContourPlot`.

```
pe[z1_, z2_] := ContourPlot[Abs[(x + I * y - z1) / (x + I * y - z2)],
    {x, -8, 8}, {y, -8, 8}, AxesOrigin → {0, 0}, ContourShading → None,
    Contours → {.2, .3, .5, .6, .7, .9, 1.7, 1.1, 1.6, 1.5, 1.3, 2}, PlotPoints → 250];


pe2[z1_, z2_] := ListPlot[{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}}];

p1 = pe[-1, 1]; p2 = pe2[-1, 1];

Show[{p1, p2}]
```

```
In[6]:= pe[z1_, z2_] := ContourPlot[Abs[(x + I * y - z1) / (x + I * y - z2)],
            {x, -8, 8}, {y, -8, 8}, AxesOrigin → {0, 0}, ContourShading → None,
            Contours → {.2, .3, .5, .6, .7, .9, 1.7, 1.1, 1.6, 1.5, 1.3, 2}, PlotPoints → 250];

In[7]:=

In[8]:= pe2[z1_, z2_] := ListPlot[{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}}];

In[9]:= p1 = pe[I, 1]; p2 = pe2[I, 1];

In[10]:= Show[{p1, p2}]
```

Out[10]=